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THESIS

A SIMULATED SINGLE-ITEM AGGREGATE
INVENTORY MODEL FOR U.S. NAVY REPAIRABLE ITEMS

by

Kevin J. Maher

September, 1993

Thesis Advisor:

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**A Simulated Single-Item Aggregate Inventory Model
for U.S. Navy Repairable Items**

by

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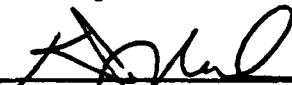
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ABSTRACT

A readiness-based sparing (RBS) model for the repair and replenishment of repairable items is needed by the Navy which considers the aggregate inventory of repaired units and new ones. This thesis presents progress in the development of such a model. In contrast to other such current repairables models in the literature, it also allows for both batch repair and procurement. A theoretical model had been developed earlier at the Naval Postgraduate School for the probability distribution of inventory position for such a model. However, no theoretical model has yet been developed for the probability distribution of net inventory because the real-world inventory management of repairables is quite complex. Therefore, a simulation model was developed of the Navy's repairables management process to explore the nature of that distribution as a function of relevant system parameters. It was then run for a range of values of a subset of those parameters. The net inventory distribution appears to be normally distributed with its mean and variance being a linear function of the product of carcass return rate and repair survival rate. The theoretical distribution for inventory position was not only validated, it was found to be quite robust. Further analyses, however, are required before the effects of all relevant parameters are well understood.

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EXECUTIVE SUMMARY

A. PURPOSE

The purpose of this thesis is to develop a simulation model which accurately describes the Navy's secondary item repairables process, and then use the simulated data to study the aggregate net inventory and inventory position distributions, compare the resulting inventory position distribution to a proposed theoretical one, analyze the effect of three parameters on the mean and variance of the net inventory, and define safety level.

The main reason for developing a mathematical model of an inventory system is to use it as an aid in developing an operating doctrine. Because the mathematics in the repairables model are so complicated, to accurately model it analytically is virtually impossible except in the limiting case where the repair induction quantity is one. However, simulation can be used to develop an approximate analytical model.

B. BACKGROUND

Both Navy ICP's use the Uniform Inventory Control Program (UICP) system for determining the replenishment of consumable and repairable secondary items. The formulas used in the UICP for determining the order quantity and reorder point for the repairables replenishment system were developed

in the late 1960's. However, no rigorous mathematical development supported them. Rather, they were direct extrapolations of the consumable model formulas. The result was two sets of formulas, one for the procurement process and one for the repair process. In particular, there were two separate sets of risk equations, safety levels, order quantities, and reorder levels. The only "improvement" to the repair model occurred in 1984 when the Integrated Repairables Model was implemented. This model contained only one risk formula and applied the same safety level computation to both processes, but continued to model the repairable inventory management process as a two-part process.

C. RESULTS

Two major results of this study have emerged. The first is that the theoretical distribution for inventory position was not only validated, but found to be quite robust. The second is that the net inventory distribution appears to be normally distributed with its mean and variance being a linear function of the product of the carcass return rate and the repair survival rate.

D. RECOMMENDATIONS.

More sensitivity analyses of the inventory position and net inventory probability distributions must be accomplished. Clearly, the effect of the low values of the carcass return rate on the net inventory distribution parameters needs to be better understood. In addition, major sensitivity analyses of

these distributions must include the effects caused by varying demand rates and distributions, and by varying turn-around time rates and distributions. It is also recommended that the Naval Postgraduate School and the Naval Supply Systems Command share in the research effort so that it is accomplished in an expeditious manner.

I. INTRODUCTION

A. BACKGROUND

As of 30 September 1992 the Navy's two Inventory Control Points (ICP's), the Aviation Supply Office in Philadelphia, PA and the Ships Parts Control Center in Mechanicsburg, PA, managed over 580,000 secondary support items which were broken down into two major classifications, consumables and repairables [Ref. 1, 2]. Consumable items are ones which are immediately disposed of at the time of failure. Repairables are items which are sent to a repair activity upon failure and are typically returned to "new" condition at a cost which is no more than that of a new item and in a time interval substantially shorter than a procurement lead time.

As a result of the Defense Management Review Decision 901, "Reducing Supply System Cost/Navy Inventory Reduction Plan", the Navy is in the process of transferring almost all of its consumable items to the Defense Logistics Agency leaving the ICP's with some 207,000 repairables to manage. To maintain its existence, the ICP's need to better manage this material, i.e, each must become a repairables "Center of Excellence".

Both Navy ICP's use the Uniform Inventory Control Program (UICP) system for determining the replenishment of consumable

and repairable secondary items. The formulas used in the UICP for determining the order quantity and reorder point for consumables were developed in the mid-1960's based on the research of Professors G. Hadley and T. M. Whitin. [Ref 3] The formulas for the repairables replenishment system were developed in the late 1960's. However, no rigorous mathematical development supported them. Rather, they were direct extrapolations of the consumable model formulas. The formulas for procurement and repair order quantities and the associated reorder points were based on the modelers' intuition from modeling consumables. The result was two sets of formulas, one for the procurement process and one for the repair process. In particular, there were two separate sets of risk equations, safety levels, order quantities, and reorder levels. The only "improvement" to the repair model occurred in 1984 when the Integrated Repairables Model was implemented. This model contained only one risk formula and applied the same safety level computation to both processes, but continued to model the repairable inventory management process as a two-part process.

The development of an analytically-based repairable item inventory system is an extremely complex process. As will be seen in the discussion of the development of the Navy's current model, when the formulas did not provide values which the inventory managers could believe in, more assumptions were made, and more approximations were developed. However, the

problems were really rooted in the basis of the model being incorrect, and subsequent "patches" did not help much.

B. OBJECTIVE

The main reason for developing a mathematical model of an inventory system is to use it as an aid in developing an operating doctrine. Because the mathematics in the repairables model are so complicated, to accurately model it analytically is virtually impossible except in the limiting case where the repair induction quantity is one. However, simulation can be used to develop an approximate analytical model. Indeed, Hadley and Whitin suggested that in very complex cases one may have to use a computer and simulation to study a small number of different operating doctrines and choose one which best suits the objective. [Ref. 3] Therefore, the objectives of this thesis are to:

1. Develop a simulation model which accurately describes the Navy's secondary item repairables process.
2. Use the simulated data to study the aggregate net inventory and inventory position distributions.
3. Compare the resulting inventory position distribution to a proposed theoretical one.
4. Analyze the effect of the Carcass Return Rate (CRR), the Repair Survival Rate (RSR), and the Repair Processing time (REP) on the mean and variance of the net inventory.
5. Define safety level.

C. SCOPE

The intent of this thesis is to take the first step in the analysis of the Navy's aggregate repairables inventory system. The simulation analysis focusses on the probability distributions for inventory position and net inventory. However, the scope of the simulation studies includes only three stochastic parameters. One stochastic parameter is the quarterly demand, and it is assumed to be generated by a Poisson process. The two other stochastic parameters are the Carcass Return Rate (CRR) and the Repair Survival Rate (RSR) which are probabilities and are generated by a Bernoulli process. All other parameters are assumed to be deterministic because the models described in Reference (1) are founded on the same assumptions and are serving as a basis for comparison.

D. PREVIEW

Chapter II is an indepth discussion of the development of the inventory system formulas for the Navy's Consumable Model and the Navy's Integrated Repairables Model. Chapter III introduces the inventory position probability mass function formulas for the proposed theoretical model, and then describes in detail the simulation model, the way data was generated, and how the data was manipulated. Chapter IV contains the analysis of the results of the simulations.

Chapter V summarizes the thesis research and findings, offers conclusions, and makes recommendations for further analyses.

II. DEVELOPMENT OF UICP INVENTORY MODELS

Inventory problems are as old as history itself, but it has only been since the beginning of this century that mathematics have been used to manage inventory systems. These attempts focused on what will be called "the operating doctrine"; namely, how much to buy (order quantity) and when.

The earliest of these mathematical "models" was developed by Ford Harris of Westinghouse Corporation and the order quantity he derived was called the "simple lot size formula" [Ref. 3]. This formula was subsequently used after World War I by R. H. Wilson and is often referred to as the "Wilson Formula" because he used it as an integral part of the inventory control scheme he sold to many organizations. The model assumed demand to be known and constant, and no stock-outs were allowed.

Following World War II, inventory models which considered demand to be a random variable were developed [Ref. 4]. Among those who developed these models were G. Hadley of the University of Chicago and T. M. Whitin of the University of California, Berkeley. They received support from the Navy's Bureau of Supplies and Accounting, the predecessor to the Navy Supply Systems Command (NAVSUP), for much of their work. In 1963 this research was consolidated in their definitive text, "Analysis of Inventory Systems", which was published by

Prentice Hall, Inc., Englewood Cliffs, New Jersey. Their research formed the cornerstone of the Navy's wholesale inventory models, [Ref. 5], and in particular, the Uniform Inventory Control Program (UICP) consumable item inventory model, [Ref. 6]. The next section presents the derivation of this model and, because it forms the basis of the Navy's current wholesale inventory system, it will serve as a reference for the models described later for managing repairable items.

A. THE NAVY'S UICP CONSUMABLE MODEL

Basic to the Navy's UICP consumable model are the definitions of Inventory Position (IP), Net Inventory (NI), and the average annual Total Variable Costs (TVC) where,

$$\begin{aligned} IP &= \text{On Hand} + \text{On Order} - \text{Backorders} \\ &= O/H + O/O - B/O; \end{aligned} \tag{1}$$

$$\begin{aligned} NI &= \text{On Hand} - \text{Backorders} \\ &= O/H - B/O; \end{aligned} \tag{2}$$

and

$$\begin{aligned} TVC &= \text{Average Annual Order Costs} + \\ &\quad \text{Average Annual Holding Costs} + \\ &\quad \text{Average Annual Backorder Costs.} \end{aligned} \tag{3}$$

The major assumptions of the Navy's consumable model are:

1. There exists a steady state environment. This means that demand, while being a random variable, has a fixed mean, variance, and probability distribution which do not change over time.
2. There is a transaction item reporting system; whenever a demand occurs, it is immediately recorded. The result is a continuous review of demand.
3. Units are demanded one at a time. This assumption is used to develop the order costs and holding costs terms of the model (they are taken directly from Reference 3). The UICP model violates this assumption in the backorder costs term and allows a requisition size of any number of units.
4. Ordering occurs when the inventory position reaches a value called the reorder level or reorder point. That value is non-negative.
5. The amount to be procured is called the order quantity and has a constant value. Procurement of the order quantity is not constrained by budget restrictions.
6. The cost of placing an order is constant and independent of the order quantity. Procurement lead time can be either deterministic or a random variable. In this section it will be assumed to be known and constant.
7. Inventory holding costs are proportional to the unit cost.
8. Demands are either filled or backordered.
9. The cost of a backorder is quantifiable.
10. A factor, known as Military Essentiality, is quantifiable. This factor attempts to measure an item's worth in terms of operational readiness. An item such as an air/search radar would have a large factor because loss of one would severely cripple a ship's mission, whereas, night-vision goggles would have a smaller factor because loss of one would not cripple a ship's mission.
11. An entire order quantity is received at the same time; i.e., it is not split up into two or more subsets arriving at different times.

The objective of the model is to determine non-negative values of the order quantity, Q , and the reorder point, R , which minimize equation (3).

B. DERIVATION OF THE TVC FORMULA

1. Average Annual Order Costs

The average annual ordering costs are simply the product of the cost of placing an order and the expected number of orders placed per year. The cost of placing an order is denoted as A . The expected number of orders placed per year is the forecasted average annual demand, $4D$, (where D is the forecasted quarterly demand), divided by Q . Thus, the average annual ordering costs can be written as equation (4),

$$\text{Average Annual Order Costs} = \frac{4DA}{Q}. \quad (4)$$

2. Average Annual Holding Costs

The average annual holding costs are computed as the product of the average on-hand inventory and the annual holding cost rate per unit, IC , where C is the unit purchase cost and I is a percentage established by the Fleet Material Support Office and is composed of four estimated factors, a "value of money" rate of 10%, an obsolescence rate of 10%, a shrinkage/pilferage rate of 2% and a "handling" rate of 1%,

for a total of 23%. The average on-hand quantity, $E(O/H)$, is found by rewriting equation (1) as

$$E(O/H) = E(IP) - E(O/O) + E(B/O). \quad (5)$$

The model assumes that every time a demand occurs, the IP is reduced by one unit. At the instant when the inventory position reaches the reorder point, R , an order of size Q is generated and the inventory position, IP , increases to $R + Q$ because Q has been ordered. Hadley and Whiten [Ref. 1:p 183] have shown that, under the assumption of Poisson demand, the probability of IP being $R + x$ is

$$p(IP = R + x) = \frac{1}{Q} \quad \text{for } 1 \leq x \leq Q. \quad (6)$$

Thus, the expected IP at any point in time is

$$\begin{aligned} E(IP) &= \frac{Q + R + R + 1}{2} \\ &= \frac{Q + 1}{2} + R. \end{aligned} \quad (7)$$

The expected on-order quantity can be found in the following way. The time between generations of an order is known as the order cycle. Let M be the expected number of order cycles in a procurement lead time, L . Then,

$$L = (M) (\text{order cycle}). \quad (8)$$

Multiplying both sides by the forecasted quarterly demand, D , results in

$$(D)(L) = (M)(\text{order cycle})(D). \quad (9)$$

If the order cycle begins with the generation of an order, then another order will be generated when the demand reaches Q . Therefore,

$$DL = MQ, \quad (10)$$

where MQ is the expected on-order quantity. The result is that the expected on-order quantity is equivalent to the expected lead time demand.

Substituting equations (7) and (10) into (5) and denoting $E(B/O)$ as $B(Q,R)$, the resulting equation for $E(O/H)$ is

$$E(O/H) = \frac{Q+1}{2} + R - DL + B(Q,R). \quad (11)$$

The average annual holding costs are, therefore,

$$\text{Average Annual Holding Costs} = IC \left(\frac{Q+1}{2} + R - DL + B(Q,R) \right). \quad (12)$$

3. Average Annual Backorder Costs

The UICP determines the cost of backorders based on the expected time-weighted number of requisitions short per year. This is done by determining the expected number of backorders on the books at any time t , and then dividing that value by the average requisition size, S .

To find the expected number of backorders at any time t , let $R + x$ be the inventory position at the beginning of a procurement lead time preceding t (i.e., at time $t - L$) and let y equal the number of backorders occurring during that lead time. To incur y backorders, the demand over the lead time must be $R + x + y$ since any quantity on order prior to the beginning of a lead time will have been received by the end of the lead time, and any order placed after the beginning of an order cycle will be received after the end of the lead time (as noted above, lead time is assumed to be known and constant). Let the Poisson probability mass function of lead time demand u be denoted by $p(u;DL)$ with the corresponding cumulative distribution function, $P(u;DL)$. The combined probability that $R + x + y$ demands occur during a lead time and the inventory position was $R + x$ at the start of the lead time is the product

$$p(R + x + y ;DL) p(IP = R + x), \quad (13)$$

and, because of equation (6), (13) reduces to

$$\frac{1}{Q} p(R + x + y;DL). \quad (14)$$

Equation (14) is valid for all x such that $1 \leq x \leq Q$, and all y such that $0 \leq y \leq \infty$.

To find the probability $f(y)$ of y backorders for all possible IP values, equation (14) must be summed over all the possible values of x :

$$\begin{aligned}
f(y) &= \sum_{x=1}^Q \frac{1}{Q} P(R + x + y; DL) \\
&= \frac{1}{Q} [P(R + Q + y; DL) - P(R + y; DL)]; \quad (15)
\end{aligned}$$

and the expected number of backorders at any time t , $B(Q, R)$, is the expected value of y :

$$\begin{aligned}
B(Q, R) &= \sum_{y=0}^{\infty} y f(y) \\
&= \sum_{y=0}^{\infty} y \left(\frac{1}{Q} \right) [P(R + Q + y; DL) - P(R + y; DL)] \\
&= \frac{1}{Q} \sum_{y=0}^{\infty} y [P(R + Q + y; DL) - P(R + y; DL)] . \quad (16)
\end{aligned}$$

If we let $u = R + y$, then (16) can be rewritten as

$$B(Q, R) = \frac{1}{Q} \sum_{u=R}^{\infty} (u - R) [P(u + Q; DL) - P(u; DL)] . \quad (17)$$

As noted in Reference (3), $B(Q, R)$ is also the average unit-years of backordered units per year. To get the average time-weighted annual costs of requisitions short, the UICP model first divides equation (17) by the average requisition size, S , and then multiplies the result by the product of a shortage cost per requisition, λ , and the military essentiality factor, E . The result is

$$\text{Average Annual B/O Costs} = \left(\frac{\lambda E}{S} \right) B(Q, R) . \quad (18)$$

4. Average Annual Total Variable Costs

Substituting equations (4), (12) and (18) into equation (3) yields the following formula for the average annual Total Variable Costs:

$$TVC = \frac{4DA}{Q} + IC\left(\frac{Q+1}{2} + R - DL + B(Q,R)\right) + \frac{\lambda E}{S}B(Q,R). \quad (19)$$

5. Determining the Optimal Values of Q and R

To simplify discussion of the optimization process and the development of the associated formula for Q and R, lead time demand will be assumed to be large enough that a Normal distribution can be used to approximate the Poisson. This will allow us the use of calculus to determine Q and R. In the process, $F(u;DL)$ will denote the cumulative distribution function for the Normal distribution. In this continuous case,

$$B(Q,R) = \frac{1}{Q} \int_R^{\infty} (u-R) [F(u+Q;DL) - F(u;DL)] du. \quad (20)$$

Taking the partial derivative of (19) with respect to Q results in

$$\frac{\partial TVC}{\partial Q} = \frac{-4DA}{Q^2} + \frac{IC}{2} + \left(IC + \frac{\lambda E}{S}\right) \frac{\partial}{\partial Q} B(Q,R). \quad (21)$$

Unfortunately, $\frac{\partial}{\partial Q} B(Q,R)$ is impossible to write out as a simple general function of Q because Q is contained in

$F(u + Q; DL)$. As a consequence, this term is ignored in UICP and the remainder of the partial derivative is set to zero to get

$$\frac{-4DA}{Q^2} + \frac{IC}{2} = 0, \quad (22)$$

which, when solved for Q^* , gives:

$$Q^* = \sqrt{\frac{8DA}{IC}}. \quad (23)$$

After Q^* is computed, the UICP constrains the order quantity to at least one quarter's worth of demand and no more than six quarters' demand. It is further constrained to avoid any possible deterioration due to shelf-life.

The next step is to determine the optimal reorder point. This is done by first determining the optimal probability of stockout (the UICP calls it "RISK") and comparing it to the acceptable range constrained in the UICP. Once the RISK value has been determined, the reorder point can be computed based on the probability distribution of the lead time demand.

The RISK formula is developed by taking the partial derivative of equation (19) with respect to R and setting the result equal to zero,

$$\frac{\partial TVC}{\partial R} = IC + IC \left(\frac{\partial}{\partial R} B(Q, R) \right) + \frac{\lambda E}{S} \left(\frac{\partial}{\partial R} B(Q, R) \right) = 0. \quad (24)$$

Equation (24) reduces to

$$IC = - \left(IC + \frac{\lambda E}{S} \right) \frac{\partial}{\partial R} B(Q, R). \quad (25)$$

The partial derivative $\frac{\partial}{\partial R} B(Q, R)$ is obtained using Leibnitz's rule for differentiation of an integral. The result is

$$\frac{\partial}{\partial R} B(Q, R) = -\frac{1}{Q} \int_R^{\infty} [F(u + Q; DL) - F(u; DL)] du. \quad (26)$$

Substituting (26) into (25) results in

$$IC = \left(IC + \frac{\lambda E}{S} \right) \left(\frac{1}{Q} \right) \int_R^{\infty} [F(u + Q; DL) - F(u; DL)] du. \quad (27)$$

Multiplying both sides by the product, $(S)(Q)$, and dividing by $(SIC + \lambda E)$ results in

$$\int_R^{\infty} [F(u + Q; DL) - F(u; DL)] du = \frac{ICQS}{SIC + \lambda E} \quad (28)$$

Equation (28) is impossible to solve analytically for R since the left-hand side cannot be reduced to a simple function of R . An alternative approach is to use an approximation technique, described in Reference (7), to obtain the RISK formula. Using some limiting arguments, it can be shown that

$$Q(1 - F(R;DL)) \geq \frac{SICQ}{SIC + \lambda E} \geq Q(1 - F(R + Q;DL)), \quad (29)$$

which reduces to

$$(1 - F(R;DL)) \geq \frac{SIC}{SIC + \lambda E} \geq (1 - F(R + Q;DL)) \quad (30)$$

Then, instead of solving for the smallest R that satisfies equation (30), UICP merely uses the equation

$$1 - F(R;DL) = \frac{SIC}{SIC + \lambda E} \quad (31)$$

to solve for R. This expression describes the probability of being out of stock during a lead time, or RISK. The final step is to let

$$S = \frac{D}{W}, \quad (32)$$

where W is the average quarterly frequency of requisitions. Substituting (32) into (31) gives:

$$RISK = 1 - F(R;DL) = \frac{DIC}{DIC + \lambda EW}. \quad (33)$$

UICP then places both a lower and upper bound on the value that RISK can take. The bounds depend upon how active the item is; for instance, at the Navy's Ships Parts Control Center, most of the DLR items they manage have bounds of 0.15 and 0.4, respectively.

The reorder level, R, is obtained by solving equation (33), constraining RISK with an upper and lower bound, and

then determining the probability distribution of the lead time demand. The distribution assumed is determined by two methods. One method involves the category of the material. UICP divides all wholesale material into categories called Marks. A Mark 0 item is one in which the yearly demand is very small, usually less than one. The lead time demand for all Mark 0 items is assumed to be Poisson distributed. For all items in the remaining categories, the ICP's have established a value known as the probability break-point. If the lead time demand is greater than or equal to this value, it is assumed to be Normal. Otherwise, it is assumed to be Negative Binomial.

There are three methods in calculating R. When the lead time demand is assumed to be Normal, the reorder point becomes the lead time demand plus the safety level:

$$R = DL + z\sigma, \quad (34)$$

where σ is the standard deviation of demand during lead time, and z is the standard Normal deviate associated with the value for RISK. When the lead time demand is assumed to be Poisson, R is the smallest integer which satisfies the following inequality:

$$F(R) \geq 1 - RISK. \quad (35)$$

This same method is employed when lead time demand distribution is assumed to be Negative Binomial.

C. THE NAVY'S DEPOT LEVEL REPAIRABLES MODEL

As noted, the Navy's consumable inventory model conforms to the requirements of Reference (5). However, there is no such guidance for the procurement or repair of Depot Level Repairables (DLR) because the group that developed Reference (5) realized there would be little hope for consensus on a repairables document. That is because each service had developed its own model with no two models being exactly alike.

This section will focus on the Navy's model. The objective of the Navy's model is to determine how much to buy, when to buy, how much to repair, and when to repair so that the average annual total variable costs are minimized.

The Navy's DLR model is based on the consumable model with a couple of major differences. First, the UICP views the DLR system as two separate systems in the modeling process; one for the procurement of new material and the other for the repair of the not-ready-for-issue (NRFI) but repairable carcasses. Thus, the system receives ready-for-issue (RFI) material from two sources, repair and procurement, where the repair rate G (called the "regeneration" rate in UICP) is forecasted and the attrition rate is $(D - G)$. The regeneration rate is computed as

$$G = (CRR) (RSR) (D), \quad (36)$$

where CRR is the estimated carcass return rate, RSR is the forecasted carcass repair survival rate, and D is the forecasted quarterly demand.

Second, after a certain number of attritions occur, a procurement is needed to buy replacements for the lost units. The lead time attrition demand for the "procurement system" is assumed to be composed of a weighted average lead time L_2 and the quarterly demand rate D. It is referred to as the procurement problem variable (Z). From Reference (6), L_2 is given by equation (37):

$$L_2 = \left(\frac{D - G}{D} \right) L + \frac{G}{D} T_2, \quad (37)$$

where T_2 is the repair turn-around time. The ratio $\frac{(D - G)}{D}$ is the fraction of the time that a demand for an RFI unit has no associated repairable carcass being returned to a depot. Conversely, the ratio $\frac{G}{D}$ represents the fraction of the time that a demand is accompanied by a repairable carcass.

If a repairable carcass is not returned, a new unit will have to be purchased. That unit will take a procurement lead time to be received by the system. Thus, L is associated with that fraction of demands having no repairable carcasses. When a carcass is returned and is repairable, it will take a time

T_2 , called the repair turn-around time, for it to be processed through repair.

The equation for the Procurement Problem Variable, Z , is:

$$Z = DL_2 = (D - G)(L) + GT_2. \quad (38)$$

The lead time demand for the "repair system", Z_2 , is denoted by the product of D and the repair turn-around time,

$$Z_2 = DT_2. \quad (39)$$

Repair turn-around time is measured from the time a failed carcass is inducted into the repair system (it changes from condition code F to M) until it is successfully repaired and returned to RFI condition (from M to A condition).

Since the UICP views the DLR system as two separate systems in the modeling process (one for the procurement of new material and the other for the repair of the NRFI but repairable carcasses), two inventory positions are used, [Ref. 6]. The inventory position for the procurement problem, IP_p , (ignoring any planned requirements and war reserves) is:

$$\begin{aligned} IP_p = & \text{(On hand RFI material)} \\ & + \text{(RFI on order through procurement)} \\ & + \text{(RSR) (On hand NRFI carcasses)} \\ & - \text{(Backorders)} \\ & - \text{(RSR) (Unfilled NRFI requirements for repair)}. \end{aligned} \quad (40)$$

The inventory position of the repair system, IP_r , is:

$$\begin{aligned}
 IP_r = & \text{(On hand RFI material)} \\
 & + \text{(On order procured material} \\
 & \quad \text{scheduled to be received within} \\
 & \quad \text{a depot level repair turn-around time)} \\
 & + \text{(RSR) (Inducted material)} \\
 & - \text{(Backorders)}.
 \end{aligned} \tag{41}$$

1. Procurement Problem

The average annual total variable cost equation associated with procurement was adapted from the consumable model by incorporating the attrition demand (D-G) and the Procurement Problem Variable, Z, into equation (19). The result is

$$TVC = \frac{4(D-G)A}{Q} + IC \left(\frac{Q}{2} + R - Z + B_1 \right) + \frac{\lambda E}{S} \left(\frac{4(D-G)}{Q} \right) B_1, \tag{42}$$

where B_1 is the expected number of backorders at the end of a lead time. The formula for B_1 is given by Reference (6) as

$$B_1 = \int_R^{\infty} (u - R) f(u; DL) du \tag{43}$$

because, when the model was developed, there was no requirement for the expected backorders to be time-weighted. Taking the partial derivative of (42) with respect to Q gives

$$\frac{\partial TVC}{\partial Q} = \frac{-4(D-G)A}{Q^2} + \frac{IC}{2} + \frac{-4\lambda E(D-G)B_1}{SQ^2}. \tag{44}$$

When equation (44) is set equal to zero, the result is:

$$Q = \sqrt{\frac{8(D - G)A}{IC} + \frac{8\lambda E(D - G)B_1}{ICS}}. \quad (45)$$

Unfortunately, B_1 is a function of the procurement problem reorder point R . Thus, to solve (45) would require an iterative procedure which includes determining optimal R as a function of Q . Because of this, the UICP approach is to ignore the second term and use

$$Q^* = \sqrt{\frac{8(D - G)A}{IC}}. \quad (46)$$

The order quantity constraints mentioned before for the consumable model are then applied after replacing D by $(D - G)$.

2. Repair Induction Quantity

The average annual total variable costs associated with the repair system were only "surmised" as indicated in Reference (6). The formula used is:

$$\begin{aligned} TVC_2 = \frac{4[Min(D, G)]}{Q_2} A_2 + IC_2 \left(\frac{Q_2}{2} + R_2 - Z_2 + B_3 \right) \\ + \frac{4\lambda E[Min(D, G)]}{Q_2} B_4, \end{aligned} \quad (47)$$

where C_2 is the unit cost to repair an item, A_2 is the cost to prepare a repair order, Q_2 is the repair quantity, R_2 is the repair reorder point, B_3 is the expected number of units backordered during T_2 , B_4 is the expected number of

requisitions backordered during T_2 , and $\text{Min}(D,G)$ is the smaller of the forecasted quarterly demand or the quarterly regeneration rate. The equations for B_3 and B_4 are not presented here because neither is used in the integrated repairable model.

The reason presented for using $\text{Min}(D,G)$ is that it is possible that forecasted regenerations exceed forecasted demand. (Examples include a phase-out of a weapon system where there exists turn-ins but little or no demand, or where an extreme decrease in one quarter's demand coincides with the receipt of a large number of carcasses from demands of the previous quarter.)

Following the same approach used in the consumable and DLR procurement models, the optimal repair order quantity is approximated as

$$Q_2 = \sqrt{\frac{8 [\text{Min}(D,G)] A_2}{IC_2}}, \quad (48)$$

and is constrained with the same limits.

3. Integrated RISK Formula

In the past, the procedure for computing reorder levels separately for the DLR procurement and the DLR repair quantities resulted in the procurement levels for many items not being sufficient to provide carcasses so that the repair quantity Q_2 could be inducted. [Ref. 6] This was due, in

part, to the computation of two different safety levels and, in certain cases, the repair safety level exceeding the procurement safety level. This motivated the integration of the computation of the safety levels for the procurement and repair levels. The first step of the process was to compute a weighted average unit/repair cost, C_3 , from C and C_2 in the same way as L_2 was developed, and then substitute C_3 for C in equation (33). The result is

$$RISK = 1 - F(R; DL) = \frac{DIC_3}{DIC_3 + \lambda EW}. \quad (49)$$

As with the consumable case, the RISK is constrained by UICP set minimum and maximum values.

The procurement reorder point, R , is computed in the same way it was for the consumable model. The associated safety level follows from the fact that:

$$R = Z + \text{safety level}. \quad (50)$$

Once the safety level is determined, the repair reorder point, R_2 , is calculated using the same safety level:

$$R_2 = DT_2 + \text{safety level}. \quad (51)$$

Finally, it should be noted that UICP assumes DLR items are never Mark 0 items. Therefore, only the Normal and Negative Binomial distributions are used to compute these reorder points.

III. THE AGGREGATE REPAIRABLES MODELS

This chapter first discusses the proposed "Aggregate" Depot Level Repairables Model developed by Professor McMaster at the Naval Postgraduate School. Then it describes a simulation model which attempts to recreate the "real-world" repairables system. Finally, it will describe the steps of this model's simulation analyses. The results of the analyses will be presented in Chapter IV.

A. THE AGGREGATE MODEL

The basic approach in the model's development is to first divide the RFI inventory, inventory position, demand and reorder point into two separate categories, as is done in the current UICP integrated repairables model. The first category contains the inventory when the anticipated demand is for a unit with a returned carcass which will eventually be successfully repaired. The other category will contain the inventory when the carcass will be either lost or determined to be uneconomical to repair.

1. Definition of terms

Define Q_p as the procurement quantity of new material each time a procurement order is generated, and Q_r as the quantity inducted into a depot every time a repair order is

generated. Let D_1 be the quarterly demand rate of those units for which a carcass is turned in and will eventually be repaired, and call it the "repairable" demand rate. Let D_2 be the quarterly demand rate of those units for which a carcass is either not turned in or, if turned in, cannot be repaired. D_2 is called the "attrition" demand rate. Then D_1 and D_2 can be computed as

$$D_1 = (CRR) (RSR) (D) = \left(\frac{G}{D}\right)D \quad (52)$$

and

$$D_2 = [1 - (CRR) (RSR)] D = \left(\frac{D - G}{D}\right)D. \quad (53)$$

Adding equations (52) and (53) results in

$$D = D_1 + D_2, \quad (54)$$

where D is the total forecasted quarterly demand. Assume that both D_1 and D_2 are Poisson distributed. It is then also true that D is Poisson distributed. [Ref. 8]

Next, let the RFI inventory position for each category be denoted as IP_R and IP_P , respectively, and let IP denote the aggregate RFI inventory position. It follows that

$$IP = IP_P + IP_R. \quad (55)$$

Finally, let the maximum RFI inventory position of each category be denoted as SW_R and SW_P , respectively, and let SW

denote the maximum aggregate inventory position. Then, from equation (55),

$$SW = SW_R + SW_P. \quad (56)$$

2. Inventory Position Ranges and Probability Distributions

As demands of each type occur, the corresponding inventory positions decrease one unit at a time until either there are Q_R carcasses or Q_P attritions. Assume that a repair order or an order for procurement will be generated instantaneously whenever needed. Each will then instantaneously increase the corresponding inventory position by Q_R or Q_P .

Figure 1 shows the state transition diagram for the inventory position of the attrition demand category. Note that IP_P ranges from SW_P down to $SW_P - (Q_P - 1)$. Because of the Poisson process, it follows from Reference 3 that

$$p_P(SW_P - w) = \frac{1}{Q_P} \quad \text{for } w = 0, 1, 2, \dots, Q_P - 1, \quad (57)$$

where w represents the number of units below the maximum inventory position, SW_P .

Similarly, Figure 2 shows the state transition diagram for the inventory position of the repair demand category. From the same arguments used in the attrition demand case,

$$p_R(SW_R - v) = \frac{1}{Q_R} \quad \text{for } v = 0, 1, 2, \dots, Q_R - 1, \quad (58)$$

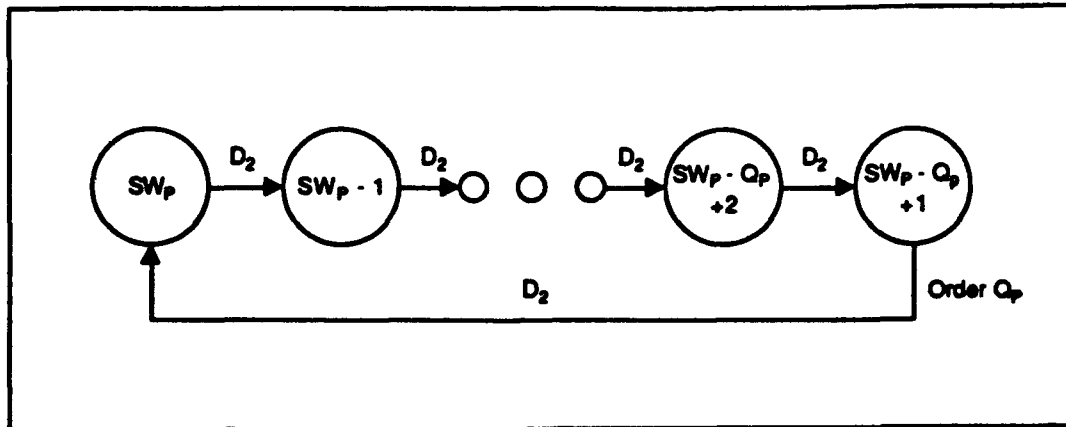


Figure 1. State Transition Diagram for Inventory Position of Attrition Demand.

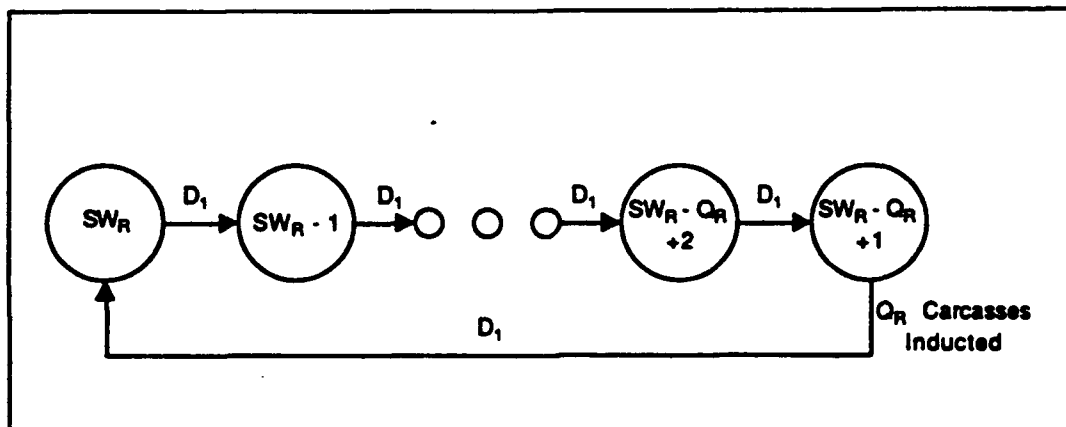


Figure 2. State Transition Diagram for Inventory Position of the Repair Demand.

where v is the number of units below the maximum inventory position, SW_R .

In reality, the attritions from the inducted batch are actually not known until after the batch is inducted and repair is attempted. However, in the steady state situation, the previously inducted batch can be considered as generating some of the current attrition demands while carcasses are

accumulating for the next batch. In addition, when inducted carcasses are determined to be not repairable, the failure is usually quickly reported to the ICP's so that attrition demand does not wait until the entire batch is processed through a depot. Thus, while D_2 was assumed to represent continuous attrition demand, in reality only that part corresponding to demands without carcass turn-ins is continuous. The attrition from repair is only approximately continuous.

. The Aggregate Inventory Position Probability Distribution

Define x as the aggregate number of units below the maximum total inventory position, SW . Then,

$$SW - x = SW_P - w + SW_R - v, \quad (59)$$

or

$$x = w + v. \quad (60)$$

From equations (57) and (58) the ranges of w and v are respectively,

$$0 \leq w \leq Q_P - 1; \quad (61)$$

and

$$0 \leq v \leq Q_R - 1. \quad (62)$$

Adding (61) and (62) yields

$$0 \leq w + v \leq Q_P + Q_R - 2; \quad (63)$$

or

$$0 \leq x \leq Q_P + Q_R - 2. \quad (64)$$

From the definition of x and equations (56), (57), (58), and (60), the SW term will no longer be used in the arguments. Rather, only w , v , and x will be considered. Counting from SW downward, the probability distribution for the total inventory position can be derived as the convolution of the two Uniform distributions, ρ_P and ρ_R . That is,

$$\rho(x) = \sum_{w=0}^x \rho_P(w) \rho_R(v = x - w), \quad (65)$$

for $x = 0, 1, 2, \dots, (Q_P + Q_R - 2)$. Applying equations (57) and (58) to (65), subject to the inequalities (61), (62), and (64) results in:

$$\rho(x) = \begin{bmatrix} \frac{x+1}{Q_P Q_R} & \text{for } 0 \leq x \leq x_1 \\ \frac{\text{Min}(Q_P, Q_R)}{Q_P Q_R} & \text{for } x_1 < x \leq x_2 \\ \frac{x_{\text{Max}} + 1 - x}{Q_P Q_R} & \text{for } x_2 < x \leq x_{\text{Max}} \\ 0 & \text{otherwise.} \end{bmatrix}; \quad (66)$$

where x_{Max} , x_1 , and x_2 are defined as

$$\begin{aligned}x_{Max} &= Q_P + Q_R - 2; \\x_1 &= \text{Min} (Q_P, Q_R) - 1; \\x_2 &= x_{Max} - x_1.\end{aligned}\tag{67}$$

The next step is to model the aggregate net inventory level. Unfortunately, this is analytically difficult at present. Arguments such as those presented on pages 28 through 31 have been attempted. However, the assumption was that a batch of carcasses inducted into repair were all finished in a total time T_2 . In reality, this is not true. T_2 is the time an individual carcass is in repair, while carcasses are inducted serially. Thus, reality is a single-server queue for each batch. Therefore, this simulation model has been developed to provide an understanding of the nature of the net inventory as a function of various model parameter values.

B. THE SIMULATED REPAIRABLES MODEL

This simulation has tried to build a system which reflects reality as closely as possible. It takes a different view of the Depot Level Repairables (DLR) inventory management system than does UICP. It recognizes only one "aggregate" inventory position rather the dual view of the procurement and repair inventory positions of UICP. It also views attrition differently. Attritions occur in two different time frames.

First, they can occur if no turn-in accompanies the demand or the turn-in is lost in shipment to a supply center or other holding point. When this happens, it is assumed to occur instantly at the time of the demand. The other way to incur an attrition is for the unit to be condemned at the depot after it has been inducted for repair. Thus, the process generating each of these types of attrition are quite different. Because this simulation model can monitor attritions from the repair process, it allows the second type of attrition to occur at a different time. Once attritions reach a fixed level, called Q_p in the model, an order is generated to buy the Q_p quantity. Although UICP does not track attritions, it implicitly assumes that both types of attrition occur instantaneously upon demand of a unit. As described in Chapter II, it also uses a procurement reorder point to determine when to procure the next order.

This model also takes a different view of the carcass repair process. UICP makes the assumption that at a certain point in time a batch of Q_r units are inducted into the depot, and that all of them are repaired and returned to the supply system a Repair Turn-Around Time (RTAT or T_r) later. This, in fact, does not actually happen. Carcasses are sent to the depot in batches. When a batch reaches the depot, it enters a serial queue. The first unit is examined to determine if it is repairable or not. If it is not, it is condemned immediately, and the next carcass is examined. If it is

determined to be repairable, it is inducted into the repair process and is returned as an RFI unit to the supply center a time T_2 later. This carcass passes through the first work station in a processing time, REP. When processing is complete the next carcass is examined.

To illustrate this process, assume five units are sent to a depot at time T . Also, assume the third and fourth units are not repairable. The first unit is examined at time T and returned in RFI condition at time $T + T_2$. The second carcass is examined at time $T + REP$ and returned in RFI condition at time $T + REP + T_2$. The third unit is examined at time $T + 2(REP)$, and condemned immediately. Because the third unit had been condemned, the fourth carcass is also examined at time $T + 2(REP)$. It, too, is condemned immediately. Finally, the fifth unit is examined at time $T + 2(REP)$ and found to be repairable. It is then returned in RFI condition at time $T + 2(REP) + T_2$.

1. Model Assumptions

The Simulated Repairables Model makes some of the same basic assumptions as the UICP DLR model. The model assumptions are listed below.

1. Demand follows a Poisson distribution.
2. Carcass return rate, CRR, repair survival rate, RSR, procurement lead time, L , repair turn-around time, T_2 , and the time it takes a repairable unit to pass through the first work station, REP, are known and constant.

3. Demands occur at a rate of one unit per requisition.
4. Carcass turn-ins occur instantaneously when an associated demand occurs.
5. Shipments of carcasses from a supply center or other holding point to the depot occurs instantaneously.
6. Inventory position changes with each demand, attrition, reorder of new material, and the induction of carcasses into repair at the depot.
7. Carcasses are inducted in batches. Once in the repair system, it is assumed that the first carcass in the batch is immediately reviewed to determine if it can be successfully repaired. If it cannot, it is condemned instantly and the next carcass is reviewed. If that carcass is determined to be repairable, it begins the repair process. The next carcass is then reviewed a REP time later.
8. Once a carcass begins repair, it is returned to ready-for-issue inventory (INV) a time T_2 later.
9. Purchases are received sequentially in the order they were placed.
10. A purchase order is generated when the number of attritions, ATTR, reaches a specified order quantity, Q_p .
11. A repair order is generated when the number of carcasses accumulated in a repair queue, RQUE, reaches a specified value Q_R .
12. The safety level is defined as the average net inventory when a procurement of new material is received or a repaired item is returned to the supply system.

C. DISCRETE EVENT MODELING

The simulation software package used for this simulation model is called SIGMA, the SIMulation Graphical Modeling and Analysis system, developed by L. Schruben, Cornell University, and distributed by Scientific Press, South San Francisco,

California. It was originally developed to study the dynamic behavior of systems; i.e., how systems change over time, and is ideally suited for simulating the Navy's repairables inventory management process. It is an interactive graphics approach for building, testing, and experimenting with discrete event simulations on personal computers. A powerful feature of SIGMA is that it can automatically translate a simulation model into portable source codes for Pascal and C, and can be compiled and run on a wide variety of computers. [Ref. 9]

1. Discrete Event Simulation Modeling Terms

To discuss how discrete event simulation works, certain terms need to be described. A "system" is defined as, "...a collection of entities that interact with a common purpose according to sets of laws and policies." [Ref 9] Systems are characterized by their function or purpose such as a transportation system or an inventory control system. The "entities" are the elements which are used to perpetuate or "fuel" the system. Examples of entities are components on an assembly line system or variables in a system of equations. They are described by their "attributes" or characteristics.

A discrete event system is one in which changes in the system occur at particular instances of time. The "state" of a system is simply a description of that system. In a computer simulation of a discrete event system, the state is

defined by the numerical values of its variables (referred to as the state variables) and the schedule of future "events" (state changes within the system). Finally, "laws and policies" determine the conditions under which future events will occur, the time they are scheduled to occur, and the priority of execution between two or more events scheduled at the same time.

2. Discrete Event Simulation

In a discrete event system simulation time is advanced in discrete steps until it reaches a value at which the state is scheduled to change. At this point, one or more variables change value, and one or more events may be scheduled or canceled. An excellent example is the Navy's repairable management system. The system is changed when a demand of one unit occurs. This event causes the inventory position and the on-hand inventory to reduce their respective values by one unit. Other events may be scheduled, such as another demand, a reorder, or an event which determines if a failed carcass is turned in to the supply system. These events can only occur if certain conditions of the new state are present. They can be scheduled (or canceled) after a fixed, or random, time period interval following the current time.

In SIGMA a main controller executes event routines. This controller operates from a master appointment list of scheduled events referred to as the "future events list"

because, at any given time, it contains all events which have been scheduled and the times they are to be executed. During a simulation, the main control program will advance the simulated time to the time of the next scheduled event, remove the event from the list and execute it. The main control program continues calling the next scheduled event until a condition for stopping the simulation is met. SIGMA will stop the simulation either after the simulation clock reaches a user designated time, or after a user designated event occurs a given number of times. This parameter is established in SIGMA's RUN menu just prior to executing the simulation. A general flow chart summarizing this process is illustrated in Figure 3.

3. Modeling with Event Graphs

The major thrust of SIGMA is to model or simulate a discrete event system using an event graph. As mentioned above, the three elements of a discrete event system are the state variables, the events that change state variables, and the relationships between events. An event graph organizes these into the simulation model. In SIGMA, events are represented by vertices, and the relationships between the events are represented by directed edges. A picture of the event graph used in this thesis research is shown in Figure 4.

To develop a simulation model, the three elements need to be defined in mathematical terms. SIGMA provides the means

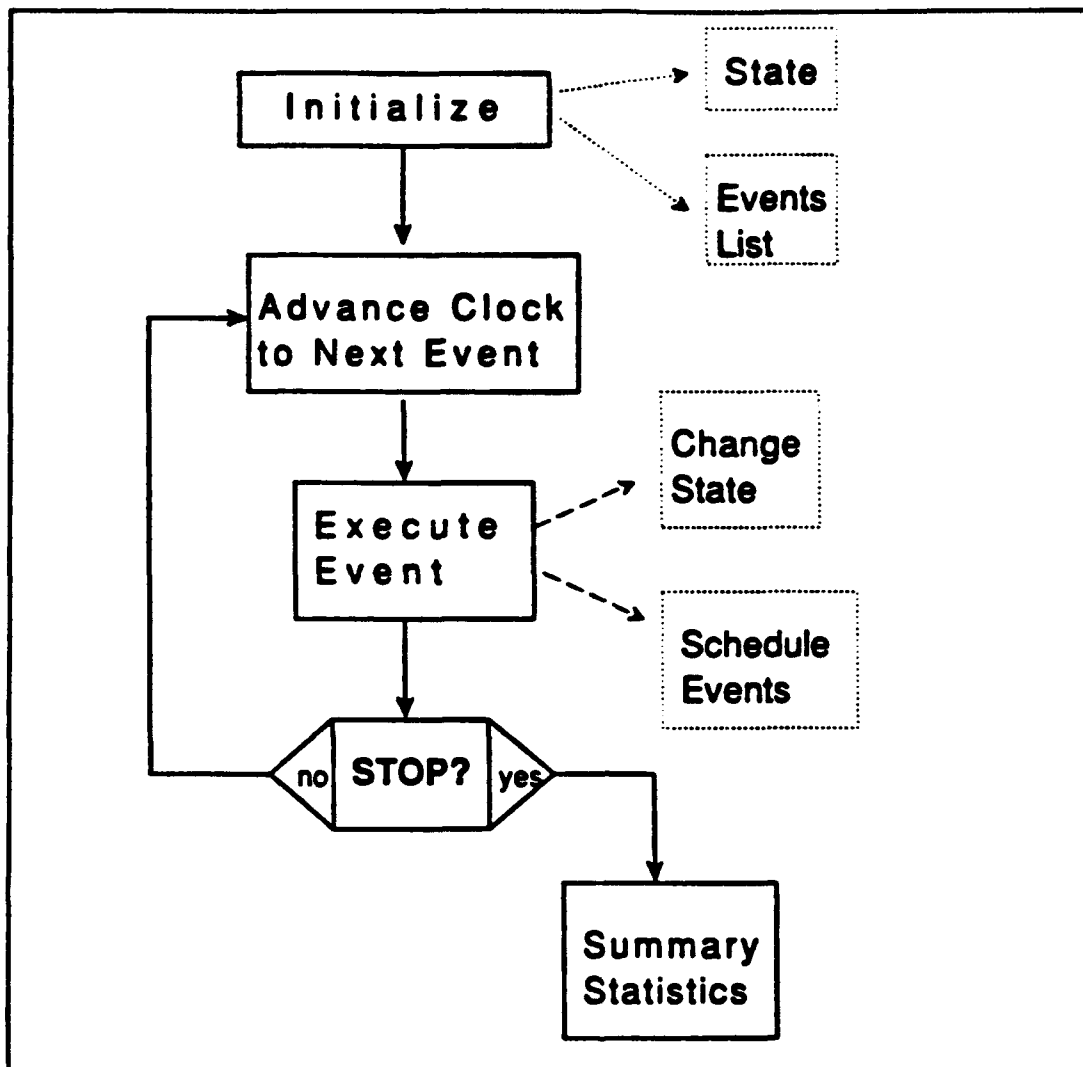


Figure 3. Main Event-Scheduling Algorithm.

to define the terms using "pop-up" menus. State variables can be single variables, arrays, or matrices. They are defined as either real or integer. The events (vertices) contain formulas which determine how state variables change their values. The edges represent "laws and policies"; i.e., the conditions, time delay, and priority, under which one event schedules (or cancels) the connecting event. These conditions

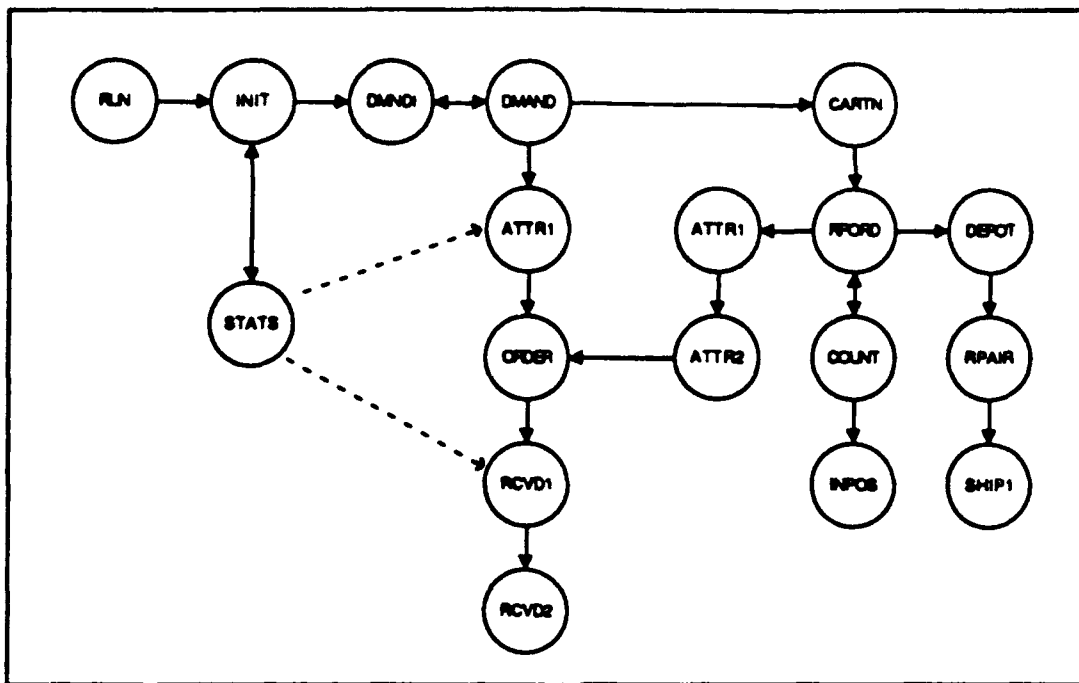


Figure 4. Event graph of THSVER4.MOD

are implicit "If..., then" statements which either schedule the connecting event if the statement is true or do nothing if the statement is false.

In SIGMA, the default edge condition is " $1=1$ ". (The use of the double equal sign is peculiar to SIGMA and is only used in condition statements on the edges.) The edges meeting this condition are labeled as "unconditional" edges because they automatically schedule the connecting event. The time delay can be a number, a variable, a random number, or a function of any of these. SIGMA has its own random number generator which can be called from either an event or an edge. It generates an uniform random variable, RND, whose

range is the set of all real numbers between zero and one, i.e., $0.0 < \text{RND} < 1.0$.

The use of priorities on an edge is the way in which SIGMA executes events scheduled to occur at the same time. When the scheduling condition is met by two or more out-edges which have the same time delay, SIGMA executes the connecting event with the highest priority first. The priority range is from one to ten with one the highest and ten the lowest priority. They are assigned to each event by the user.

Descriptions of the state variables, events, and edges for the three models used in this thesis research (THSVER4.MOD, THSVER7.MOD, and THSVER8.MOD) are provided in Appendices A, D, and F, respectively. A short synopsis of the relationships of each vertex with its out-edges and connecting vertices is automatically generated by SIGMA. These synopses are provided in Appendices B, E, and G, respectively, for each of the three models. A detailed description of THSVER4.MOD, which illustrates these relationships, is discussed in section D below.

4. Output

Obtaining data from a SIGMA simulation is not a difficult task. The user simply designates the state variables and events which need to be "traced". Once control passes to a traced event and state changes occur, SIGMA will write the name of the event, the number of times the event

occurred, and the value of all traced variables to an external file. An example of an output file is provided in Appendix C.

D. THE MODEL

1. Model Description

The model begins with the RUN event. Its purpose is to initialize various state variables. Variables not given specific values by the user in this event are automatically initialized as zero. [Ref. 9:p. 30] For the repairables simulation model analyzed in this thesis, certain state variables are never changed. Their names and values are:

L = 8.2 quarters
T2 = 1.3 quarters
D = 9 units per quarter
QP = 10 units
QR = 5 units
TIME = 850 quarters
SW = 72 units.

Three other variables initialized in the RUN event will be changed from one simulation run to another. These variables are CRR and RSR, which range from 0.0 to 1.0 in increments of 0.2, and REP, which ranges from 0.0 to the value of T2 (1.3) in increments of 0.25 quarters.

Following initialization of the model variables, the RUN event schedules the INIT event. In the INIT event, IP and INV are set to their maximum values. From Figure 4, there are two edges originating from INIT connecting with DMND1 and STATS. As noted in Appendix D, both are "unconditional"

edges. The edge going to STATS has a time delay of TIME quarters which was used in this model to stop the simulation. The user must designate STATS as the event which stops the simulation, and must also specify that it occurs only once. This ensures that the simulation will stop when the time of the simulation reaches TIME.

The event DMND1 schedules all demands. Thus, it unconditionally passes control to the DMAND event. Once control passes to DMAND, the state immediately changes by reducing the state variables IP and INV by one value, (i.e., mathematically, $INV = INV - 1$ and $IP = IP - 1$), and by assigning the state variable X[1] the first value of the uniform random variable, RND, to be used in the decision associated with the implicit Bernoulli trial from the conditional statements of two of DMAND's out-edges.

Once changes occur, the main control program schedules events based on the conditions described on the edges. There are three out-edges from DMAND connecting with events CARTN, DMND1, and ATTRI, and denoted by edges (1), (6), and (12), respectively, in Appendix A. Edges (1) and (6) model a Bernoulli trial to determine if a failed carcass is returned to the supply system when a demand occurs. If the value of X[1] is less than or equal to CRR, then the carcass has been returned and CARTN will be scheduled at a time equal to the current time. If X[1] is greater than CRR, then the carcass was not returned and the condition of edge (6) is met. ATTRI

is, therefore, scheduled to occur immediately. If the carcass is returned, the variable, RQUE, will increase by one unit. If not, the variable, ATTRI, will increase by one unit.

The other directed edge from DMAND connects with DMND1. Its condition is "1==1", making it an unconditional edge. In other words, the event DMND1 will always be scheduled whenever DMAND occurs. The time delay of scheduling DMND1 is $(-1/D) * \ln(RND)$, which is the exponentially distributed random time between demands and is a consequence of the Poisson process. Therefore, DMND1 will be scheduled at the current time plus the delay. In the case where the time delay is equal to zero, there will be two events, DMND1 and either CARTN or ATTRI, scheduled at the same time. However, the edges scheduling CARTN and ATTRI have been assigned a priority of three by the author, and the edge scheduling DMND1 has been assigned a priority of five so that CARTN or ATTRI will be executed before DMND1.

Demands continue to occur in the above manner until the simulation stops. As demands occur, the variables ATTR and RQUE increase in size. When either reaches a given level, the respective procurement and repair processes begin.

2. Repair Process

Once RQUE reaches the value QR, the batch of carcasses in RQUE are inducted into the repair process. As a consequence, IP and QUEREP are increased by QR, and the event

RPORD is immediately scheduled. This event is the focus of the repair process because it is the event which determines whether a carcass can be repaired or must be condemned. The event RPORD, coupled with the COUNT event, will actually analyze each carcass sequentially (but at the same simulated clock time) and make a decision on each carcass. Then, as a result of the edge time delays, each carcass' delay in entering attrition or repair categories will be assigned a repair processing time within the range of zero to $(QR - 1) * REP$.

Just prior to the occurrence of RPORD, the CARTN event will have just increased IP and QUEREP by QR, and set the variable COUNT equal to zero. Then the event RPORD is executed in a string of QR distinct occurrences at the same simulated clock time. As this sequence proceeds, RQUE empties one at a time as the string of Bernoulli trials is executed and the decision is made to either repair or condemn the carcass. If the carcass can be repaired, the event DEPOT is scheduled. If it cannot, the event ATTRI is scheduled. The time delay effect of REP is then accessed through the COUNT event.

The time delay on the edges from RPORD to ATTRI and DEPOT models the repair processing time. As each carcass is analyzed, a time delay is assigned based on the product of REP and the variable COUNT. After the first carcass is examined in RPORD, the time delay in scheduling either ATTRI or DEPOT

is $COUNT * REP = 0$, since initially the variable COUNT equals zero. The return to RFI condition for this carcass, then, would be the current time plus T2. At the same time, but after either ATTRI or DEPOT is scheduled, the event, COUNT, is scheduled and executed. The only state change in this event is to change the variable, COUNT. If the first carcass is repairable, then the variable COUNT increases by one. If not, COUNT will remain zero. If there are any more carcasses in the batch, (i.e., if RQUE is greater than zero) then control passes immediately back to the event RPORD where the next carcass is processed. (Notice, no simulated clock time has expired.)

Since only one carcass has been processed at this point (and assuming $QR > 0$), RPORD examines the second carcass. RQUE is reduced by one and another random number is generated for the Bernoulli trial on the edges leading to the events ATTRI and DEPOT. The time delay will now have been affected by the first carcass. If the first carcass was repairable, then COUNT would have been increased to one, and the time delay would be REP times the quantity COUNT, or a total of REP quarters. The carcass would then be returned to RFI condition at the current time plus REP plus T2. If the first carcass was condemned, COUNT would not increase, and the time delay for this carcass would be zero. This process continues until RQUE equals zero and control passes to the next scheduled event.

An illustration of how this simulation models the repair process is presented here using typical values for the variables. Assume RSR equals 0.8, REP equals 0.25, QUEREP equals zero, IP equals 60, and the simulated clock time is 18. Also, assume the constant variables mentioned above in Section D.1 retain their designated values.

Suppose at time 18, the variable RQUE reaches $QR = 5$ in event CARTN. This will cause the variables QUEREP and IP to be increased to five and 65, respectively. The condition on the out-edge to RPORD is met; therefore, RPORD is immediately scheduled and executed. This causes RQUE to be reduced by one unit, to four. Suppose that $X[0]$'s random variable value is found to equal 0.6. Since $X[0]$ is less than $RSR = 0.8$, the edge condition from DMAND to DEPOT is met and event DEPOT is scheduled to be executed at time $18 + 0 = 18$ since COUNT is zero and the product of COUNT and REP is zero. Control moves to the event, DEPOT, since its edge has a lower numbered priority than the edge leading to the event COUNT. There are no state variable changes here, but the event, RPAIR is scheduled to occur in T2 time units (i.e., the edge from DEPOT to RPAIR simulates the repair turn-around time). Control will pass to RPAIR at the current time plus T2 and the first carcass will be simulated to return to RFI condition at time $18 + 0 + 1.3 = 19.3$ quarters.

Immediately following this execution of DEPOT, the event COUNT is executed. Since $X[0]$ is less than RSR, the

variable COUNT increases to a value of one and schedules RPORD with no delay. (At this point in the example, control in the repair process will fluctuate between events RPORD and COUNT until RQUE becomes zero.) RQUE is reduced by one more unit to three and another value for X[0] is generated, assumed here to be 0.4. Because X[0] is less than RSR, DEPOT will be scheduled with a time delay of $REP * COUNT = .25 * 1 = 0.25$ quarters. Once DEPOT is executed, the event RPAIR will be scheduled at a time delay of T2. This second carcass will be returned to RFI condition at time $18 + .25 + 1.3 = 19.55$ quarters.

The event, COUNT, will be executed again, increasing the variable COUNT by one unit to a total of two, and for the third time in this sequence, RPORD is executed. RQUE is reduced by one unit to two, and a new value for X[0] is generated, assumed here to be 0.85. Since X[0] is now greater than RSR, the event ATTR1 will be scheduled to occur at the current time plus $REP * COUNT = 0.25 * 2 = 0.5$ quarters, or at clock time $18 + 0.5 = 18.5$ quarters. At that time, the repair attrition will be simulated, reducing IP and QUEREP by one unit.

The event COUNT is again executed at clock time equal to 18. However, since X[0] was greater than RSR, COUNT will not be increased, retaining its value at two. RPORD will then be executed.

This is the fourth time in this sequence that RPORD has been executed. RQUE is now reduced one unit to the value one and another value for X[0] is generated and assumed to be 0.92. As in the previous case, ATTR1 is again scheduled to occur at time 18.5 quarters, resulting in the reduction of QUEREP and IP by one unit.

The event, COUNT, is executed a fourth time at time 18 quarters, and again, because this carcass was not repairable, the variable COUNT remains at two, and RPORD is executed. In RPORD, RQUE is reduced by one unit to zero and the next value for X[0] is generated; say, 0.2. This implies that DEPOT will be scheduled at time $18 + .5 = 18.5$ quarters and it will schedule the event RPAIR to occur at time $18.5 + 1.3 = 19.8$ quarters.

The event COUNT is scheduled and executed once again (still at time 18 quarters), increasing the variable COUNT to three. This event becomes meaningless, however, as RPORD will not be scheduled since RQUE is equal to zero and no longer meets the condition on the edge from COUNT to RPORD. Thus, all that is left in the repair process for this batch is the occurrence of the scheduled repairs and attritions.

To complete the repair process, every time RPAIR occurs QUEREP is reduced by one unit and the event SHIP1 is immediately executed, allowing the net inventory, INV, to be increased by one unit. Likewise, every time ATTR1 occurs, QUEREP decreases by one unit and the event ATTR2 is executed

increasing the attrition variable, ATTR, by one unit. As explained previously in this example, two attritions occur simultaneously at time 18.5 quarters, and three carcasses will be returned to the supply system in RFI condition at times 19.3, 19.55, and 19.8 quarters.

One of the purposes of THSVER4.MOD was to record the inventory position, IP, each time it changed its value. IP changes only in events DMAND, ATTR2, CARTN, and ORDER. However, in simulations where CRR is greater than zero and QR is greater than one, it does not always change in the event CARTN. IP changes in the event CARTN only when the value RQUE increases to the value QR. This leads to difficulties when recording only IP changes. Tracing CARTN would be impractical because each time CARTN is executed, IP would be recorded whether it had changed or not.

To avoid this problem, a "dummy" event, named INPOS, was created. INPOS was connected to the event COUNT where its condition was "RQUE == 0" and its delay time was also zero. This resulted in control passing to the event INPOS only after all carcasses had been examined. Thus, tracing INPOS for IP would result in the recording of IP only when the change in IP is due to the induction of QR carcasses.

3. Procurement Process

The procurement process is much less complex than the repair process. procurement is generated whenever the

inventory system loses a given number of assets from attrition. Attritions occur whenever an asset fails to enter the repair process (e.g., lost in shipment or surveyed by the end user) or is condemned at the repair facility. Attrition due to loss or survey is modeled as a Bernoulli trial in event DMAND, based on the carcass return rate, CRR. Attrition due to condemnation is modeled as a Bernoulli trial in the event RPORD, based on the repair survival rate, RSR. Every time a carcass is lost or condemned, the variable ATTR is increased by one unit. Once ATTR reaches the value QP, an order is generated, IP is increased, reflecting the order, the variable QUEORD is also increased by QP, and ATTR is reset to zero. The receipt of the order is scheduled for $L = 8.3$ quarters later in time. INV will increase by QP at that time, while QUEORD will decrease by QP.

4. Steady State Determination and the Termination Process

As in many time series simulations, this model needs to run for a period of time to "settle" into a steady state. Appendix H contains various graphs of the net inventory versus time with widely different values of CRR and RSR. From a visual inspection it appears that steady state is achieved after approximately 10 quarters. As a precautionary measure in this study, an additional ten quarters were allowed to pass before data were recorded for the analyses to be presented in

the next chapter (i.e., the first 20 quarters of data were ignored).

To determine when to stop the simulation runs, the limits of the APL2 editor, called Editor 3, needed to be taken into account. This editor is provided by International Business Machines as part of the PC version of its APL 232 software. It was used initially to edit each simulation output. Originally, a simulation was stopped after 360 simulated quarters. However, when graphing the resulting simulated data into a histogram, the shape of the graph appeared "ragged". To "smooth" the shape, the time of the run was extended to 1,200 quarters, however, Editor 3 was unable to capture all the data when the edited version was saved. It was discovered that if the simulation stopped after 850 simulated quarters, the edited data would be retained in full. Therefore, 850 quarters were used as the ending time of each simulation. (Later, the MS-DOS editor, Edlin, was suggested as an alternative editor for the purpose of this research. It was found to be a better tool than any other editor available. It can easily edit the 1,200 quarter data sets. In fact, it was used to edit the largest single file (39 megs of data) generated from a batch run. Unfortunately, by the time this discovery was made, the vast majority of the simulation runs had been made with the output edited using the time of 850 quarters. Thus, to remain consistent, 850 quarters was used as the ending time for all simulation runs.)

5. Revisions to the Model

SIGMA, as noted, is a powerful simulation software package. It has two major features which make it powerful. First, it has the ability to read data from an external file. This feature is important when running batch simulations because variables can easily be assigned a range of values in an external file. Second, it has the ability to automatically reset all variables to zero, reset the random number seed to the original value, cancel all events on the future events list, and restart the simulated clock time by using the SIGMA function, "SET { }", on an edge. The combination of both of these features allows the running of batched simulations.

Initially, individual simulation runs of THSVER4.MOD were needed to graph and examine the resulting histograms associated with inventory position and net inventory for each combination of the ranges of CRR, RSR, and REP listed above. These histograms were used to identify the nature of the probability distributions of the Inventory Position and Net Inventory and to examine the effect of the three variables on them. Later analyses did not require these graphs, and a model, THSVER7.MOD, designed for batch runs was developed. This model includes an exact duplicate of THSVER4.MOD and, in addition, the SET { } function is introduced on an out-edge from event STATS to a new event, RINIT. When the event STATS occurs, the introduction of SET { } on the edge automatically resets and restarts the model as noted above. RINIT

reinitializes the variables L, T2, QP, QR, and TIME to their original values. The out-edge from RINIT to INIT is unconditional with no delay. Thus, INIT is scheduled and reads the next set of values of D, CRR, RSR, and REP from an external file, THS.DAT. (For each simulation run of this thesis, the value, D, remained at nine units per quarter. Future thesis efforts can use this feature to study the effects of changing D.)

This model was first used to determine inventory safety levels for all values of CRR, RSR and REP. As mentioned in the model assumptions, the safety level is defined as the average net inventory at the time either a repaired carcass returns to the supply system in RFI condition, or a procurement order is received. The model also introduced two new variables, SAFTY and COUNT[1]. Both were used in the events where INV changed as a result of these types of receipts (i.e., in the events SHIP1 and RCVD2). In each event, SAFTY summed the value of net inventory; i.e., $SAFTY = SAFTY + INV$, while COUNT[1] accumulated the total number of times net inventory increased. When 850 quarters was reached, the event STATS computed the average safety level using the formula, $SAFTY = SAFTY / COUNT[1]$. Appendices D and E provide the details of the THSVER7.MOD model.

To study the average time-weighted values of net inventory, on hand inventory, and backorders, an additional model was developed. Using the batch model discussed above,

the new model, THSVER8.MOD, introduced an array of variables to accumulate this data. A six-variable array, named C[j], was required as SIGMA limits the number of variable names to 20 and assigning six additional names rather than using an array would have resulted in 24 names, exceeding the limitation. The elements of C[j] are described as follows:

1. C[1]: the time of the last net inventory change;
2. C[2]: the time of the current net inventory change;
3. C[3]: the difference between C[2] and C[1]. It is the time which elapsed before the net inventory changed value;
4. C[4]: accumulates the total time-weighted segments of net inventory at any time INV changed value for the run;
5. C[5]: accumulates the total time-weighted segments of INV whenever INV was greater than or equal to zero at any time it changed value during the run;
6. C[6]: accumulates the total time-weighted segments of INV whenever INV was less than zero at any time it changed value during the run.

Because INV changes values in only three events (DMAND, SHIP1, and RCVD2), extracting the time-weighted data was not difficult. The relationships between the above variables were formulated in the event just prior to the change in INV (i.e., in any of the events DMND1, RPAIR, and RCVD2). The relationships are as follows:

1. C[1] = C[2]; sets C[1] to previous clock time.
2. C[2] = CLK; sets C[2] to current clock time.

3. $C[3] = (C[2] - C[1])$, if $CLK \geq 20$; computes the time weighting factor for time intervals after the 20th quarter.
4. $C[4] = C[4] + C[3] * INV$; computes the total time weighted value of INV.
5. $C[5] = C[5] + C[3] * INV$, if $INV \geq 0$; computes the total time-weighted value of O/H inventory.
6. $C[6] = C[6] + C[3] * INV$, if $INV < 0$; computes the total time-weighted value of backorders.

In the STATS event, the averages were written using the following formulas:

$$C[4] = C[4] / 830,$$

$$C[5] = C[5] / 830,$$

$$C[6] = C[6] / (-830).$$

Appendices F and G together provide a detailed description of THSVER8.MOD.

IV. RESULTS

The purpose of this chapter is to discuss the significant results of the simulation study. The first section provides a comparison of the simulated results with the theoretical trapezoidal distribution of the time-weighted inventory position presented in Chapter III. The next section discusses the safety level analysis and compares the safety level calculated by the UICP model with the proposed method of defining the safety level described in Chapter III. Finally, the third section presents a discussion of the results of the simulated aggregate net inventory distribution.

A. TIME-WEIGHTED INVENTORY POSITION PROBABILITY DISTRIBUTION

To validate the probability distribution of the time-weighted inventory position which was described in Chapter III, a set of batched simulations were run using THSVER8.MOD. The variable IP was traced on all the events where its value changed; i.e., DMAND, ATTR2, INPOS, and ORDER. There were a total of 181 runs, one run with CRR equalling zero and the remaining 180 were the combinations of varying CRR from 0.2 to 1.0 in steps of 0.2, RSR from 0.0 to 1.0 in steps of 0.2, and REP from 0.0 to 1.3 in steps of 0.25. Then, for each set of parameters, the theoretical trapezoidal probability distribution discussed in Chapter III was compared to the

results. A Chi-square goodness of fit test was attempted to determine if the simulation results would validate the theoretical distribution. For this test the "observed frequencies" resulting from the simulation model were generated by determining the total length of time that each value of the inventory position occurred during the 830 periods. Thus, the frequency values are not integer. The Chi-square test procedure assumes integer frequency values. As a consequence, this use of the Chi-square goodness of fit test "stretches" the theory and is not exact. However, no other goodness of fit test exists for this type of frequency determination.

The results of the Chi-square test for "goodness of fit" strongly supports the theoretical model. Only the simulation runs where CRR equalled zero, or when CRR equalled one and RSR equalled zero or one, did the results fail the test (the Chi-square statistics were greater than 500 resulting in a p-value of zero). For these cases, the inventory system reduces to the consumable model of Reference 3, and as such, the inventory position is Uniformly distributed.

The remaining simulation runs resulted in a Chi-square statistic for each case well below the value of 27.7 or 22.4 which are the Chi-square critical values at 0.01 and 0.05 significance levels, respectively, with 13 degrees of freedom. (The 13 degrees of freedom were derived from the fact that there were 14 possible values IP could take and none of the

expected frequencies were computed using any estimated data. Therefore, the degrees of freedom were $14 - 1 = 13$.) This leads to the non-rejection of the null hypothesis that the empirical distribution of the simulated data follows this trapezoidal distribution. The test results are presented in Appendix I.

An important result of this analysis is that while the Chi-square statistic changes with REP, it is not statistically significant. As described in Chapter III, the theoretical distribution assumes REP equals zero in deriving the formulas for D_1 and D_2 . The variable REP introduces a delay in attrition accumulation. However, for these simulations, REP does not seem to have an effect on the inventory position distribution. Even when REP has its strongest influence on the inventory system (i.e., when all the carcasses are repaired ($RSR = 1.0$) and REP equals the repair turnaround-time (T_2) of 1.3 quarters) the Chi-square statistic did not change significantly. This would suggest, then, that the model discussed in Chapter III is robust. However, further analysis involving changes in Q_p , Q_R , D , L , and T_2 are needed before this claim can be fully justified.

B. SAFETY LEVELS

As noted in assumption (12) of Chapter III, safety level is defined as the expected value of the net inventory at the instant a procurement order or successfully repaired carcass

is received at a stock point. To find this aggregate safety level for the inventory system being modeled, THSVER7.MOD was used for a set of batched runs which varied the parameters CRR, RSR, and REP as mentioned above. The resulting simulated safety levels are found in Appendix J.

A "theoretical" integrated repairables model safety level using the Navy's UICP formulas was also computed using the same range of values for CRR and RSR. These values take into account the dual inventory positions established in UICP as described in Chapter III, section A. As discussed in Chapter III, equation (56) shows the maximum inventory position is, in fact, equal to the sum of the maximum inventory positions of the repair and procurement processes:

$$SW = SW_R + SW_P. \quad (56)$$

Under the UICP model, the maximum values of SW_R and SW_P occur at the instant a repair order or a procurement order is generated, respectively. The UICP model describes the maximum inventory positions for each part as

$$SW_R = Z_2 + \text{SAFETY LEVEL} + Q_R \quad (68)$$

and

$$SW_P = Z + \text{SAFETY LEVEL} + Q_P, \quad (69)$$

where the safety level is the same in both cases for the integrated repairables model, as explained in Chapter III [Ref. 6].

By substituting equations (68) and (69) into equation (56) one can compute the UICP total safety level, which is the sum of the two safety levels shown above, as:

$$TOTAL\ SAFETY\ LEVEL = SW - (Z + Z_2 + Q_P + Q_R); \quad (70)$$

when SW , Q_P , and Q_R are fixed, and Z and Z_2 are computed using the formulas from Chapter II. These results are compared with the simulated results for $REP = 0.0$ in Table 1. The $REP = 0.0$ case results in all repaired carcasses in a batch being returned after a time interval of T_2 . Interestingly, in almost all cases, the simulated safety levels exceeded the UICP safety levels. They even exceeded the values of the UICP safety levels divided by two when $RSR \geq 0.2$. This suggests that the UICP does not accurately reflect the "real world" of the depot level repairables inventory system and, in fact, would understate the actual aggregate safety levels.

C. TIME-WEIGHTED NET INVENTORY

1. Net Inventory Distribution

The Hadley-Whitin Consumable item inventory system described in Chapter II and Reference 3 allows backorders and, in particular, incorporates the expected number of unit years of shortage (i.e., time-weighted backorders) incurred per year in the backorders cost term of the Total Variable Cost equation. Of course, the expected time-weighted on-hand

D = 9 QR = 5
T2 = 1.3 QP = 10
L = 8.2 SW = 72
Z2 = 11.7 REP = 0

RSR	CRR	Z	AGGREGATE SAFETY LEVEL	(HALF) SAFETY LEVEL	SIMULATED SAFETY LEVEL
0	0	73.8	-28.5	-14.25	-12.4
0	0.2	73.8	-28.5	-14.25	-15.1
0	0.4	73.8	-28.5	-14.25	-14.5
0	0.6	73.8	-28.5	-14.25	-14.8
0	0.8	73.8	-28.5	-14.25	-12.8
0	1.0	73.8	-28.5	-14.25	-13.2
0.2	0	73.8	-28.5	-14.25	-12.4
0.2	0.2	71.316	-26.016	-13.008	-11.7
0.2	0.4	68.832	-23.532	-11.766	-8.14
0.2	0.6	66.348	-21.048	-10.524	-5.56
0.2	0.8	63.864	-18.564	-9.282	-0.55
0.2	1	61.38	-16.08	-8.04	0.38
0.4	0	73.8	-28.5	-14.25	-12.4
0.4	0.2	68.832	-23.532	-11.766	-8.82
0.4	0.4	63.864	-18.564	-9.282	-2.91
0.4	0.6	58.896	-13.596	-6.798	2.21
0.4	0.8	53.928	-8.628	-4.314	9.66
0.4	1	48.96	-3.66	-1.83	13.4
0.6	0	73.8	-28.5	-14.25	-12.4
0.6	0.2	66.348	-21.048	-10.524	-6.36
0.6	0.4	58.896	-13.596	-6.798	2.32
0.6	0.6	51.444	-6.144	-3.072	9.5
0.6	0.8	43.992	1.308	0.0654	18.8
0.6	1	36.54	8.76	4.38	25.6
0.8	0	73.8	-28.5	-14.25	-12.4
0.8	0.2	63.864	-18.564	-9.282	-3.78
0.8	0.4	53.928	-8.628	-4.314	7.3
0.8	0.6	43.992	1.308	0.654	16.9
0.8	0.8	34.056	11.244	5.622	28.6
0.8	1	24.12	21.18	10.59	38.3
1	0	73.8	-28.5	-14.25	-12.4
1	0.2	61.38	-16.08	-8.04	-1.05
1	0.4	48.96	-3.66	-1.83	12.3
1	0.6	36.54	8.76	4.38	24.5
1	0.8	24.12	21.18	10.59	38.1
1	1	11.7	33.6	16.8	55.0

TABLE 1. COMPARISON OF UICP COMPUTED SAFETY LEVELS WITH SIMULATED SAFETY LEVELS.

inventory is included in the holding costs term. These terms affect both the determination of the Order quantity and Reorder Level.

The expected unit years of on-hand and shortages can be obtained from the probability distribution of time-weighted

net inventory (i.e., $\psi(x)$ for all $-\infty < x < x_{\max}$). The formulas for the time-weighted expected on-hand inventory (O/H) and time-weighted expected number of backorders (B/O) are given by equations (71) and (72), respectively:

$$O/H = \sum_{x=0}^{SW} x\psi(x) \quad (71)$$

$$B/O = -\sum_{x=-\infty}^0 x\psi(x) . \quad (72)$$

For the consumable model, when demand is generated by a Poisson process, lead time is deterministic, and the lead time demand is large, the probability distribution of demand during lead time can be approximated using the Normal distribution. [Ref. 3:p. 192] The probability density function for the net inventory is then,

$$\psi(x) = \frac{1}{Q} \left[\phi \left(\frac{r - x - \mu}{\sigma} \right) - \phi \left(\frac{r - Q - x - \mu}{\sigma} \right) \right], \quad (73)$$

for all $-\infty \leq x \leq SW$,

where Q is the order quantity, r is the reorder level, μ is the mean lead time demand, $\sigma^2 = \mu$, and $\phi(x)$ is the complementary cumulative distribution for the Standard Normal function. Thus, $\psi(x)$ is not Normally distributed.

Equation (73) can then be used to compute the expected on-hand ($E(O/H)$) and expected backorder ($E(B/O)$) quantities. Hadley and Whitin derived the equation for expected on-hand. [Ref. 3:p. 194] This equation is given as

$$\begin{aligned}
 E(O/H) &= \int_0^{x_{MAX}} x \psi(x) \\
 &= \frac{Q}{2} + r - \mu + E(B/O) .
 \end{aligned}
 \tag{74}$$

Because

$$E(N/I) = E(O/H) - E(B/O) , \tag{75}$$

the expected net inventory can be computed as

$$E(N/I) = \frac{Q}{2} + r - \mu \tag{76}$$

by substituting equation (74) into (76).

Equation (76) was then used to calculate the theoretical mean of net inventory for the two limiting cases (i.e., when CRR = 0.0, or when CRR = RSR = 1.0 and REP = 0.0), for the purpose of comparison with the simulated results. The values used in equation (76) when CRR = 0.0 (the completely consumable case) were

$$\begin{aligned}
 Q &= Q_P = 10 \\
 r &= x_{MAX} - Q_P = 72 - 10 = 62 \\
 \mu &= D * L = 9 * 8.2 = 73.2 .
 \end{aligned}$$

Therefore, $E(N/I) = -6.80$. When CRR = RSR = 1.0 and REP = 0.0 (the completely repairable case), the values used in equation (76) were

$$\begin{aligned}
 Q &= Q_R = 10 \\
 r &= x_{MAX} - Q_R = 72 - 5 = 67 \\
 \mu &= D * T_2 = 9 * 1.3 = 11.7 .
 \end{aligned}$$

Therefore, $E(N/I) = 57.8$.

a. Simulated Expected Value of Net Inventory

The expected values of the simulated time-weighted net inventory for the various values of the parameters CRR, RSR, and REP were generated using THSVER8.MOD as the simulation vehicle and are presented in Appendix K. The resulting values for the limiting cases were then compared with the calculated ones using equation (76). When CRR was zero, the value of the simulated expected or mean net inventory was -6.92 while the calculated theoretical value was -6.80; when CRR and RSR were one and REP was zero, the simulated mean net inventory was 58.1 while the calculated value was 57.8. This validates the simulation model results for these cases.

b. Simulated Net Inventory Distribution

Net inventory distributions for various values of CRR, RSR and REP were generated and graphed as histograms. The initial graphs appeared bell-shaped (see Appendix L), so a Normal probability distribution was "fitted" using the AGSS software. The Chi-square results are displayed in Table 2. Rather than showing the critical Chi-square values for $\alpha = 0.05$ and 0.01 , the AGSS software provides the alpha value which would be needed to give the computed Chi-square statistic. An α value of 1.0 means that virtually all the Chi-square distribution for the given degrees of freedom was to the right of the computed statistic. Thus, the probability

CRR	RSR	REP	χ^2	α	ν
0	1	1.0	10.608	1.00	44
.2	0	0	13.176	1.00	44
.2	0	.25	13.176	1.00	44
.2	.6	0	14.82	.99996	42
.2	.6	.5	14.431	.99997	42
.2	.6	1.3	15.642	.99995	43
.2	.8	0	15.345	.9999	41
.2	.8	.5	17.364	.99953	41
.4	0	0	10.886	1.0	41
.4	.4	0	10.586	1.0	38
.4	.4	.5	7.883	1.0	38
.4	.4	1.3	9.019	1.0	38
.4	.6	0	13.08	.99994	38
.4	.6	.5	16.54	.99846	37
.4	.6	1.3	14.77	.99956	37
.4	.8	0	9.52	1.0	36
.4	.8	.5	9.757	1.0	36
.4	.8	1.3	11.511	.99998	37
.6	0	0	21.165	.99362	40
.6	.2	0	20.539	.99057	38
.6	.2	1.3	18.174	.99814	39
.6	.4	0	16.288	.99916	38
.6	.4	.5	21.946	.9873	39
.6	.4	1.3	26.656	.91639	38
.8	0	0	12.768	1.0	44
.8	.2	0	9.021	1.0	41
.8	.2	.5	9.864	1.0	41
.8	.2	1.3	8.435	1.0	42
.8	.4	0	24.661	.95339	38
.8	.4	.5	17.237	.99897	39
.8	.4	1.3	10.818	1.0	40
.8	.8	0	11.578	.99894	30
.8	.8	.5	18.258	.96607	30
.8	.8	1.3	10.723	.9999	33
1	0	0	9.414	1.0	43
1	.2	0	16.328	.99945	39
1	.2	.5	14.691	.99991	40
1	.2	1.3	19.504	.9973	40
1	.8	0	10.782	.99768	27
1	.8	.5	16.037	.96528	28
1	.8	1.3	21.677	.91587	32
1	1	0	23.66	.20951	19
1	1	.5	23.342	.32601	21
1	1	1.3	16.449	.9008	25

TABLE 2. SUMMARY OF CHI-SQUARE STATISTICS FOR FITTING NORMAL CURVE TO SIMULATED TIME-WEIGHTED NET INVENTORY DISTRIBUTIONS.

of making a Type I error is virtually zero in accepting the null hypothesis that the data is Normally distributed at any α level. The lowest value of α in Table 1 was 0.20951, which

means that approximately 21% of the area of the Chi-square distribution was to the right of the computed statistic. Thus, the null hypothesis is again strongly accepted at the $\alpha = 0.05$ and 0.01 level since it would be accepted for any significance level less than 0.20951.

In conclusion, the results of Chi-square goodness of fit tests in all cases suggest that one cannot reject the null hypothesis that the net inventory distribution fits the Normal distribution.

What does this conclusion imply about the distribution given by equation (73) for the limiting case of $CRR = 0.0$ (the completely consumable case)? As noted earlier, equation (73) is not a Normal distribution. It would only be Normal if $Q = 1.0$, and that was not the case in the simulation since $Q_p = 10.0$. Perhaps it is close to Normal because of the parameter values selected for the simulation. At this time nothing further can be concluded. However, the result suggests that further analysis of that formula is appropriate to see under what general conditions it can be approximated by a Normal distribution.

2. Sensitivity Analysis of CRR, RSR, and REP.

a. Mean and Variance as a function of CRR, RSR, and REP

Assuming, from the above results, that net inventory is Normally distributed, the next step is to examine

the values of the two distribution parameters, the mean and the variance, as a function of the parameters which were varied; namely CRR, RSR, REP. The data was generated using THSVER8.MOD. The values of the means and variances of the simulated distributions for net inventory are summarized in Appendix K. Using the data from Appendix K, graphs were first produced for the distribution means; graphs for the variances were then added. These graphs are paired in Appendix M. In these graphs, one parameter is fixed; the second parameter represents the independent variable for plots of curves for fixed values of the third. Each line or curve is identified by an integer which represents a unique value of this third parameter. When the third parameter is REP, a line or curved identified with a "0" means $REP = 0.0$; "1" means $REP = 0.25$; "2" means $REP = 0.5$; "3" means $REP = 0.75$; "4" means $REP = 1.0$; "5" means $REP = 1.3$. When the third parameter is CRR or RSR, the integer identifying the line or curve represents the value of CRR or RSR where a "0" means 0; "1" means 1.0; "8" means 0.8; "6" means 0.6; etc.

In addition to these graphs, because the UICP model uses the product of CRR and RSR to forecast attrition demand, plots of the mean and variance of the net inventory distribution for various values of the product of CRR and RSR were made. These graphs are provided in Appendix N.

The first 23 graphs of Appendix M examine the influence of REP for fixed values of CRR and RSR. As REP

increases, the mean of the distribution for net inventory decreases because the carcasses of an inducted batch are delayed longer before returning to RFI, and its variance increases because of the same delay. No delay occurs, of course, when $REP = 0$. The graphs show a completely linear relationship between REP and the distribution mean and a fairly linear relationship for the variance as a function of REP .

The REP curves for the different RSR values show that as RSR increases so does the mean of the net inventory because the repair process is providing more and more of the RFI units, and it has a much shorter "lead time" (repair turnaround time) than procurement. The variance of net inventory decreases for the same reason. The REP curves for different CRR values show that as CRR increases for $RSR \geq 0.4$ the net inventory mean and variance behave similarly for the same reason.

In the next 22 graphs of Appendix M the relationship between RSR and the distribution mean and variance appears to be almost linear except in the case where $CRR = 1.0$ and RSR approaches 1.0. As noted above, when RSR increases so does the mean, while the variance decreases. Interestingly, the variance curves for various REP values are quite close. A review of the graphs for both the mean and variance when REP is fixed shows the crossing of RSR curves for various CRR values when RSR is less than 0.4 and 0.6 for

the mean and variance of the net inventory, respectively. This CRR effect is examined next.

The final 24 graphs of Appendix M show plots of the distribution mean and variance as a function of CRR. The first 12 are for fixed REP. No monotonistically increasing behavior was observed for the means and the variances until $RSR \geq 0.8$. Below RSR of 0.8 the variances are very erratic. This is more noticeable in the second 12 graphs where RSR is fixed. The distribution means and variances are not linear in CRR even when $RSR = 1.0$. The worst case is when $RSR = 0.0$. Notice that when $RSR = 0.0$, the largest mean occurs when $CRR = 0.0$, and the smallest occurs when $CRR = 0.2$. Also, a larger mean is achieved when $CRR = 0.8$ than when $CRR = 1.0$. In addition, the distribution mean is not an increasing function of CRR until $RSR \geq 0.4$. The distribution variance has a similar behavior to the mean when $CRR \geq 0.6$ and the opposite for $CRR < 0.6$.

A review of the graphs of the distribution mean as a function of CRR when RSR is small suggests an explanation which may have some credibility. When $CRR = 0$, there is only one queue affecting the system; i.e., the attrition queue. When carcasses are returned ($CRR > 0$), both the repair and the attrition queues affect the system. The drop in the value of the mean and the increase in the variance values for $RSR < 0.6$ can be attributed to the fact that because the repair queue fills at such a slow rate, the delay in identifying non-

repairable carcasses which then pass to the attrition queue allows so much time to pass that demands "eat up" the net inventory. In other words, attritions are waiting in the repair queue and will not be counted as such until identified in the induction process. From an operational point of view these results suggest that it is better not to return any carcasses when the repair survival rate is very low.

As the carcass return rate increases for $RSR \geq 0.4$, the repair process has a greater effect on the system and, because repair turn-around time is shorter than procurement lead time, the value of the mean increases. However, RSR of at least 0.8 is needed before the values of the variances decrease as a function of CRR over its full range of values. When $CRR = 1.0$, again only one queue affects the system; i.e., the repair queue.

b. Distribution Mean and Variance as a function of the product of CRR and RSR

The sensitivity analysis of the effect of the product of CRR and RSR on the net inventory's mean and variance was also conducted using the data of Appendix K. Because UICP uses $(1 - CRR * RSR)$ to compute the expected attrition demand, plots of the distribution mean and variance as a function of the product of CRR and RSR were generated for fixed values of REP. Because these plots appeared "linear", a regression line was fitted to each plot using the AGSS

software and, as previously stated, AGSS generated the associated ANOVA tables. The plots are provided in Appendix N. A summary of the test results are shown in Tables 3 and 4.

REP	F_R	R^2	F_{LOF}
0.00	6690.	.996	2.548
0.25	5936.	.995	2.453
0.50	5237.	.995	2.301
0.75	4462.	.994	2.126
1.00	3703.	.992	1.973
1.30	2608.	.989	1.903

TABLE 3. STATISTICAL SUMMARY RESULTS OF THE MEAN VS PRODUCT OF CRR AND RSR.

REP	F_R	R^2	F_{LOF}
0.00	332.6	.922	.230
0.25	310.9	.917	.244
0.50	277.7	.908	.267
0.75	252.0	.900	.281
1.00	218.5	.886	.326
1.30	175.7	.863	.374

TABLE 4. STATISTICAL SUMMARY RESULTS OF THE VARIANCE VS THE PRODUCT OF CRR AND RSR.

Two tests were performed by the AGSS software, one for the regression and the other for the "Lack of Fit".

A quick look at the fitted lines for the various values of REP indicate that there does indeed appear to be a linear relationship between the distribution mean and the product of CRR and RSR. This conclusion is supported, though not strongly, by the results of the regression and lack of fit

tests. In the regression test the null hypothesis is that the distribution mean is not a function of the product; namely, the slope of the regression line, B_1 , is zero. At $\alpha = .05$ and $.01$, and with the combined degrees of freedom of 1 and 28, the critical F-statistics are $F_{95,1,28} = 4.196$ and $F_{99,1,28} = 7.636$, respectively. Note in Table 3 that in all cases the F-statistic (F_R), which ranges from 2,608 to 6,690, is much greater than these critical values. Therefore, we would reject the null hypothesis.

For the test of lack of fit, the null hypothesis becomes $H_0: E(y) = B_0 + B_1x$, a straight line. The alternative hypothesis, H_1 , is that $E(y)$ is some other function of x . At $\alpha = .05$ and $.01$ with the combined degrees of freedom of 13 and 15 the critical values are $F_{95,13,15} = 2.45$ and $F_{99,13,15} = 3.615$, respectively. Referring to Table 3, we would accept the null hypothesis at the $.01$ significance level because all the F-statistics (F_{LOF}) are less than the critical value. However, for a significance level of $.05$, we would reject the null hypothesis for the cases where $REP = 0$ and $REP = .25$ because their F-statistics are greater than the critical value. We would not reject the null hypothesis in the cases where $REP = 0.5, 0.75, 1.0$ and 1.3 because their F-statistics were less than the critical value. The results of these tests indicate that the mean is very close to being a linear function of the product of CRR and RSR (indeed, the R^2 values are all very close to one), but there is likely at least one other term

involved for low REP values. Notice that in all plots the $CRR * RSR = 1.0$ point is the point furthest from the fitted line. The reason for this is not obvious.

The same hypothesis testing as was done for the mean was performed for the variance, and the results are presented in Table 4. The regression null hypothesis (i.e., slope of the regression line is zero) for all values of REP is rejected because all the observed F-statistics are greater than the critical statistic, $F_{9,1,28} = 7.636$ for the 0.01 significance level. The null hypothesis that the variance is a linear function of the $CRR * RSR$ product is accepted for all values of REP because the observed F-statistics are all greater than the critical F-statistics, $F_{9,13,15} = 3.615$.

Finally, the regression lines of the means and variances for each value of REP was plotted in the same graph to determine if there was any relationship between the mean and variance. These graphs are presented in Appendix O. The key relationship indicated from these observations is that the variance to mean ratio is extremely large when the product of CRR and RSR is small; and as the product increases, the ratio decreases. Also noted is the fact that the product value where the lines intersect (and the variance to mean ratio is 1.0) increases as REP increases.

V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A. SUMMARY

The current Integrated Repairables Model which is used by the Navy's Inventory Control Points was first conceived in the late 1970's and implemented in 1984. Its development was intended as an interim "patch" to the previous model (developed in the late 1960's) because there tended to be a problem of not having enough carcasses available to induct for repair when the economic repair quantity and the old repair reorder point were used. Its approach was intuitively simple; i.e., it provided a common safety stock for both the repair and procurement reorder points. This safety stock was derived from a RISK formula which was based on a weighted average of the unit costs to procure and repair. It also combined the procurement lead time and the repair turn-around time with the same weights to generate an average lead time for the procurement part of the model. A Total Variable Cost equation was postulated but the details of all of the terms were never worked out so that optimization could justify the formulas for the reorder points. In addition, the order quantity formulas remained the same as those from the model used prior to the integrated model.

The problem with the old and integrated repairables models was that the models as detailed in the Functional Description of the CARES (Computation and Research Evaluation System) III of UICP were really two models. One model was for procurement of new units and the other was for repair of old ones. These models had separate reorder points associated with "fictitious" inventory positions. The reality is that there is only one aggregate inventory made up of new procured and old repaired units.

This thesis takes the first step in modelling that single inventory problem. It takes a different approach from the classical continuous review inventory model which uses reorder points to trigger repair and procurement orders. Instead, it collects demands for two "economic order quantities", one for the procurement of new items, and the other for the repair of the broken ones. It does this by creating two queues; the repair queue, which accumulates NRFI carcasses, and the attrition queue, which accumulates attritions from either the non-receipt of a carcass from a customer or a condemnation of a carcass at a maintenance activity. A repair order is generated when the repair queue reaches a level corresponding to the economic repair quantity, then the contents of that queue are reset to zero. The attrition queue behaves similarly. Thus, there is no need for a "reorder level" per se.

The major tasking of this study was to develop a simulation program which accurately models the repair process and incorporates the queues just mentioned. This simulation models the repair process as it really is at an organic depot. A carcass is inducted, determined to be repairable or not, and then sent either on through repair or sent immediately to the attrition queue. Each carcass going through repair takes the same specified repair turn-around time to complete the process. However, when a batch of carcasses is "inducted", each unit must, in fact, wait its turn for induction and that wait is not part of its repair turn-around time.

The purpose of the simulation was to generate the appropriate data for use in analyzing the effects of the procurement and repair processes on the aggregate inventory position probability distribution, the net inventory probability distribution, and the safety level. In particular, three separate analyses were conducted. In the first, inventory position values were simulated for different values of the three parameters, the Carcass Return Rate (CRR), the Repair Survival Rate (RSR), and the Repair Processing Time (REP). The resulting probability distributions were compared to the theoretical trapezoidal distribution described in Chapter III, which had been developed by Professor A. W. McMasters of the Naval Postgraduate School. Next, a way to define the aggregate safety level was discovered from studying simulation plots of net inventory. Then simulated safety

levels were compared to the UICP computed safety levels. Finally, net inventory distributions were generated for various values of the three previously mentioned parameters as a first step in developing an analytical model of the aggregate net inventory probability distribution.

B. CONCLUSIONS.

1. Inventory Position.

The results of the comparison of the simulated inventory position distributions with the proposed trapezoidal distribution were quite revealing. For the limiting cases (i.e., when the Carcass Return Rate (CRR) equals zero; CRR equals one and the Repair Survival Rate (RSR) is zero; or when both CRR and RSR are one.) the trapezoidal model does not fit the data. This, however, was expected because under those conditions the system parallels the consumable inventory system where the inventory position distribution is known to be uniformly distributed. [Ref. 3] However, for the "in between" cases, (i.e., those where repair and procurement processes are both active) the trapezoidal distribution fits the data extremely well even though the simplifying assumptions under which it was derived are not met. Thus, the trapezoidal model is quite robust.

2. Safety Level.

The results of the comparison of the simulated safety levels with those of the UICP model indicate that in all but

two cases, the simulated safety level was greater than the total safety level (i.e., the sum of the two reorder point safety levels of UICP). Thus, the total safety level computed in CARES III understates reality.

3. Net Inventory

As noted previously, in the Hadley-Whitin consumable model which assumes demand to be a Poisson process and lead time to be deterministic, one can approximate the demand during lead time by using the Normal Distribution with μ equal to mean lead time demand and $\sigma^2 = \mu$. The formula for the net inventory distribution is then given in Hadley and Whitin [Ref. 3] and it is not Normal. Interestingly though, the fitting of the Normal distribution to the various simulated net inventory distributions proved to be very good in all cases. Although one cannot conclude yet that the net inventory distribution can be approximated by the Normal distribution for the aggregate repairables model for all parameter values affecting the model, this result certainly whets the appetite for further investigation.

From the sensitivity analysis of the three parameters, CRR, RSR and REP, it appears that REP has a linear effect on the mean and variance of the net inventory distribution; as REP increases, the mean decreases, while the variance increases most of the time, particularly when CRR and RSR are greater than or equal to 0.8. The effects on the mean and

variance of the net inventory distribution of the individual parameters, CRR and RSR, were not linear. As expected, the mean was an increasing function of each parameter except for small values of CRR. Similarly, the variance was a decreasing function of each parameter except for small values of CRR.

Because the UICP model uses CRR and RSR as a product, a sensitivity analysis was conducted to analyze the effect of that product on the mean and variance of the net inventory distribution. The results showed a statistically stable linear form for both, with the mean increasing and the variance decreasing as the product value increased.

C. RECOMMENDATIONS.

This thesis represents the "tip of the iceberg" in the analysis of the aggregate repairables inventory system and the development of an analytical model of it. Because the Navy's ICP's intend to become the Repairables "Center of Excellence", they must be able to manage such items effectively. The model for the theoretical aggregate repairables inventory system can aid in that process.

Now that a simulation model exists, more sensitivity analyses of the inventory position and net inventory probability distributions must be accomplished. Clearly, the effect of low values of CRR on the net inventory distribution parameters needs to be better understood. In addition, major

sensitivity analyses of these distributions must include the effects caused by the following:

1. High, medium and low quarterly demand rates;
2. Actual empirical demand distributions;
3. Long, medium, and short procurement lead times;
4. Long, medium, and short repair turn-around times; and,
5. Stochastic procurement lead times and repair turn-around times.

It is also recommended that the Naval Postgraduate School and the Naval Supply Systems Command share in the research effort so that it is accomplished in an expeditious manner.

APPENDIX A

MODEL DEFAULTS

Model Name: THESIS
Model Description: DEPOT LEVEL REPAIRABLES
Output File: THSVER4.OUT
Run Mode: HIGH SPEED
Trace Vars: IP, INV

Random Number Seed: 18645
Initial Values: 8.2, 1, .8, 1.3, 36, 10, 5, 9, .50
Ending Condition: STOP_ON_EVENT
Event: RUN Number to Run: 1
Trace Events: ALL EVENTS TRACED

STATE VARIABLES

State Variable #1

Name: L
Description: PROCUREMENT LEAD TIME
Type: REAL
Size: 1

State Variable #2

Name: CRR
Description: CARCASS RETURN RATE
Type: REAL
Size: 1

State Variable #3

Name: RSR
Description: REPAIR SURVIVAL RATE OF A CARCASS.
Type: REAL
Size: 1

State Variable #4

Name: T2
Description: REPAIR TURNAROUND TIME
Type: REAL
Size: 1

State Variable #5
Name: D
Description: AVERAGE DEMAND
Type: REAL
Size: 1

State Variable #6
Name: QP
Description: PROCUREMENT ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #7
Name: QR
Description: REPAIR ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #8
Name: INV
Description: ACTUAL ON HAND INVENTORY
Type: INTEGER
Size: 1

State Variable #9
Name: IP
Description: INVENTORY POSITION
Type: INTEGER
Size: 1

State Variable #10
Name: QUEREP
Description: NUMBER OF UNITS IN QUEUE FOR REPAIR
Type: INTEGER
Size: 1

State Variable #11
Name: QUEORD
Description: NUMBER OF UNITS ON ORDER
Type: INTEGER
Size: 1

State Variable #12
Name: RQUE
Description: NUMBER OF UNITS IN QUEUE FOR A REPAIR ORDER
Type: INTEGER
Size: 1

State Variable #13

Name: TIME
Description: LENGTH OF TIME FOR ONE RUN OF THE SIMULATION.
Type: REAL
Size: 1

State Variable #14

Name: RUN
Description: COUNTER FOR THE NUMBER OF RUNS
Type: REAL
Size: 1

State Variable #15

Name: ATTR
Description: CUMULATIVE TOTAL OF CONDEMN/LOST CARCASSES
Type: INTEGER
Size: 1,1

State Variable #16

Name: X
Description: HOLDER OF A RANDOM VARIABLE FOR A STATE.
Type: REAL
Size: 3

State Variable #17

Name: REP
Description: EVALUATION PROCESSING TIME FOR REPAIR.
Type: REAL
Size: 1,1

State Variable #18

Name: COUNT
Description: NUMBER OF REPAIRABLE CARCASSES IN A BATCH
Type: INTEGER
Size: 1,1

VERTICES

Vertex #1

Name: RUN
Description: START OF THE SIMULATION
State Changes: RUN=1
Parameter(s): L,CRR,RSR,T2,TIME,QP,QR,D,REP

Vertex #2
Name: DMAND
Description: DEMAND OF A DEPOT LEVEL REPAIRABLE
State Changes: INV=INV-1, X[1]=RND, IP=IP-1
Parameter(s):

Vertex #3
Name: ORDER
Description: ORDERING OF QP UNITS OF NEW MATERIAL
State Changes: IP=IP+QP, QUEORD=QUEORD+QP, ATTR=0
Parameter(s):

Vertex #4
Name: RCVD2
Description: RECEIPT OF AN ORDER OF QP UNITS
State Changes: INV=INV+QP, QUEORD=QUEORD-QP
Parameter(s):

Vertex #5
Name: CARTN
Description: CARCASS RETURNING TO THE STOCK POINT
State Changes: RQUE=RQUE+1, COUNT=1, IP=IP+RQUE*(RQUE==QR),
QUEREP=QUEREP+(RQUE==QR)*RQUE, X[2]=0
Parameter(s):

Vertex #6
Name: RPORD
Description: DECISION TO REPAIR OR CONDEMN A CARCASS
State Changes: X[0]=RND, RQUE=RQUE-1
Parameter(s):

Vertex #7
Name: DEPOT
Description: DEPOT REPAIRING A CARCASS
State Changes:
Parameter(s):

Vertex #8
Name: ATTRI
Description: LOSS OR CONDEMNATION OF A CARCASS
State Changes: ATTR=ATTR+1
Parameter(s):

Vertex #9
Name: STATS
Description: ACCUMULATION OF SIMULATION RUNS
State Changes: RUN=RUN+1
Parameter(s):

Vertex #10

Name: INIT
Description: INITIALIZE IP AND INV FOR EACH RUN
State Changes: INV=72, IP=INV
Parameter(s):

Vertex #11

Name: ATTR1
Description: CAPTURING OF THE IP JUST BEFORE AN ATTRITION
State Changes: QUEREP=QUEREP-1, X[2]=1
Parameter(s):

Vertex #12

Name: RCVD1
Description: RECORDING OF THE INV VALUE JUST BEFORE A RECEIPT
State Changes:
Parameter(s):

Vertex #13

Name: SHIP1
Description: RECEIPT OF A REPAIRED CARCASS
State Changes: INV=INV+1.
Parameter(s):

Vertex #14

Name: DMND1
Description: CAPTURING OF THE INV AND IP JUST BEFORE A DEMAND
State Changes:
Parameter(s):

Vertex #15

Name: RPAIR
Description: INV VALUE JUST BEFORE RECEIPT OF A REPAIRED UNIT
State Changes: QUEREP=QUEREP-1
Parameter(s):

Vertex #16

Name: ATTR2
Description: REDUCTION OF IP DUE TO A REPAIR ATTRITION
State Changes: ATTR=ATTR+1, IP=IP-1
Parameter(s):

Vertex #17

Name: COUNT
Description: NUMBER OF SUCCESSFUL REPAIRS IN A BATCH
State Changes: COUNT=COUNT+ (X[0] <=RSR)
Parameter(s):

Vertex #18

Name: INPOS
Description: INCREASE OF IP BY INDUCTING A BATCH IN REPAIR
State Changes: IP=IP
Parameter(s):

EDGES

Edge #1

Description: A CARCASS IS SUCCESSFULLY RETURNED
Type: Scheduling
Origin: DMAND
Destination: CARTN
Condition: $X[1] \leq CRR$
Delay: 0
Priority: 3
Attributes:

Edge #2

Description: BEGIN THE SIMULATION
Type: Scheduling
Origin: RUN
Destination: INIT
Condition: $1 == 1$
Delay: 0
Priority: 5
Attributes:

Edge #3

Description: START THE NEXT SIMULATION
Type: Scheduling
Origin: STATS
Destination: INIT
Condition: $1 == 1$
Delay: 0
Priority: 5
Attributes:

Edge #4

Description: END THE RUN OF THE SIMULATION
Type: Scheduling
Origin: INIT
Destination: STATS
Condition: $1 == 1$
Delay: TIME
Priority: 5
Attributes:

Edge #5
Description: CANCEL ALL FUTURE DEMANDS
Type: Cancelling
Origin: STATS
Destination: DMAND
Condition: 1==1
Delay: 0
Priority: 5
Attributes: *

Edge #6
Description: A CARCASS IS NOT RETURNED TO THE STOCK POINT
Type: Scheduling
Origin: DMAND
Destination: ATTRI
Condition: CRR<X[1]
Delay: 0
Priority: 3
Attributes:

Edge #7
Description: PROCURE QP UNITS
Type: Scheduling
Origin: ATTRI
Destination: ORDER
Condition: ATTR>=QP
Delay: 0
Priority: 3
Attributes:

Edge #8
Description: SCHEDULE THE RECEIPT OF QP UNITS
Type: Scheduling
Origin: ORDER
Destination: RCVD1
Condition: 1==1
Delay: L
Priority: 5
Attributes:

Edge #9
Description: ADD QP TO INVENTORY (INV)
Type: Scheduling
Origin: RCVD1
Destination: RCVD2
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #10

Description:
Type: Cancelling
Origin: STATS
Destination: RCVD1
Condition: 1==1
Delay: 0
Priority: 5
Attributes: *

Edge #11

Description: BEGIN THE DEMAND CYCLE OF THE SIMULATION
Type: Scheduling
Origin: INIT
Destination: DMND1
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #12

Description: SCHEDULE ANOTHER DEMAND
Type: Scheduling
Origin: DMAND
Destination: DMND1
Condition: 1==1
Delay: $-(1/D) * \text{LN}\{\text{RND}\}$
Priority: 5
Attributes:

Edge #13

Description: RECORD INV AND IP JUST PRIOR TO A DEMAND
Type: Scheduling
Origin: DMND1
Destination: DMAND
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #14

Description: SHIP THE RFI CARCASS TO THE STOCK POINT
Type: Scheduling
Origin: DEPOT
Destination: RPAIR
Condition: 1==1
Delay: T2
Priority: 3
Attributes:

Edge #15
Description: RECEIVE A JUST REPAIRED UNIT
Type: Scheduling
Origin: RPAIR
Destination: SHIP1
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #16
Description: SCHEDULE THE REPAIR ATTRITION
Type: Scheduling
Origin: RPORD
Destination: ATTR1
Condition: X[0]>RSR
Delay: REP*COUNT
Priority: 3
Attributes:

Edge #17
Description: SCHEDULE THE REPAIR OF A NRFI UNIT
Type: Scheduling
Origin: RPORD
Destination: DEPOT
Condition: X[0]<=RSR
Delay: REP*COUNT
Priority: 3
Attributes:

Edge #18
Description: SEND A BATCH OF QP NRFI CARCASSES TO A DEPOT
Type: Scheduling
Origin: CARTN
Destination: RPORD
Condition: RQUE>=QR
Delay: 0
Priority: 5
Attributes:

Edge #19
Description: INCREASE ATTR AND REDUCE IP BY ONE
Type: Scheduling
Origin: ATTR1
Destination: ATTR2
Condition: 1==1
Delay: 0
Priority: 7
Attributes:

Edge #20
Description: INCREASE THE COUNT OF RFI UNITS IN THE BATCH
Type: Scheduling
Origin: RPORD
Destination: COUNT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #21
Description: SCHEDULE REVIEW OF THE NEXT CARCASS IN THE BATCH
Type: Scheduling
Origin: COUNT
Destination: RPORD
Condition: RQUE>0
Delay: 0
Priority: 5
Attributes:

Edge #22
Description: PROCURE QP UNITS
Type: Scheduling
Origin: ATTR2
Destination: ORDER
Condition: ATTR>=QP
Delay: 0
Priority: 2
Attributes:

Edge #23
Description: RECORD THE IP AS A RESULT OF THIS BATCH
Type: Scheduling
Origin: CARTN
Destination: INPOS
Condition: RQUE==QR
Delay: 0
Priority: 8
Attributes:

APPENDIX B

The SIGMA Model, THESIS, is a discrete event simulation. It models depot level repairables.

I. STATE VARIABLE DEFINITIONS.

For this simulation, the following state variables are defined:

- L: procurement lead time (real valued)
- CRR: carcass return rate (real valued)
- RSR: repair survival rate of a carcass. (real valued)
- T2: repair turnaround time (real valued)
- D: average demand (real valued)
- QP: procurement order quantity (integer valued)
- QR: repair order quantity (integer valued)
- INV: actual on hand inventory (integer valued)
- IP: inventory position (integer valued)
- QUEREP: number of units in queue for repair (integer valued)
- QUEORD: number of units on order (integer valued)
- RQUE: number of units in queue for a repair order (integer valued)
- TIME: length of time for one run of the simulation. (real valued)
- RUN: counter for the number of runs (real valued)
- ATTR: cumulative total of condemn/lost carcasses (integer valued)
- X[3]: holder of a random variable for a state. (real valued)
- REP: evaluation processing time for repair. (real valued)
- COUNT: number of repairable carcasses in a batch (integer valued)

II. EVENT DEFINITIONS.

Simulation state changes are represented by event vertices (nodes or balls) in a SIGMA graph. Event vertex parameters, if any, are given in parentheses.

Logical and dynamic relationships between pairs of events are represented in a SIGMA graph by edges (arrows) between event vertices. Unless otherwise stated, vertex execution priorities, to break time ties, are equal to 5.

1. The RUN(L,CRR,RSR,T2,TIME,QP,QR,D,REP) event models the start of the simulation.

Initial values for, L, CRR, RSR, T2, TIME, QP, QR, D, REP, are needed for each run.

This event causes the following state change(s):

RUN=1

After every occurrence of the RUN event:

Unconditionally, begin the simulation;

that is, schedule the INIT() event to occur without delay.

2. The DMAND() event models the demand of a depot level repairable.

This event causes the following state change(s):

INV=INV-1

X[1]=RND

IP=IP-1

After every occurrence of the DMAND event:

If $X[1] \leq CRR$, then a carcass is successfully returned;
that is, schedule the CARTN() event to occur without delay.

(Time ties are broken by an execution priority of 3.)

If $CRR < X[1]$, then a carcass is not returned to the stock point;

that is, schedule the ATTRI() event to occur without delay.

(Time ties are broken by an execution priority of 3.)

Unconditionally, schedule another demand;

that is, schedule the DMND1() event to occur in
 $-(1/D) * \text{LN}\{RND\}$ time units.

3. The ORDER() event models the ordering of QP units of new material.

This event causes the following state change(s):

IP=IP+QP

QUEORD=QUEORD+QP

ATTR=0

After every occurrence of the ORDER event:

Unconditionally, schedule the receipt of QP units ;
that is, schedule the RCVD1() event to occur in L
time units.

4. The RCVD2() event models the receipt of an order of QP units.

This event causes the following state change(s):

INV=INV+QP

QUEORD=QUEORD-QP

No additional events are scheduled here.

5. The CARTN() event models the carcass returning to the stock point.
 This event causes the following state change(s):
 RQUE=RQUE+1
 COUNT=1
 IP=IP+RQUE*(RQUE==QR)
 QUEREP=QUEREP+(RQUE==QR)*RQUE
 X[2]=0
 After every occurrence of the CARTN event:
 If RQUE>=QR, then send a batch of QP NRFI carcasses to a depot;
 that is, schedule the RPORD() event to occur without delay.
 If RQUE==QR, then record the IP as a result of this batch;
 that is, schedule the INPOS() event to occur without delay.
 (Time ties are broken by an execution priority of 8.)

6. The RPORD() event models the decision to repair or condemn a carcass.
 This event causes the following state change(s):
 X[0]=RND
 RQUE=RQUE-1
 After every occurrence of the RPORD event:
 If X[0]>RSR, then schedule the repair attrition ;
 that is, schedule the ATTR1() event to occur in REP*(COUNT-1)
 time units.
 (Time ties are broken by an execution priority of 3.)
 If X[0]<=RSR, then schedule the repair of a NRFI unit;
 that is, schedule the DEPOT() event to occur in REP*(COUNT-1)
 time units.
 (Time ties are broken by an execution priority of 3.)
 Unconditionally, increase the count of RFI units in the batch;
 that is, schedule the COUNT() event to occur without delay.

7. The DEPOT() event models the depot repairing a carcass.
 After every occurrence of the DEPOT event:
 Unconditionally, ship the RFI carcass to the stock point;
 that is, schedule the RPAIR() event to occur in T2
 time units.
 (Time ties are broken by an execution priority of 3.)

8. The ATTRI() event models the loss or condemnation of a carcass.

This event causes the following state change(s):

ATTR=ATTR+1

After every occurrence of the ATTRI event:

If ATTR>=QP, then procure QP units;

that is, schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 3.)

9. The STATS() event models the accumulation of simulation runs.

This event causes the following state change(s):

RUN=RUN+1

After every occurrence of the STATS event:

Unconditionally, start the next simulation;

that is, schedule the INIT() event to occur without delay.

Unconditionally, cancel all future demands;

that is, immediately cancel all scheduled occurrences of the DMAND event.

Unconditionally, immediately cancel all scheduled occurrences

of the RCVD1 event.

10. The INIT() event models the initialize IP and INV for each run.

This event causes the following state change(s):

INV=72

IP=INV

After every occurrence of the INIT event:

Unconditionally, end the run of the simulation.;

that is, schedule the STATS() event to occur in TIME time units.

Unconditionally, begin the demand cycle of the simulation;

that is, schedule the DMND1() event to occur without delay.

11. The ATTR1() event models the capturing of the IP just before an attrition.

This event causes the following state change(s):

QUEREP=QUEREP-1

X[2]=1

After every occurrence of the ATTR1 event:

Unconditionally, increase ATTR and reduce IP by one;

that is, schedule the ATTR2() event to occur without delay.

(Time ties are broken by an execution priority of 7.)

12. The RCVD1() event models the recording of the INV value just before a receipt.

After every occurrence of the RCVD1 event:

Unconditionally, add QP to inventory (INV);

that is, schedule the RCVD2() event to occur without delay.

13. The SHIP1() event models the receipt of a repaired carcass.

This event causes the following state change(s):

INV=INV+1

No additional events are scheduled here.

14. The DMND1() event models the capturing of the INV and IP just before a demand.

After every occurrence of the DMND1 event:

Unconditionally, record INV and IP just prior to a demand;

that is, schedule the DMAND() event to occur without delay.

15. The RPAIR() event models the INV value just before receipt of a repaired unit.

This event causes the following state change(s):

QUEREP=QUEREP-1

After every occurrence of the RPAIR event:

Unconditionally, receive a just repaired unit;

that is, schedule the SHIP1() event to occur without delay.

16. The ATTR2() event models the reduction of IP due to a repair attrition.

This event causes the following state change(s):

ATTR=ATTR+1

IP=IP-1

After every occurrence of the ATTR2 event:

If ATTR>=QP, then procure QP units;

that is, schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 2.)

17. The COUNT() event models the number of successful repairs in a batch.

This event causes the following state change(s):

COUNT=COUNT+(X[0]<=RSR)

After every occurrence of the COUNT event:

If RQUE>0, then schedule review of the next carcass in the batch;

that is, schedule the RPORD() event to occur without delay.

18. The INPOS() event models the increase of IP by inducting a batch in repair.

This event causes the following state change(s):

IP=IP

No additional events are scheduled here.

APPENDIX C

MODEL DEFAULTS

```

Model Name:          THESIS
Model Description:   DEPOT LEVEL REPAIRABLES
Output File:        THSVER4.OUT
Run Mode:           HIGH SPEED
Trace Vars:         IP,INV
Random Number Seed: 18645
Initial Values:      8.2, .8, .6, 1.3,36, 10, 5, 9, .25
Ending Condition:    STOP_ON_EVENT
Event: STATS        Number to Run: 1
Trace Events:        ALL EVENTS TRACED
  
```

Time	Event	Count	IP	INV
0.000	RUN	1	0	0
0.000	INIT	1	72	72
0.000	DMND1	1	72	72
0.000	DMAND	1	71	71
0.000	CARTN	1	71	71
0.072	DMND1	2	71	71
0.072	DMAND	2	70	70
0.072	CARTN	2	70	70
0.142	DMND1	3	70	70
0.142	DMAND	3	69	69
0.142	CARTN	3	69	69
0.221	DMND1	4	69	69
0.221	DMAND	4	68	68
0.221	CARTN	4	68	68
0.314	DMND1	5	68	68
0.314	DMAND	5	67	67
0.314	CARTN	5	72	67
0.314	RPORD	1	72	67
0.314	DEPOT	1	72	67
0.314	COUNT	1	72	67
0.314	RPORD	2	72	67
0.314	COUNT	2	72	67
0.314	RPORD	3	72	67
0.314	COUNT	3	72	67
0.314	RPORD	4	72	67
0.314	COUNT	4	72	67
0.314	RPORD	5	72	67
0.314	COUNT	5	72	67
0.314	INPOS	1	72	67
0.431	DMND1	6	72	67
0.431	DMAND	6	71	66
0.431	CARTN	6	71	66
0.491	DMND1	7	71	66
0.491	DMAND	7	70	65
0.491	CARTN	7	70	65
0.518	DMND1	8	70	65
0.518	DMAND	8	69	64
0.518	CARTN	8	69	64

0.564	DEPOT	2	69	64
0.814	DEPOT	3	69	64
0.924	DMND1	9	69	64
0.924	DMAND	9	68	63
0.924	ATTRI	1	68	63
1.008	DMND1	10	68	63
1.008	DMAND	10	67	62
1.008	ATTRI	2	67	62
1.035	DMND1	11	67	62
1.035	DMAND	11	66	61
1.035	CARTN	9	66	61
1.064	DEPOT	4	66	61
1.077	DMND1	12	66	61
1.077	DMAND	12	65	60
1.077	CARTN	10	70	60
1.077	RPORD	6	70	60
1.077	ATTR1	1	70	60
1.077	COUNT	6	70	60
1.077	RPORD	7	70	60
1.077	ATTR1	2	70	60
1.077	COUNT	7	70	60
1.077	RPORD	8	70	60
1.077	DEPOT	5	70	60
1.077	COUNT	8	70	60
1.077	RPORD	9	70	60
1.077	COUNT	9	70	60
1.077	RPORD	10	70	60
1.077	COUNT	10	70	60
1.077	ATTR2	1	69	60
1.077	ATTR2	2	68	60
1.077	INPOS	2	68	60
1.208	DMND1	13	68	60
1.208	DMAND	13	67	59
1.208	CARTN	11	67	59
1.216	DMND1	14	67	59
1.216	DMAND	14	66	58
1.216	CARTN	12	66	58
1.314	ATTR1	3	66	58
1.314	ATTR2	3	65	58
1.327	DEPOT	6	65	58
1.360	DMND1	15	65	58
1.360	DMAND	15	64	57
1.360	ATTRI	3	64	57
1.394	DMND1	16	64	57
1.394	DMAND	16	63	56
1.394	CARTN	13	63	56
1.442	DMND1	17	63	56
1.442	DMAND	17	62	55
1.442	CARTN	14	62	55
1.569	DMND1	18	62	55
1.569	DMAND	18	61	54
1.569	CARTN	15	66	54
1.569	RPORD	11	66	54
1.569	ATTR1	4	66	54
1.569	COUNT	11	66	54
1.569	RPORD	12	66	54
1.569	DEPOT	7	66	54
1.569	COUNT	12	66	54
1.569	RPORD	13	66	54
1.569	COUNT	13	66	54
1.569	RPORD	14	66	54
1.569	COUNT	14	66	54
1.569	RPORD	15	66	54

1.569	COUNT	15	66	54
1.569	ATTR2	4	65	54
1.569	INPOS	3	65	54
1.577	ATTR1	5	65	54
1.577	ATTR2	5	64	54
1.614	RPAIR	1	64	54
1.614	SHIP1	1	64	55
1.733	DMND1	19	64	55
1.733	DMAND	19	63	54
1.733	CARTN	16	63	54
1.819	ATTR1	6	63	54
1.819	ATTR1	7	63	54
1.819	DEPOT	8	63	54
1.819	ATTR2	6	62	54
1.819	ATTR2	7	61	54
1.819	ORDER	1	71	54
1.864	RPAIR	2	71	54
1.864	SHIP1	2	71	55
1.865	DMND1	20	71	55
1.865	DMAND	20	70	54
1.865	CARTN	17	70	54
1.943	DMND1	21	70	54
1.943	DMAND	21	69	53
1.943	CARTN	18	69	53
2.114	RPAIR	3	69	53
2.114	SHIP1	3	69	54
2.182	DMND1	22	69	54
2.182	DMAND	22	68	53
2.182	ATTR1	4	68	53
2.364	RPAIR	4	68	53
2.364	SHIP1	4	68	54
2.369	DMND1	23	68	54
2.369	DMAND	23	67	53
2.369	CARTN	19	67	53
2.377	RPAIR	5	67	53
2.377	SHIP1	5	67	54
2.520	DMND1	24	67	54
2.520	DMAND	24	66	53
2.520	ATTR1	5	66	53
2.627	RPAIR	6	66	53
2.627	SHIP1	6	66	54
2.668	DMND1	25	66	54
2.668	DMAND	25	65	53
2.668	CARTN	20	70	53
2.668	RPORD	16	70	53
2.668	DEPOT	9	70	53
2.668	COUNT	16	70	53
2.668	RPORD	17	70	53
2.668	COUNT	17	70	53
2.668	RPORD	18	70	53
2.668	COUNT	18	70	53
2.668	RPORD	19	70	53
2.668	COUNT	19	70	53
2.668	RPORD	20	70	53
2.668	COUNT	20	70	53
2.668	INPOS	4	70	53
2.675	DMND1	26	70	53
2.675	DMAND	26	69	52
2.675	CARTN	21	69	52
2.798	DMND1	27	69	52
2.798	DMAND	27	68	51
2.798	CARTN	22	68	51
2.869	RPAIR	7	68	51

2.869	SHIP1	7	68	52
2.918	DEPOT	10	68	52
3.028	DMND1	28	68	52
3.028	DMAND	28	67	51
3.028	CARTN	23	67	51
3.086	DMND1	29	67	51
3.086	DMAND	29	66	50
3.086	CARTN	24	66	50
3.109	DMND1	30	66	50
3.109	DMAND	30	65	49
3.109	CARTN	25	70	49
3.109	RPORD	21	70	49
3.109	ATTR1	8	70	49
3.109	COUNT	21	70	49
3.109	RPORD	22	70	49
3.109	DEPOT	11	70	49
3.109	COUNT	22	70	49
3.109	RPORD	23	70	49
3.109	COUNT	23	70	49
3.109	RPORD	24	70	49
3.109	COUNT	24	70	49
3.109	RPORD	25	70	49
3.109	COUNT	25	70	49
3.109	ATTR2	8	69	49
3.109	INPOS	5	69	49
3.119	RPAIR	8	69	49
3.119	SHIP1	8	69	50
3.168	ATTR1	9	69	50
3.168	DEPOT	12	69	50
3.168	ATTR2	9	68	50
3.194	DMND1	31	68	50
3.194	DMAND	31	67	49
3.194	ATTR1	6	67	49
3.205	DMND1	32	67	49
3.205	DMAND	32	66	48
3.205	CARTN	26	66	48
3.218	DMND1	33	66	48
3.218	DMAND	33	65	47
3.218	CARTN	27	65	47
3.227	DMND1	34	65	47
3.227	DMAND	34	64	46
3.227	CARTN	28	64	46
3.270	DMND1	35	64	46
3.270	DMAND	35	63	45
3.270	ATTR1	7	63	45
3.359	ATTR1	10	63	45
3.359	DEPOT	13	63	45
3.359	ATTR2	10	62	45
3.399	DMND1	36	62	45
3.399	DMAND	36	61	44
3.399	CARTN	29	61	44
3.418	DEPOT	14	61	44
3.555	DMND1	37	61	44
3.555	DMAND	37	60	43
3.555	ATTR1	8	60	43
3.609	DEPOT	15	60	43
3.651	DMND1	38	60	43
3.651	DMAND	38	59	42
3.651	CARTN	30	64	42
3.651	RPORD	26	64	42
3.651	DEPOT	16	64	42
3.651	COUNT	26	64	42

APPENDIX D

MODEL DEFAULTS

Model Name: THESIS
Model Description: DEPOT LEVEL REPAIRABLES
Output File: THSVER7.OUT
Run Mode: HIGH SPEED
Trace Vars: CRR, RSR, REP, IP
Random Number Seed: 18645
Initial Values: 8.2, 1.3, 850, 10, 5, 10
Ending Condition: STOP_ON_EVENT
Event: STATS Number to Run: 1
Trace Events: ALL EVENTS TRACED

STATE VARIABLES

State Variable #1

Name: L
Description: PROCUREMENT LEAD TIME
Type: REAL
Size: 1

State Variable #2

Name: CRR
Description: CARCASS RETURN RATE
Type: REAL
Size: 1

State Variable #3

Name: RSR
Description: REPAIR SURVIVAL RATE OF A CARCASS.
Type: REAL
Size: 1

State Variable #4

Name: T2
Description: REPAIR TURNAROUND TIME
Type: REAL
Size: 1

State Variable #5

Name: D
Description: AVERAGE QUARTERLY DEMAND
Type: REAL
Size: 1

State Variable #6

Name: QP
Description: PROCUREMENT ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #7

Name: QR
Description: REPAIR ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #8

Name: INV
Description: ACTUAL ON HAND INVENTORY
Type: INTEGER
Size: 1

State Variable #9

Name: IP
Description: INVENTORY POSITION
Type: INTEGER
Size: 1

State Variable #10

Name: QUEREP
Description: NUMBER OF UNITS IN QUEUE FOR REPAIR
Type: INTEGER
Size: 1

State Variable #11

Name: QUEORD
Description: NUMBER OF UNITS ON ORDER
Type: INTEGER
Size: 1

State Variable #12

Name: RQUE
Description: NUMBER OF CARCASSES AWAITING REPAIR ORDER
Type: INTEGER
Size: 1

State Variable #13
Name: TIME
Description: LENGTH OF TIME FOR ONE RUN OF THE SIMULATION.
Type: REAL
Size: 1

State Variable #14
Name: SAFTY
Description: COUNTER FOR THE SAFETY LEVELS
Type: REAL
Size: 1

State Variable #15
Name: ATTR
Description: CUMULATIVE TOTAL OF CONDEMN/LOST CARCASSES
Type: INTEGER
Size: 1,1

State Variable #16
Name: X
Description: HOLDER OF A RANDOM VARIABLE FOR A STATE.
Type: REAL
Size: 3

State Variable #17
Name: REP
Description: EVALUATION PROCESSING TIME FOR REPAIR.
Type: REAL
Size: 1,1

State Variable #18
Name: COUNT
Description: COUNTER FOR THE REPAIR PROCESS
Type: INTEGER
Size: 1,1

State Variable #19
Name: COUNT1
Description: COUNTER FOR THE NUMBER OF INV CHANGES
Type: INTEGER
Size: 1

VERTICES -----

Vertex #1
Name: RUN
Description: START OF THE SIMULATION
State Changes:
Parameter(s): L, T2, TIME, QP, QR

Vertex #2
Name: DMAND
Description: DEMAND OF A DEPOT LEVEL REPAIRABLE
State Changes: INV=INV-1, X[1]=RND, IP=IP-1
Parameter(s):

Vertex #3
Name: ORDER
Description: PROCURING OF QP UNITS
State Changes: IP=IP+QP, QUEORD=QUEORD+QP, ATTR=0
Parameter(s):

Vertex #4
Name: RCVD2
Description: RECEIPT OF AN ORDER OF QP UNITS
State Changes: INV=INV+QP, QUEORD=QUEORD-QP
Parameter(s):

Vertex #5
Name: CARTN
Description: CARCASS RETURNING TO THE STOCK POINT
State Changes: RQUE=RQUE+1, COUNT=0, IP=IP+RQUE*(RQUE==QR),
QUEREP=QUEREP+(RQUE==QR)*RQUE, X[2]=0
Parameter(s):

Vertex #6
Name: RPORD
Description: DECISION TO REPAIR OR CONDEMN A CARCASS
State Changes: X[0]=RND, RQUE=RQUE-1
Parameter(s):

Vertex #7
Name: DEPOT
Description: DEPOT REPAIRING A CARCASS.
State Changes:
Parameter(s):

Vertex #8
Name: ATTRI
Description: LOSS OR CONDEMNATION OF A CARCASS
State Changes: ATTR=ATTR+1
Parameter(s):

Vertex #9
Name: STATS
Description: COMPUTATION OF AVERAGE SAFETY LEVEL
State Changes: SAFTY=SAFTY/COUNT1
Parameter(s):

Vertex #10

Name: INIT

Description: INITIALIZATION OF IP, INV, D, CRR, RSR, AND REP

S t a t e C h a n g e s :
INV=72, IP=INV, D=DISK{THS.DAT;0}, CRR=DISK{THS.DAT;0},
RSR=DISK{THS.DAT;0}, REP=DISK{THS.DAT;0}

Parameter(s):

Vertex #11

Name: ATTR1

Description: CAPTURING OF IP JUST BEFORE A REPAIR ATTRITION

State Changes: QUEREP=QUEREP-1, X[2]=1

Parameter(s):

Vertex #12

Name: RCVD1

Description: ACCUMULATION OF INV JUST BEFORE A RECEIPT

S t a t e C h a n g e s :
SAFTY=SAFTY+(CLK>=20)*INV, COUNT1=COUNT1+(CLK>=20.0)

Parameter(s):

Vertex #13

Name: SHIP1

Description: RECEIPT OF A REPAIRED CARCASS

State Changes: INV=INV+1

Parameter(s):

Vertex #14

Name: DMND1

Description: CAPTURING OF INV AND IP JUST PRIOR TO A DEMAND

State Changes:

Parameter(s):

Vertex #15

Name: RPAIR

Description: INV VALUE JUST BEFORE RECEIPT OF A REPAIRED UNIT

State Changes: QUEREP=QUEREP-1, SAFTY=SAFTY+(CLK>=20)*INV,
COUNT1=COUNT1+(CLK>=20)

Parameter(s):

Vertex #16

Name: ATTR2

Description: ATTRITION DUE TO REPAIR

State Changes: ATTR=ATTR+1, IP=IP-1

Parameter(s):

Vertex #17

Name: COUNT
Description: NUMBER OF SUCCESSFUL REPAIRS IN A BATCH
State Changes: COUNT=COUNT+ (X[0] <=RSR)
Parameter(s):

Vertex #18

Name: INPOS
Description: INDUCTION OF A BATCH OF CARCASSES INTO REPAIR
State Changes: IP=IP
Parameter(s):

Vertex #19

Name: RINIT
Description: REINITIALIZATION OF TIME, L, T2, QP, AND QR
State Changes: TIME=850, L=8.2, T2=1.3, QP=10, QR=5
Parameter(s):

EDGES

Edge #1

Description: A CARCASS IS SUCCESSFULLY RETURNED
Type: Scheduling
Origin: DMAND
Destination: CARTN
Condition: X[1] <=CRR
Delay: 0
Priority: 3
Attributes:

Edge #2

Description: BEGIN THE SIMULATION
Type: Scheduling
Origin: RUN
Destination: INIT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #3

Description: END THE RUN OF THE SIMULATION
Type: Scheduling
Origin: INIT
Destination: STATS
Condition: 1==1
Delay: TIME
Priority: 4
Attributes:

Edge #4
 Description: A CARCASS IS NOT RETURNED TO THE STOCK POINT
 Type: Scheduling
 Origin: DMAND
 Destination: ATTRI
 Condition: CRR<X[1]
 Delay: 0
 Priority: 3
 Attributes:

Edge #5
 Description: ORDER AN ECONOMIC ORDER QUANTITY.
 Type: Scheduling
 Origin: ATTRI
 Destination: ORDER
 Condition: ATTR>=QP
 Delay: 0
 Priority: 3
 Attributes:

Edge #6
 Description: MARK THE INVENTORY QTY JUST BEFORE A RECEIPT.
 Type: Scheduling
 Origin: ORDER
 Destination: RCVD1
 Condition: 1==1
 Delay: L
 Priority: 5
 Attributes:

Edge #7
 Description: ADD AN ORDER QTY TO NET INVENTORY
 Type: Scheduling
 Origin: RCVD1
 Destination: RCVD2
 Condition: 1==1
 Delay: 0
 Priority: 5
 Attributes:

Edge #8
 Description: SCHEDULE THE NEXT DEMAND
 Type: Scheduling
 Origin: DMAND
 Destination: DMND1
 Condition: 1==1
 Delay: $-(1/D) * \text{LN}\{\text{RND}\}$
 Priority: 5
 Attributes:

Edge #9
Description: RECORD INV AND IP JUST PRIOR TO DEMAND
Type: Scheduling
Origin: DMND1
Destination: DMAND
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #10
Description: SHIP THE RFI CARCASS TO THE STOCK POINT.
Type: Scheduling
Origin: DEPOT
Destination: RPAIR
Condition: 1==1
Delay: T2
Priority: 3
Attributes:

Edge #11
Description: INCREASE ATTR AND DECREASE QUEREP BY ONE
Type: Scheduling
Origin: RPORD
Destination: ATTR1
Condition: X[0]>RSR
Delay: REP*COUNT
Priority: 3
Attributes:

Edge #12
Description:
Type: Scheduling
Origin: RPORD
Destination: DEPOT
Condition: X[0]<=RSR
Delay: REP*COUNT
Priority: 3
Attributes:

Edge #13
Description: SUPPLY CENTER SENDS ALL ITS CARCASSES TO
DEPOT.
Type: Scheduling
Origin: CARTN
Destination: RPORD
Condition: RQUE>=QR
Delay: 0
Priority: 5
Attributes:

Edge #14
Description: INCREASE ATTR AND REDUCE IP BY ONE
Type: Scheduling
Origin: ATTR1
Destination: ATTR2
Condition: 1==1
Delay: 0
Priority: 7
Attributes:

Edge #15
Description:
Type: Scheduling
Origin: RPORD
Destination: COUNT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #16
Description:
Type: Scheduling
Origin: COUNT
Destination: RPORD
Condition: RQUE>0
Delay: 0
Priority: 5
Attributes:

Edge #17
Description:
Type: Scheduling
Origin: COUNT
Destination: INPOS
Condition: RQUE==0&X[2]==0
Delay: 0
Priority: 8
Attributes:

Edge #18
Description:
Type: Scheduling
Origin: ATTR2
Destination: ORDER
Condition: ATTR>=QP
Delay: 0
Priority: 2
Attributes:

Edge #19
Description:
Type: Scheduling
Origin: RPAIR
Destination: SHIP1
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #20
Description: RESET THE SIMULATION MODEL TO ZERO
Type: Scheduling
Origin: STATS
Destination: RINIT
Condition: 1==1&SET{18645}
Delay: 0
Priority: 8
Attributes:

Edge #21
Description: BEGIN THE NEXT RUN OF THE SIMULATION
Type: Scheduling
Origin: RINIT
Destination: INIT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #22
Description: BEGIN THE DEMAND CYCLE
Type: Scheduling
Origin: INIT
Destination: DMND1
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

APPENDIX E

The SIGMA Model, THESIS, is a discrete event simulation. It models depot level repairables.

I. STATE VARIABLE DEFINITIONS.

For this simulation, the following state variables are defined:

- L: procurement lead time (real valued)
- CRR: carcass return rate (real valued)
- RSR: repair survival rate of a carcass. (real valued)
- T2: repair turnaround time (real valued)
- D: average quarterly demand (real valued)
- QP: procurement order quantity (integer valued)
- QR: repair order quantity (integer valued)
- INV: actual on hand inventory (integer valued)
- IP: inventory position (integer valued)
- QUEREP: number of units in queue for repair (integer valued)
- QUEORD: number of units on order (integer valued)
- RQUE: number of carcasses awaiting repair order (integer valued)
- TIME: length of time for one run of the simulation. (real valued)
- SAFTY: counter for the safety levels (real valued)
- ATTR: cumulative total of condemn/lost carcasses (integer valued)
- X[3]: holder of a random variable for a state. (real valued)
- REP: evaluation processing time for repair. (real valued)
- COUNT: counter for the repair process (integer valued)
- COUNT1: counter for the number of inv changes (integer valued)

II. EVENT DEFINITIONS.

Simulation state changes are represented by event vertices (nodes or balls) in a SIGMA graph. Event vertex parameters, if any, are given in parentheses.

Logical and dynamic relationships between pairs of events are represented in a SIGMA graph by edges (arrows) between event vertices. Unless otherwise stated, vertex execution priorities, to break time ties, are equal to 5.

1. The RUN(L,T2,TIME,QP,QR) event models the start of the simulation.
Initial values for, L,T2,TIME,QP,QR, are needed for each run.
After every occurrence of the RUN event:
Unconditionally, begin the simulation;
that is, schedule the INIT() event to occur without delay.
2. The DMAND() event models the demand of a depot level repairable.
This event causes the following state change(s):
INV=INV-1
X[1]=RND
IP=IP-1
After every occurrence of the DMAND event:
If X[1]<=CRR, then a carcass is successfully returned;
that is, schedule the CARTN() event to occur without delay.
(Time ties are broken by an execution priority of 3.)
If CRR<X[1], then a carcass is not returned to the stock point;
that is, schedule the ATTRI() event to occur without delay.
(Time ties are broken by an execution priority of 3.)
Unconditionally, schedule the next demand;
that is, schedule the DMND1() event to occur in $-(1/D)*LN\{RND\}$ time units.
3. The ORDER() event models the procuring of QP units.
This event causes the following state change(s):
IP=IP+QP
QUEORD=QUEORD+QP
ATTR=0
After every occurrence of the ORDER event:
Unconditionally, mark the inventory qty just before a receipt;
that is, schedule the RCVD1() event to occur in L time units.
4. The RCVD2() event models the receipt of an order of QP units.
This event causes the following state change(s):
INV=INV+QP
QUEORD=QUEORD-QP
No additional events are scheduled here.

5. The CARTN() event models the carcass returning to the stock point.

This event causes the following state change(s):

```
RQUE=RQUE+1
COUNT=0
IP=IP+RQUE*(RQUE==QR)
QUEREP=QUEREP+(RQUE==QR)*RQUE
X[2]=0
```

After every occurrence of the CARTN event:

If $RQUE \geq QR$, then supply center sends all its carcasses to depot; that is, schedule the RPORD() event to occur without delay.

6. The RPORD() event models the decision to repair or condemn a carcass.

This event causes the following state change(s):

```
X[0]=RND
RQUE=RQUE-1
```

After every occurrence of the RPORD event:

If $X[0] > RSR$, then increase ATTR and decrease QUEREP by one; that is, schedule the ATTR1() event to occur in

REP*COUNT

time units.

(Time ties are broken by an execution priority of 3.)

If $X[0] \leq RSR$, then schedule the DEPOT() event to occur in REP*COUNT time units.

(Time ties are broken by an execution priority of 3.)

Unconditionally, schedule the COUNT() event to occur without delay.

7. The DEPOT() event models the depot repairing a carcass.

After every occurrence of the DEPOT event:

Unconditionally, ship the RFI carcass to the stock point; that is, schedule the RPAIR() event to occur in T2 time units.

(Time ties are broken by an execution priority of 3.)

8. The ATTR1() event models the loss or condemnation of a carcass.

This event causes the following state change(s):

```
ATTR=ATTR+1
```

After every occurrence of the ATTR1 event:

If $ATTR \geq QP$, then order QP units;

that is, schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 3.)

9. The STATS() event models the computation of average safety level.
 This event causes the following state change(s):
 SAFTY=SAFTY/COUNT1
 After every occurrence of the STATS event:
 If 1=1 and SET{18645}, then reset the simulation model to zero;
 that is, schedule the RINIT() event to occur without delay.
 (Time ties are broken by an execution priority of 8.)
10. The INIT() event models the initialization of IP, INV, D, CRR, RSR and REP.
 This event causes the following state change(s):
 INV=72
 IP=INV
 D=DISK{THS.DAT;0}
 CRR=DISK{THS.DAT;0}
 RSR=DISK{THS.DAT;0}
 REP=DISK{THS.DAT;0}
 After every occurrence of the INIT event:
 Unconditionally, end the run of the simulation;
 that is, schedule the STATS() event to occur in TIME time units.
 (Time ties are broken by an execution priority of 4.)
 Unconditionally, begin the demand cycle;
 that is, schedule the DMND1() event to occur without delay.
11. The ATTR1() event models the capturing of IP just before a repair attrition.
 This event causes the following state change(s):
 QUEREP=QUEREP-1
 X[2]=1
 After every occurrence of the ATTR1 event:
 Unconditionally, increase ATTR and reduce IP by one;
 that is, schedule the ATTR2() event to occur without delay.
 (Time ties are broken by an execution priority of 7.)
12. The RCVD1() event models the accumulation of INV just before a receipt.
 This event causes the following state change(s):
 SAFTY=SAFTY+(CLK>=20)*INV
 COUNT1=COUNT1+(CLK>=20.0)
 After every occurrence of the RCVD1 event:
 Unconditionally, add an order qty to net inventory;
 that is, schedule the RCVD2() event to occur without delay.

13. The SHIP1() event models the receipt of a repaired carcass.

This event causes the following state change(s):

INV=INV+1

No additional events are scheduled here.

14. The DMND1() event models the capturing of INV and IP just prior to a demand.

After every occurrence of the DMND1 event:

Unconditionally, record INV and IP just prior to demand;
that is, schedule the DMAND() event to occur without delay.

15. The RPAIR() event models the INV value just before receipt of a repaired unit.

This event causes the following state change(s):

QUEREP=QUEREP-1

SAFTY=SAFTY+(CLK>=20)*INV

COUNT1=COUNT1+(CLK>=20)

After every occurrence of the RPAIR event:

Unconditionally, schedule the SHIP1() event to occur without delay.

16. The ATTR2() event models the attrition due to repair.

This event causes the following state change(s):

ATTR=ATTR+1

IP=IP-1

After every occurrence of the ATTR2 event:

If ATTR>=QP, then schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 2.)

17. The COUNT() event models the number of successful repairs in a batch.

This event causes the following state change(s):

COUNT=COUNT+(X[0]<=RSR)

After every occurrence of the COUNT event:

If RQUE>0, then schedule the RPORD() event to occur without delay.

If RQUE==0 and X[2]==0, then schedule the INPOS() event to occur without delay.

(Time ties are broken by an execution priority of 8.)

18. The INPOS() event models the induction of a batch of carcasses into repair.

This event causes the following state change(s):

IP=IP

No additional events are scheduled here.

19. The RINIT() event models the reinitialization of TIME, L, T2, QP, and QR.

This event causes the following state change(s):

TIME=850

L=8.2

T2=1.3

QP=10

QR=5

After every occurrence of the RINIT event:

Unconditionally, begin the next run of the simulation;
that is, schedule the INIT() event to occur without
delay.

APPENDIX F

MODEL DEFAULTS

Model Name: THESIS
Model Description: DEPOT LEVEL REPAIRABLES
Output File: THSVR8.OUT
Run Mode: HIGH_SPEED
Trace Vars: IP
Random Number Seed: 18645
Initial Values: 8.2, 1.3, 850, 10, 5, 10
Ending Condition: STOP_ON_EVENT
Event: RUN Number to Run: 1
Trace Events: ALL EVENTS TRACED

STATE VARIABLES

State Variable #1

Name: L
Description: PROCUREMENT LEAD TIME
Type: REAL
Size: 1

State Variable #2

Name: CRR
Description: CARCASS RETURN RATE
Type: REAL
Size: 1

State Variable #3

Name: RSR
Description: REPAIR SURVIVAL RATE OF A CARCASS
Type: REAL
Size: 1

State Variable #4

Name: T2
Description: REPAIR TURNAROUND TIME
Type: REAL
Size: 1

State Variable #5
Name: D
Description: AVERAGE QUARTERLY DEMAND
Type: REAL
Size: 1

State Variable #6
Name: QP
Description: PROCUREMENT ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #7
Name: QR
Description: REPAIR ORDER QUANTITY
Type: INTEGER
Size: 1

State Variable #8
Name: INV
Description: ACTUAL ON HAND INVENTORY
Type: INTEGER
Size: 1

State Variable #9
Name: IP
Description: INVENTORY POSITION
Type: INTEGER
Size: 1

State Variable #10
Name: QUEREP
Description: NUMBER OF UNITS IN QUEUE FOR REPAIR
Type: INTEGER
Size: 1

State Variable #11
Name: QUEORD
Description: NUMBER OF UNITS ON ORDER
Type: INTEGER
Size: 1

State Variable #12
Name: RQUE
Description: NUMBER OF CARCASSES AWAITING REPAIR ORDER
Type: INTEGER
Size: 1

State Variable #13
Name: TIME
Description: LENGTH OF TIME FOR ONE RUN OF THE SIMULATION.
Type: REAL
Size: 1

State Variable #14
Name: C
Description: CLOCK TIME HOLDERS AND TIME WEIGHTED INVENTORY
Type: REAL
Size: 10

State Variable #15
Name: ATTR
Description: CUMULATIVE TOTAL OF CONDEMN/LOST CARCASSES
Type: INTEGER
Size: 1,1

State Variable #16
Name: X
Description: HOLDER OF A RANDOM VARIABLE FOR A STATE.
Type: REAL
Size: 3

State Variable #17
Name: REP
Description: EVALUATION PROCESSING TIME FOR REPAIR.
Type: REAL
Size: 1,1

State Variable #18
Name: COUNT
Description: NUMBER OF REPAIRABLE CARCASSES IN A BATCH
Type: INTEGER
Size: 1

VERTICES

Vertex #1
Name: RUN
Description: START OF THE SIMULATION
State Changes:
Parameter(s): L,T2,TIME,QP,QR

Vertex #2
Name: DMAND
Description: DEMAND OF A DEPOT LEVEL REPAIRABLE
State Changes: $C[6] = C[6] + C[3] * INV * (INV < 0)$, $INV = INV - 1$,
 $X[1] = RND$, $IP = IP - 1$
Parameter(s):

Vertex #3
Name: ORDER
Description: ORDERING OF QP UNITS OF NEW MATERIAL
State Changes: $IP = IP + QP$, $QUEORD = QUEORD + QP$
Parameter(s):

Vertex #4
Name: RCVD2
Description: RECEIPT OF AN ORDER OF QP UNITS
State Changes: $C[6] = C[6] + C[3] * INV * (INV < 0)$, $INV = INV + QP$,
 $QUEORD = QUEORD - QP$
Parameter(s):

Vertex #5
Name: CARTN
Description: CARCASS RETURNING TO THE STOCK POINT.
State Changes: $RQUE = RQUE + 1$, $COUNT = 1$, $IP = IP + RQUE * (RQUE == QR)$,
 $QUEREP = QUEREP + (RQUE == QR) * RQUE$, $X[2] = 0$
Parameter(s):

Vertex #6
Name: RPORD
Description: DECISION TO REPAIR OR CONDEMN A CARCASS
State Changes: $X[0] = RND$, $QR = QR - 1$
Parameter(s):

Vertex #7
Name: DEPOT
Description: DEPOT REPAIRING A CARCASS
State Changes:
Parameter(s):

Vertex #8
Name: ATTRI
Description: LOSS OR CONDEMNATION OF A CARCASS
State Changes: $ATTR = ATTR + 1$
Parameter(s):

Vertex #9
Name: STATS
Description: GATHERING OF AVERAGES AND END OF A SIMULATION
RUN
State Changes: $C[4] = C[4] / 830$, $C[5] = C[5] / 830$, $C[6] = C[6] / -830$
Parameter(s):

Vertex #10

Name: INIT
Description: INITIALIZES IP, INV, D, CRR, RSR AND REP
State Changes: C h a n g e s :
INV=72, IP=INV, D=DISK{THS.DAT;0}, CRR=DISK{THS.DAT;0},
RSR=DISK{THS.DAT;0}, REP=DISK{THS.DAT;0}
Parameter(s):

Vertex #11

Name: ATTR1
Description: INCREASE ATTR AND DECREASE IP BY ONE
State Changes: QUERE=QUERE-1
Parameter(s):

Vertex #12

Name: RCVD1
Description: RECORDING OF INV JUST BEFORE A RECEIPT
State Changes: C[1]=C[2], C[2]=CLK, C[3]=(C[2]-C[1])*(CLK>20),
C[4]=C[4]+C[3]*INV,
C[5]=C[5]+C[3]*INV*(INV>=0)
Parameter(s):

Vertex #13

Name: SHIP1
Description: RECEIPT OF A REPAIRED CARCASS
State Changes: C h a n g e s :
C[6]=C[6]+C[3]*INV*(INV<0), QUERE=QUERE-1, INV=INV+1
Parameter(s):

Vertex #14

Name: DMND1
Description: CAPTURING INV JUST PRIOR TO A DEMAND
State Changes: C[1]=C[2], C[2]=CLK, C[3]=(C[2]-C[1])*(CLK>20),
C[4]=C[4]+C[3]*INV,
C[5]=C[5]+C[3]*INV*(INV>=0)
Parameter(s):

Vertex #15

Name: RPAIR
Description: ACCUMULATION OF TIME-WEIGHTED INV VALUES
State Changes: C[1]=C[2], C[2]=CLK, C[3]=(C[2]-C[1])*(CLK>20),
C[4]=C[4]+C[3]*INV,
C[5]=C[5]+C[3]*INV*(INV>=0)
Parameter(s):

Vertex #16

Name: ATTR2
Description: REDUCTION OF IP DUE TO A REPAIR ATTRITION
State Changes: ATTR=ATTR+1, IP=IP-1
Parameter(s):

Vertex #17

Name: COUNT
Description: NUMBER OF SUCCESSFUL REPAIRS IN A BATCH
State Changes: COUNT=COUNT+ (X[0] <=RSR)
Parameter(s):

Vertex #18

Name: INPOS
Description: INCREASE OF IP BY INDUCTING A BATCH IN REPAIR
State Changes: IP=IP
Parameter(s):

Vertex #19

Name: RINIT
Description:
State Changes: TIME=850, L=8.2, T2=1.3, QP=10, QR=5
Parameter(s):

EDGES

Edge #1

Description: A CARCASS IS SUCCESSFULLY TURNED-IN
Type: Scheduling
Origin: DMAND
Destination: CARTN
Condition: X[1] <=CRR
Delay: 0
Priority: 3
Attributes:

Edge #2

Description: BEGIN THE SIMULATION
Type: Scheduling
Origin: RUN
Destination: INIT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #3
Description: SCHEDULE THE END OF THE RUN
Type: Scheduling
Origin: INIT
Destination: STATS
Condition: 1==1
Delay: TIME
Priority: 4
Attributes:

Edge #4
Description: A CARCASS IS NOT RETURNED TO THE STOCK POINT
Type: Scheduling
Origin: DMAND
Destination: ATTRI
Condition: CRR<X[1]
Delay: 0
Priority: 3
Attributes:

Edge #5
Description: PROCURE QP UNITS
Type: Scheduling
Origin: ATTRI
Destination: ORDER
Condition: ATTR>=QP
Delay: 0
Priority: 3
Attributes:

Edge #6
Description: SCHEDULE THE RECEIPT OF QP UNITS
Type: Scheduling
Origin: ORDER
Destination: RCVD1
Condition: 1==1
Delay: L
Priority: 5
Attributes:

Edge #7
Description: ADD QP UNITS TO INVENTORY (INV)
Type: Scheduling
Origin: RCVD1
Destination: RCVD2
Condition: 1==1
Delay: *
Priority: 5
Attributes:

Edge #8
 Description: SCHEDULE ANOTHER DEMAND
 Type: Scheduling
 Origin: DMAND
 Destination: DMND1
 Condition: $1==1$
 Delay: $-(1/D) * LN\{RND\}$
 Priority: 5
 Attributes:

Edge #9
 Description: RECORD INV AND IP JUST PRIOR TO A DEMAND
 Type: Scheduling
 Origin: DMND1
 Destination: DMAND
 Condition: $1==1$
 Delay: *
 Priority: 5
 Attributes:

Edge #10
 Description: SHIP THE RFI CARCASS TO THE STOCK POINT
 Type: Scheduling
 Origin: DEPOT
 Destination: RPAIR
 Condition: $1==1$
 Delay: T2
 Priority: 3
 Attributes:

Edge #11
 Description: SCHEDULE THE REPAIR ATTRITION
 Type: Scheduling
 Origin: RPORD
 Destination: ATTR1
 Condition: $X[0] > RSR$
 Delay: $REP * COUNT$
 Priority: 3
 Attributes:

Edge #12
 Description: SCHEDULE THE REPAIR OF A NRFI UNIT
 Type: Scheduling
 Origin: RPORD
 Destination: DEPOT
 Condition: $X[0] \leq RSR$
 Delay: $REP * COUNT$
 Priority: 3
 Attributes:

Edge #13
Description: SEND A BATCH OF QP NRFI CARCASSES TO DEPOT
Type: Scheduling
Origin: CARTN
Destination: RPORD
Condition: RQUE>=QR
Delay: 0
Priority: 5
Attributes:

Edge #14
Description: INCREASE ATTR AND REDUCE IP BY ONE
Type: Scheduling
Origin: ATTR1
Destination: ATTR2
Condition: 1==1
Delay: 0
Priority: 7
Attributes:

Edge #15
Description: INCREASE THE COUNT OF REPAIRABLES IN A BATCH
Type: Scheduling
Origin: RPORD
Destination: COUNT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #16
Description: SCHEDULE THE REVIEW OF THE NEXT CARCASS
Type: Scheduling
Origin: COUNT
Destination: RPORD
Condition: RQUE>0
Delay: 0
Priority: 5
Attributes:

Edge #17
Description: PROCURE QP UNITS
Type: Scheduling
Origin: ATTR2
Destination: ORDER
Condition: ATTR>=QP
Delay: 0
Priority: 2
Attributes:

Edge #18
Description: RECEIVE A REPAIRED UNIT
Type: Scheduling
Origin: RPAIR
Destination: SHIP1
Condition: 1==1
Delay: *
Priority: 5
Attributes:

Edge #19
Description: BEGIN THE NEXT RUN OF THE SIMULATION.
Type: Scheduling
Origin: STATS
Destination: RINIT
Condition: 1==1&SET{18645}
Delay: 0
Priority: 8
Attributes:

Edge #20
Description: BEGIN THE NEXT RUN OF THE SIMULATION.
Type: Scheduling
Origin: RINIT
Destination: INIT
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #21
Description: BEGIN THE DEMAND CYCLE OF EACH RUN
Type: Scheduling
Origin: INIT
Destination: DMND1
Condition: 1==1
Delay: 0
Priority: 5
Attributes:

Edge #22
Description:
Type: Scheduling
Origin: CARTN
Destination: INPOS
Condition: RQUE==QR
Delay: 0
Priority: 8
Attributes:

APPENDIX G

The SIGMA Model, THESIS, is a discrete event simulation. It models depot level repairables.

I. STATE VARIABLE DEFINITIONS.

For this simulation, the following state variables are defined:

- L: procurement lead time (real valued)
- CRR: carcass return rate (real valued)
- RSR: repair survival rate of a carcass. (real valued)
- T2: repair turnaround time (real valued)
- D: average quarterly demand (real valued)
- QP: procurement order quantity (integer valued)
- QR: repair order quantity (integer valued)
- INV: actual on hand inventory (integer valued)
- IP: inventory position (integer valued)
- QUEREP: number of units in queue for repair (integer valued)
- QUEORD: number of units on order (integer valued)
- RQUE: number of carcasses awaiting repair order (integer valued)
- TIME: length of time for one run of the simulation. (real valued)
- C[10]: clock time holders and time weighted inventory (real valued)
- ATTR: cumulative total of condemn/lost carcasses (integer valued)
- X[3]: holder of a random variable for a state. (real valued)
- REP: evaluation processing time for repair. (real valued)
- COUNT: number of repairable carcasses in a batch (integer valued)

II. EVENT DEFINITIONS.

Simulation state changes are represented by event vertices (nodes or balls) in a SIGMA graph. Event vertex parameters, if any, are given in parentheses.

Logical and dynamic relationships between pairs of events are represented in a SIGMA graph by edges (arrows) between event vertices. Unless otherwise stated, vertex execution priorities, to break time ties, are equal to 5.

1. The RUN(L,T2,TIME,QP,QR) event models the start of the simulation.
Initial values for, L,T2,TIME,QP,QR, are needed for each run.
After every occurrence of the RUN event:
Unconditionally, begin the simulation;
that is, schedule the INIT() event to occur without delay.

2. The DMAND() event models the demand of a depot level repairable.
This event causes the following state change(s):
 $C[6] = C[6] + C[3] * INV * (INV < 0)$
 $INV = INV - 1$
 $X[1] = RND$
 $IP = IP - 1$
 After every occurrence of the DMAND event:
 If $X[1] \leq CRR$, then a carcass is successfully turned-in;
 that is, schedule the CARTN() event to occur without delay.
 (Time ties are broken by an execution priority of 3.)
 If $CRR < X[1]$, then a carcass is not returned to the stock point;
 that is, schedule the ATTRI() event to occur without delay.
 (Time ties are broken by an execution priority of 3.)
 Unconditionally, schedule another demand;
 that is, schedule the DMND1() event to occur in
 $-(1/D) * LN\{RND\}$ time units.

3. The ORDER() event models the ordering of QP units of new material.
This event causes the following state change(s):
 $IP = IP + QP$
 $QUEORD = QUEORD + QR$
 After every occurrence of the ORDER event:
 Unconditionally, schedule the receipt of QP units;
 that is, schedule the RCVD1() event to occur in L time units.

4. The RCVD2() event models the receipt of an order of QP units.
This event causes the following state change(s):
 $C[6] = C[6] + C[3] * INV * (INV < 0)$
 $INV = INV + QP$
 $QUEORD = QUEORD - QP$
 No additional events are scheduled here.

5. The CARTN() event models the carcass returning to the stock point.

This event causes the following state change(s):

RQUE=RQUE+1

COUNT=1

IP=IP+RQUE*(RQUE==QR)

QUEREP=QUEREP+(RQUE==QR)*RQUE

X[2]=0

After every occurrence of the CARTN event:

If RQUE>=QR, then send a batch of QP NRFI carcasses to depot;

that is, schedule the RPORD() event to occur without delay.

If RQUE==QR, then schedule the INPOS() event to occur without delay.

(Time ties are broken by an execution priority of 8.)

6. The RPORD() event models the decision to repair or condemn a carcass.

This event causes the following state change(s):

X[0]=RND

QR=QR-1

After every occurrence of the RPORD event:

If X[0]>RSR, then schedule the repair attrition;

that is, schedule the ATTR1() event to occur in REP*COUNT time units.

(Time ties are broken by an execution priority of 3.)

If X[0]<=RSR, then schedule the repair of a NRFI unit;

that is, schedule the DEPOT() event to occur in REP*COUNT time units.

(Time ties are broken by an execution priority of 3.)

Unconditionally, increase the count of repairables in a batch;

that is, schedule the COUNT() event to occur without delay.

7. The DEPOT() event models the depot repairing a carcass.

After every occurrence of the DEPOT event:

Unconditionally, ship the RFI carcass to the stock point;

that is, schedule the RPAIR() event to occur in T2 time units.

(Time ties are broken by an execution priority of 3.)

8. The ATTRI() event models the loss or condemnation of a carcass.

This event causes the following state change(s):

ATTR=ATTR+1

After every occurrence of the ATTRI event:

If ATTR>=QP, then procure QP units;

that is, schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 3.)

9. The STATS() event models the gathering of averages and end of a simulation run.

This event causes the following state change(s):

C[4]=C[4]/830

C[5]=C[5]/830

C[6]=C[6]/-830

After every occurrence of the STATS event:

If l==1 and SET{18645}, then begin the next run of the simulation; that is, schedule the RINIT() event to occur without delay.

(Time ties are broken by an execution priority of 8.)

10. The INIT() event models the initializes IP, INV, D, CRR, RSR, and REP.

This event causes the following state change(s):

INV=72

IP=INV

D=DISK{THS.DAT;0}

CRR=DISK{THS.DAT;0}

RSR=DISK{THS.DAT;0}

REP=DISK{THS.DAT;0}

After every occurrence of the INIT event:

Unconditionally, schedule the end of the run;

that is, schedule the STATS() event to occur in TIME time units.

(Time ties are broken by an execution priority of 4.)

Unconditionally, begin the demand cycle of each run;

that is, schedule the DMND1() event to occur without delay.

11. The ATTR1() event models the increase ATTR and decrease IP by one.

This event causes the following state change(s):

QUEREP=QUEREP-1

After every occurrence of the ATTR1 event:

Unconditionally, increase attr and reduce IP by one;

that is, schedule the ATTR2() event to occur without delay.

(Time ties are broken by an execution priority of 7.)

12. The RCVD1() event models the recording of INV just before a receipt.

This event causes the following state change(s):

C[1]=C[2]
C[2]=CLK
C[3]=(C[2]-C[1])*(CLK>20)
C[4]=C[4]+C[3]*INV
C[5]=C[5]+C[3]*INV*(INV>=0)

After every occurrence of the RCVD1 event:

Unconditionally, add QP units to inventory (INV);
that is, immediately execute the RCVD2() event.

13. The SHIP1() event models the receipt of a repaired carcass.

This event causes the following state change(s):

C[6]=C[6]+C[3]*INV*(INV<0)
QUEREP=QUEREP-1
INV=INV+1

No additional events are scheduled here.

14. The DMND1() event models the capturing INV just prior to a demand.

This event causes the following state change(s):

C[1]=C[2]
C[2]=CLK
C[3]=(C[2]-C[1])*(CLK>20)
C[4]=C[4]+C[3]*INV
C[5]=C[5]+C[3]*INV*(INV>=0)

After every occurrence of the DMND1 event:

Unconditionally, record inv and ip just prior to a demand;
that is, immediately execute the DMAND() event.

15. The RPAIR() event models the accumulation of time-weighted INV values.

This event causes the following state change(s):

C[1]=C[2]
C[2]=CLK
C[3]=(C[2]-C[1])*(CLK>20)
C[4]=C[4]+C[3]*INV
C[5]=C[5]+C[3]*INV*(INV>=0)

After every occurrence of the RPAIR event:

Unconditionally, receive a repaired unit;
that is, immediately execute the SHIP1() event.

16. The ATTR2() event models the reduction of IP due to a repair attrition.

This event causes the following state change(s):

ATTR=ATTR+1

IP=IP-1

After every occurrence of the ATTR2 event:

If ATTR>=QP, then procure QP units;

that is, schedule the ORDER() event to occur without delay.

(Time ties are broken by an execution priority of 2.)

17. The COUNT() event models the number of successful repairs in a batch.

This event causes the following state change(s):

COUNT=COUNT+(X[0]<=RSR)

After every occurrence of the COUNT event:

If RQUE>0, then schedule the review of the next carcass;
that is, schedule the RPORD() event to occur without delay.

18. The INPOS() event models the increase of IP by inducting a batch in repair.

This event causes the following state change(s):

IP=IP

No additional events are scheduled here.

19. The RINIT() event:

This event causes the following state change(s):

TIME=850

L=8.2

T2=1.3

QP=10

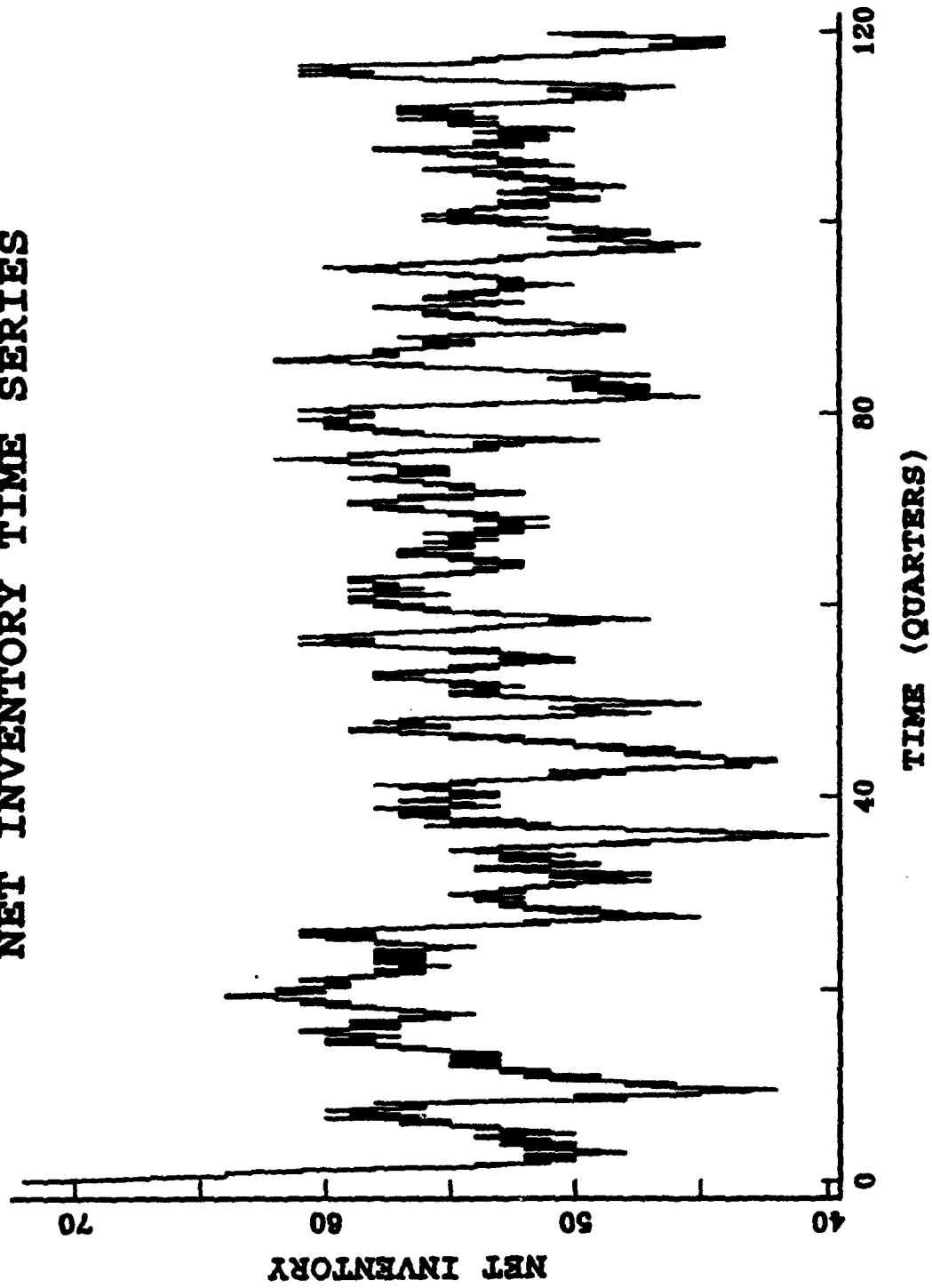
QR=5

After every occurrence of the RINIT event:

Unconditionally, begin the next run of the simulation.;
that is, schedule the INIT() event to occur without delay.

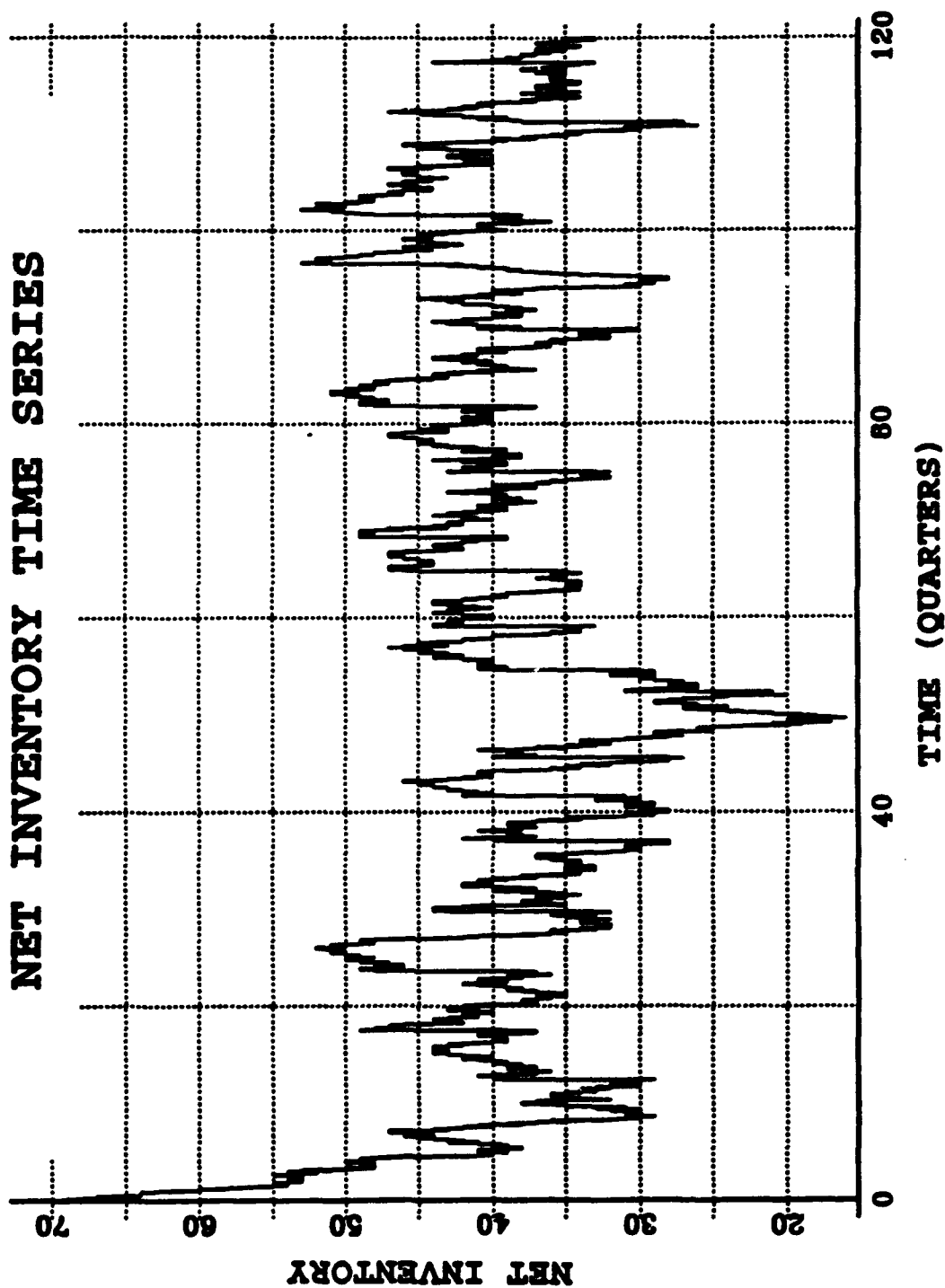
APPENDIX H

CRR = 1.0, RSR = 1.0
NET INVENTORY TIME SERIES



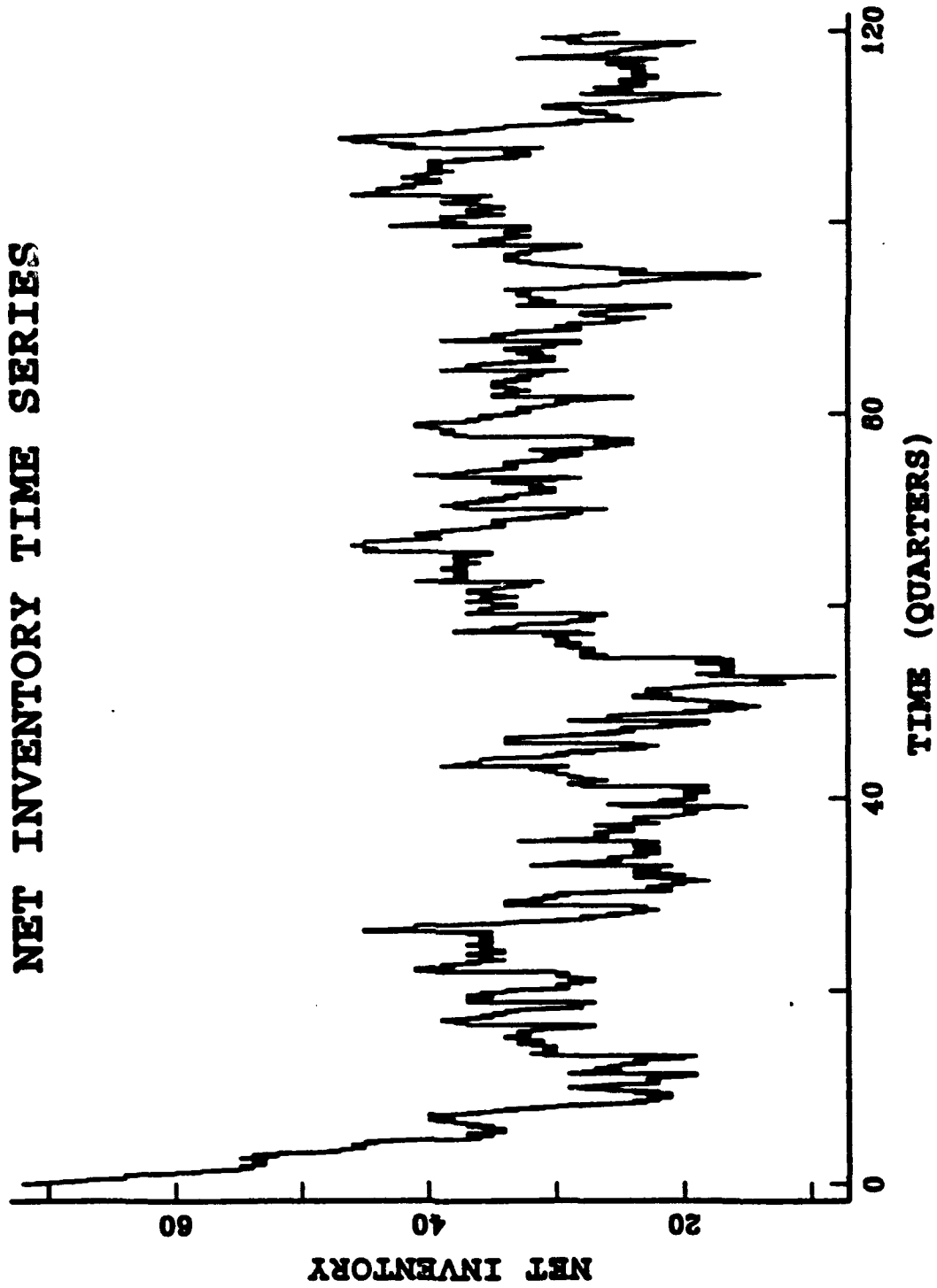
CRR = .8; RSR = 1.0

NET INVENTORY TIME SERIES



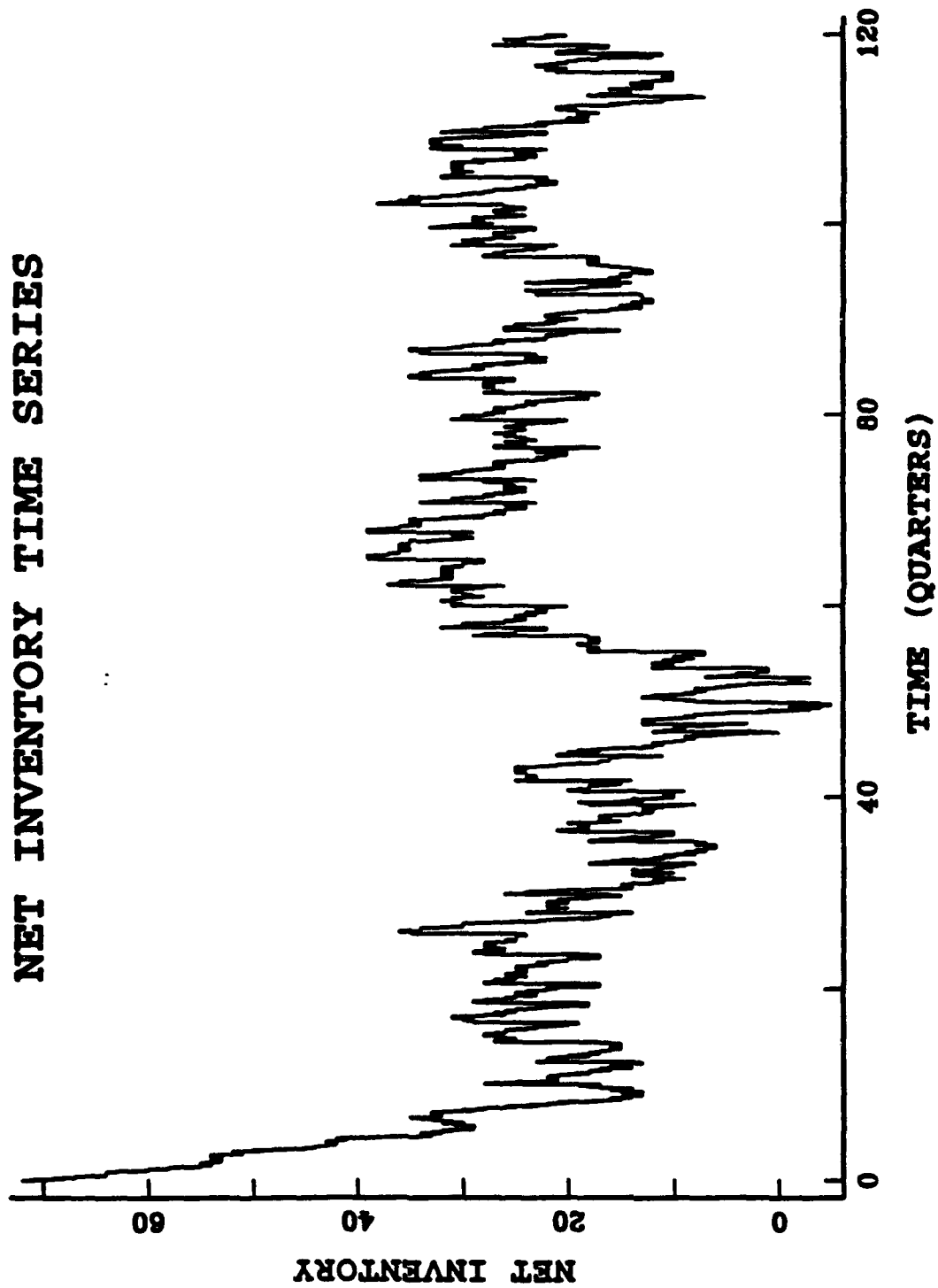
CSR = .8; RSR = .8

NET INVENTORY TIME SERIES



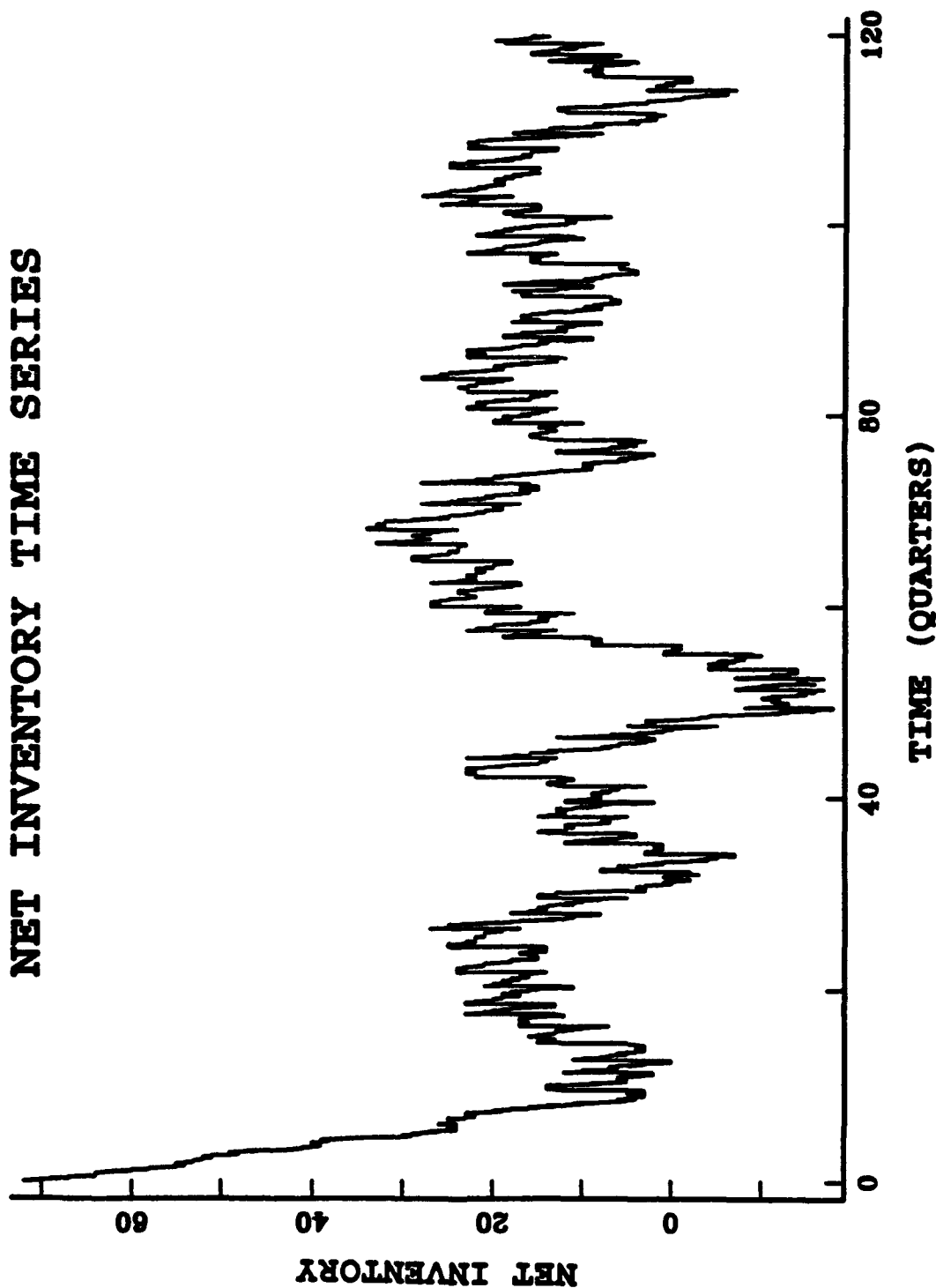
CSR = .8; RSR = .6

NET INVENTORY TIME SERIES



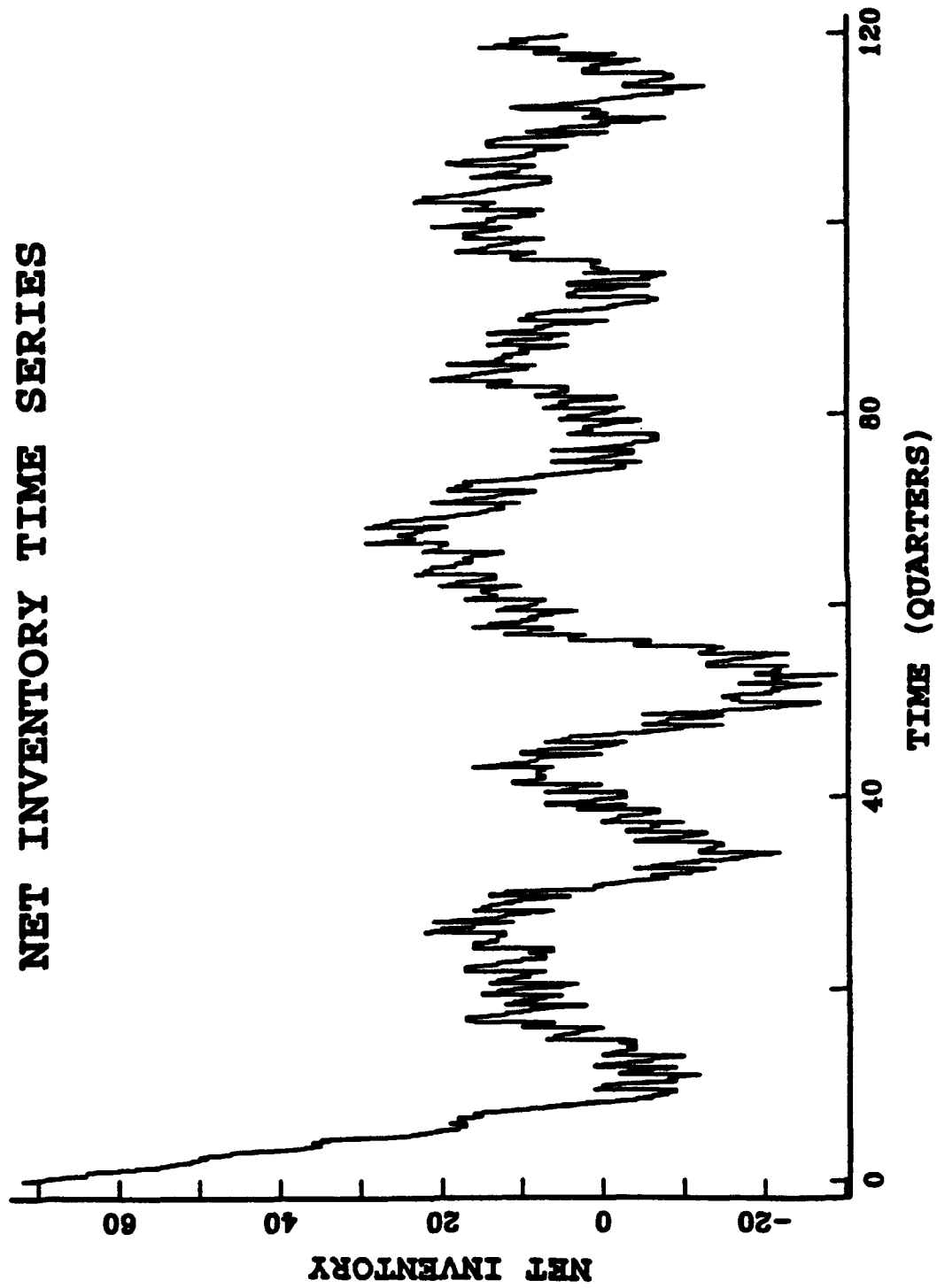
CSR = .8; RSR = .4

NET INVENTORY TIME SERIES



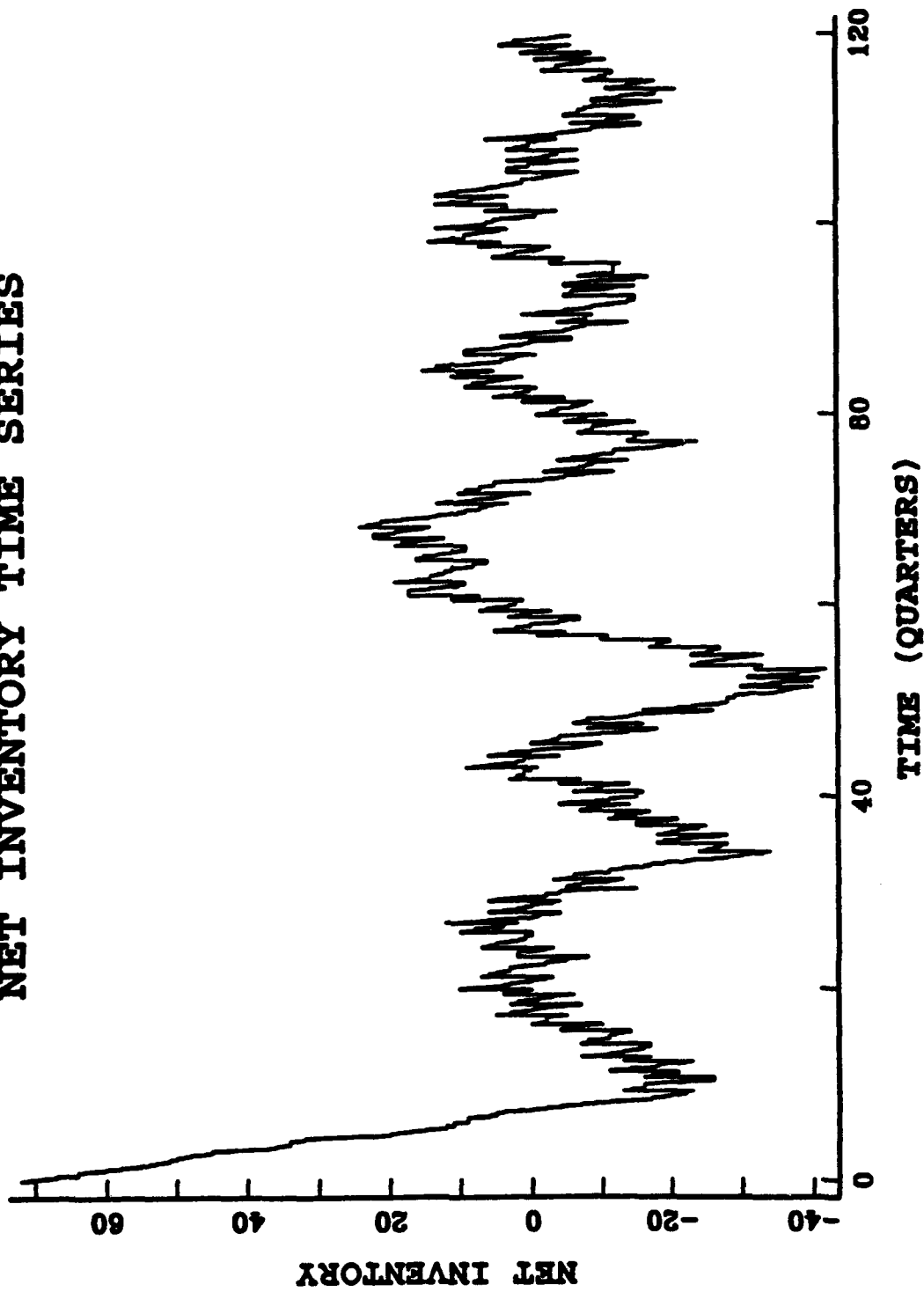
CSR = .8; RSR = .2

NET INVENTORY TIME SERIES



CSR = .8; RSR = 0.0

NET INVENTORY TIME SERIES



APPENDIX I

CRR	RSR	REP	χ^2	α
0.00	1.00	1.30	518.364	1.0000
0.20	0.00	0.00	1.384	0.0000
0.20	0.00	0.25	1.384	0.0000
0.20	0.00	0.50	1.384	0.0000
0.20	0.00	0.75	1.384	0.0000
0.20	0.00	1.00	1.384	0.0000
0.20	0.00	1.30	1.384	0.0000
0.20	0.20	0.00	1.382	0.0000
0.20	0.20	0.25	1.699	0.0001
0.20	0.20	0.50	1.719	0.0001
0.20	0.20	0.75	2.458	0.0007
0.20	0.20	1.00	2.820	0.0015
0.20	0.20	1.30	2.157	0.0003
0.20	0.40	0.00	2.681	0.0011
0.20	0.40	0.25	3.195	0.0029
0.20	0.40	0.50	3.529	0.0048
0.20	0.40	0.75	3.136	0.0026
0.20	0.40	1.00	3.874	0.0075
0.20	0.40	1.30	4.068	0.0096
0.20	0.60	0.00	1.013	0.0000
0.20	0.60	0.25	1.275	0.0000
0.20	0.60	0.50	2.117	0.0003
0.20	0.60	0.75	2.330	0.0005
0.20	0.60	1.00	2.321	0.0005
0.20	0.60	1.30	2.549	0.0009
0.20	0.80	0.00	2.304	0.0005
0.20	0.80	0.25	1.987	0.0002
0.20	0.80	0.50	2.551	0.0009
0.20	0.80	0.75	1.911	0.0002
0.20	0.80	1.00	2.370	0.0006
0.20	0.80	1.30	1.756	0.0001
0.20	1.00	0.00	2.972	0.0020
0.20	1.00	0.25	2.972	0.0020
0.20	1.00	0.50	2.972	0.0020
0.20	1.00	0.75	2.972	0.0020
0.20	1.00	1.00	2.972	0.0020
0.20	1.00	1.30	2.972	0.0020

CRR	RSR	REP	χ^2	α
0.40	0.00	0.00	1.699	0.0001
0.40	0.00	0.25	1.699	0.0001
0.40	0.00	0.50	1.699	0.0001
0.40	0.00	0.75	1.699	0.0001
0.40	0.00	1.00	1.699	0.0001
0.40	0.00	1.30	1.699	0.0001
0.40	0.20	0.00	2.254	0.0004
0.40	0.20	0.25	1.620	0.0001
0.40	0.20	0.50	1.697	0.0001
0.40	0.20	0.75	2.097	0.0003
0.40	0.20	1.00	1.744	0.0001
0.40	0.20	1.30	1.478	0.0000
0.40	0.40	0.00	2.672	0.0011
0.40	0.40	0.25	3.693	0.0060
0.40	0.40	0.50	3.557	0.0049
0.40	0.40	0.75	3.841	0.0072
0.40	0.40	1.00	4.160	0.0106
0.40	0.40	1.30	3.325	0.0035
0.40	0.60	0.00	1.270	0.0000
0.40	0.60	0.25	0.643	0.0000
0.40	0.60	0.50	1.024	0.0000
0.40	0.60	0.75	1.206	0.0000
0.40	0.60	1.00	1.151	0.0000
0.40	0.60	1.30	1.216	0.0000
0.40	0.80	0.00	1.279	0.0000
0.40	0.80	0.25	1.090	0.0000
0.40	0.80	0.50	1.200	0.0000
0.40	0.80	0.75	0.579	0.0000
0.40	0.80	1.00	0.919	0.0000
0.40	0.80	1.30	1.349	0.0000
0.40	1.00	0.00	1.517	0.0000
0.40	1.00	0.25	1.517	0.0000
0.40	1.00	0.50	1.517	0.0000
0.40	1.00	0.75	1.517	0.0000
0.40	1.00	1.00	1.517	0.0000
0.40	1.00	1.30	1.517	0.0000
0.60	0.00	0.00	1.098	0.0000
0.60	0.00	0.25	1.098	0.0000
0.60	0.00	0.50	1.098	0.0000
0.60	0.00	0.75	1.098	0.0000
0.60	0.00	1.00	1.098	0.0000
0.60	0.00	1.30	1.098	0.0000

CRR	RSR	REP	χ^2	α
0.60	0.20	0.00	1.170	0.0000
0.60	0.20	0.25	1.721	0.0001
0.60	0.20	0.50	1.813	0.0001
0.60	0.20	0.75	1.988	0.0002
0.60	0.20	1.00	0.725	0.0000
0.60	0.20	1.30	0.570	0.0000
0.60	0.40	0.00	2.174	0.0004
0.60	0.40	0.25	1.998	0.0002
0.60	0.40	0.50	2.758	0.0013
0.60	0.40	0.75	2.023	0.0002
0.60	0.40	1.00	2.308	0.0005
0.60	0.40	1.30	1.773	0.0001
0.60	0.60	0.00	2.687	0.0012
0.60	0.60	0.25	1.282	0.0000
0.60	0.60	0.50	1.689	0.0001
0.60	0.60	0.75	1.834	0.0001
0.60	0.60	1.00	1.125	0.0000
0.60	0.60	1.30	1.692	0.0001
0.60	0.80	0.00	0.809	0.0000
0.60	0.80	0.25	0.665	0.0000
0.60	0.80	0.50	1.229	0.0000
0.60	0.80	0.75	1.512	0.0000
0.60	0.80	1.00	0.664	0.0000
0.60	0.80	1.30	1.850	0.0001
0.60	1.00	0.00	2.695	0.0012
0.60	1.00	0.25	2.695	0.0012
0.60	1.00	0.50	2.695	0.0012
0.60	1.00	0.75	2.695	0.0012
0.60	1.00	1.00	2.695	0.0012
0.60	1.00	1.30	2.695	0.0012
0.80	0.00	0.00	1.661	0.0001
0.80	0.00	0.25	1.661	0.0001
0.80	0.00	0.50	1.661	0.0001
0.80	0.00	0.75	1.661	0.0001
0.80	0.00	1.00	1.661	0.0001
0.80	0.00	1.30	1.661	0.0001
0.80	0.20	0.00	2.113	0.0003
0.80	0.20	0.25	2.045	0.0003
0.80	0.20	0.50	2.063	0.0003
0.80	0.20	0.75	1.472	0.0000
0.80	0.20	1.00	1.840	0.0001
0.80	0.20	1.30	3.899	0.0078

CRR	RSR	REP	χ^2	α
0.80	0.40	0.00	1.431	0.0000
0.80	0.40	0.25	1.032	0.0000
0.80	0.40	0.50	1.202	0.0000
0.80	0.40	0.75	1.320	0.0000
0.80	0.40	1.00	1.427	0.0000
0.80	0.40	1.30	2.484	0.0008
0.80	0.60	0.00	3.350	0.0037
0.80	0.60	0.25	1.741	0.0001
0.80	0.60	0.50	0.929	0.0000
0.80	0.60	0.75	1.297	0.0000
0.80	0.60	1.00	1.541	0.0001
0.80	0.60	1.30	1.869	0.0002
0.80	0.80	0.00	0.775	0.0000
0.80	0.80	0.25	0.957	0.0000
0.80	0.80	0.50	0.855	0.0000
0.80	0.80	0.75	0.913	0.0000
0.80	0.80	1.00	1.218	0.0000
0.80	0.80	1.30	2.380	0.0006
0.80	1.00	0.00	2.321	0.0005
0.80	1.00	0.25	2.321	0.0005
0.80	1.00	0.50	2.321	0.0005
0.80	1.00	0.75	2.321	0.0005
0.80	1.00	1.00	2.321	0.0005
0.80	1.00	1.30	2.321	0.0005
1.00	0.00	0.00	540.996	1.0000
1.00	0.00	0.25	540.996	1.0000
1.00	0.00	0.50	540.996	1.0000
1.00	0.00	0.75	540.996	1.0000
1.00	0.00	1.00	540.996	1.0000
1.00	0.00	1.30	540.996	1.0000
1.00	0.20	0.00	2.358	0.0006
1.00	0.20	0.25	0.662	0.0000
1.00	0.20	0.50	1.350	0.0000
1.00	0.20	0.75	0.983	0.0000
1.00	0.20	1.00	0.498	0.0000
1.00	0.20	1.30	1.555	0.0001
1.00	0.40	0.00	0.700	0.0000
1.00	0.40	0.25	0.984	0.0000
1.00	0.40	0.50	1.460	0.0000
1.00	0.40	0.75	1.421	0.0000
1.00	0.40	1.00	1.455	0.0000
1.00	0.40	1.30	1.656	0.0001

CRR	RSR	REP	χ^2	α
1.00	0.60	0.00	1.647	0.0001
1.00	0.60	0.25	1.698	0.0001
1.00	0.60	0.50	1.588	0.0001
1.00	0.60	0.75	2.675	0.0011
1.00	0.60	1.00	1.392	0.0000
1.00	0.60	1.30	0.976	0.0000
1.00	0.80	0.00	2.770	0.0014
1.00	0.80	0.25	1.640	0.0001
1.00	0.80	0.50	1.675	0.0001
1.00	0.80	0.75	1.598	0.0001
1.00	0.80	1.00	2.360	0.0006
1.00	0.80	1.30	1.672	0.0001
1.00	1.00	0.00	2962.941	1.0000
1.00	1.00	0.25	2962.941	1.0000
1.00	1.00	0.50	2962.941	1.0000
1.00	1.00	0.75	2962.941	1.0000
1.00	1.00	1.00	2962.941	1.0000
1.00	1.00	1.30	2962.941	1.0000

APPENDIX J

Time	Event	Count	CRR	RSR	REP	SAFTY
850.000	STATS	1	0	1	1.3	-12.4
850.000	STATS	1	0.2	0	0	-15.1
850.000	STATS	1	0.2	0	0.25	-15.1
850.000	STATS	1	0.2	0	0.5	-15.1
850.000	STATS	1	0.2	0	0.75	-15.1
850.000	STATS	1	0.2	0	1	-15.1
850.000	STATS	1	0.2	0	1.3	-15.1
850.000	STATS	1	0.2	0.2	0	-11.7
850.000	STATS	1	0.2	0.2	0.25	-11.8
850.000	STATS	1	0.2	0.2	0.5	-11.9
850.000	STATS	1	0.2	0.2	0.75	-12.1
850.000	STATS	1	0.2	0.2	1	-12.2
850.000	STATS	1	0.2	0.2	1.3	-12.3
850.000	STATS	1	0.2	0.4	0	-8.82
850.000	STATS	1	0.2	0.4	0.25	-8.74
850.000	STATS	1	0.2	0.4	0.5	-8.98
850.000	STATS	1	0.2	0.4	0.75	-9.25
850.000	STATS	1	0.2	0.4	1	-9.59
850.000	STATS	1	0.2	0.4	1.3	-9.98
850.000	STATS	1	0.2	0.6	0	-6.36
850.000	STATS	1	0.2	0.6	0.25	-6.09
850.000	STATS	1	0.2	0.6	0.5	-6.52
850.000	STATS	1	0.2	0.6	0.75	-7.08
850.000	STATS	1	0.2	0.6	1	-7.59
850.000	STATS	1	0.2	0.6	1.3	-8.2
850.000	STATS	1	0.2	0.8	0	-3.78
850.000	STATS	1	0.2	0.8	0.25	-3.34
850.000	STATS	1	0.2	0.8	0.5	-4.02
850.000	STATS	1	0.2	0.8	0.75	-4.73
850.000	STATS	1	0.2	0.8	1	-5.53
850.000	STATS	1	0.2	0.8	1.3	-6.2
850.000	STATS	1	0.2	1	0	-1.02
850.000	STATS	1	0.2	1	0.25	-0.523
850.000	STATS	1	0.2	1	0.5	-1.33
850.000	STATS	1	0.2	1	0.75	-2.3
850.000	STATS	1	0.2	1	1	-3.14
850.000	STATS	1	0.2	1	1.3	-4.19

850.000	STATS	1	0.4	0	0	-14.5
850.000	STATS	1	0.4	0	0.25	-14.5
850.000	STATS	1	0.4	0	0.5	-14.5
850.000	STATS	1	0.4	0	0.75	-14.5
850.000	STATS	1	0.4	0	1	-14.5
850.000	STATS	1	0.4	0	1.3	-14.5
850.000	STATS	1	0.4	0.2	0	-8.14
850.000	STATS	1	0.4	0.2	0.25	-8.26
850.000	STATS	1	0.4	0.2	0.5	-8.57
850.000	STATS	1	0.4	0.2	0.75	-8.91
850.000	STATS	1	0.4	0.2	1	-9.18
850.000	STATS	1	0.4	0.2	1.3	-9.6
850.000	STATS	1	0.4	0.4	0	-2.91
850.000	STATS	1	0.4	0.4	0.25	-3.09
850.000	STATS	1	0.4	0.4	0.5	-3.77
850.000	STATS	1	0.4	0.4	0.75	-4.49
850.000	STATS	1	0.4	0.4	1	-5.24
850.000	STATS	1	0.4	0.4	1.3	-5.96
850.000	STATS	1	0.4	0.6	0	2.32
850.000	STATS	1	0.4	0.6	0.25	2.16
850.000	STATS	1	0.4	0.6	0.5	1.12
850.000	STATS	1	0.4	0.6	0.75	0.0438
850.000	STATS	1	0.4	0.6	1	-1.08
850.000	STATS	1	0.4	0.6	1.3	-2.28
850.000	STATS	1	0.4	0.8	0	7.3
850.000	STATS	1	0.4	0.8	0.25	7.26
850.000	STATS	1	0.4	0.8	0.5	5.79
850.000	STATS	1	0.4	0.8	0.75	4.34
850.000	STATS	1	0.4	0.8	1	2.96
850.000	STATS	1	0.4	0.8	1.3	1.21
850.000	STATS	1	0.4	1	0	12.3
850.000	STATS	1	0.4	1	0.25	12.1
850.000	STATS	1	0.4	1	0.5	10.4
850.000	STATS	1	0.4	1	0.75	8.57
850.000	STATS	1	0.4	1	1	6.82
850.000	STATS	1	0.4	1	1.3	4.75
850.000	STATS	1	0.6	0	0	-14.8
850.000	STATS	1	0.6	0	0.25	-14.8
850.000	STATS	1	0.6	0	0.5	-14.8
850.000	STATS	1	0.6	0	0.75	-14.8
850.000	STATS	1	0.6	0	1	-14.8
850.000	STATS	1	0.6	0	1.3	-14.8

850.000	STATS	1	0.6	0.2	0	-5.56
850.000	STATS	1	0.6	0.2	0.25	-5.82
850.000	STATS	1	0.6	0.2	0.5	-6.41
850.000	STATS	1	0.6	0.2	0.75	-6.85
850.000	STATS	1	0.6	0.2	1	-7.42
850.000	STATS	1	0.6	0.2	1.3	-7.96
850.000	STATS	1	0.6	0.4	0	2.21
850.000	STATS	1	0.6	0.4	0.25	1.8
850.000	STATS	1	0.6	0.4	0.5	0.799
850.000	STATS	1	0.6	0.4	0.75	-0.188
850.000	STATS	1	0.6	0.4	1	-1.22
850.000	STATS	1	0.6	0.4	1.3	-2.35
850.000	STATS	1	0.6	0.6	0	9.5
850.000	STATS	1	0.6	0.6	0.25	8.88
850.000	STATS	1	0.6	0.6	0.5	7.32
850.000	STATS	1	0.6	0.6	0.75	5.77
850.000	STATS	1	0.6	0.6	1	4.17
850.000	STATS	1	0.6	0.6	1.3	2.4
850.000	STATS	1	0.6	0.8	0	16.9
850.000	STATS	1	0.6	0.8	0.25	16
850.000	STATS	1	0.6	0.8	0.5	13.9
850.000	STATS	1	0.6	0.8	0.75	11.9
850.000	STATS	1	0.6	0.8	1	9.79
850.000	STATS	1	0.6	0.8	1.3	7.19
850.000	STATS	1	0.6	1	0	24.5
850.000	STATS	1	0.6	1	0.25	23.6
850.000	STATS	1	0.6	1	0.5	21
850.000	STATS	1	0.6	1	0.75	18.3
850.000	STATS	1	0.6	1	1	15.6
850.000	STATS	1	0.6	1	1.3	12.5
850.000	STATS	1	0.8	0	0	-12.8
850.000	STATS	1	0.8	0	0.25	-12.8
850.000	STATS	1	0.8	0	0.5	-12.8
850.000	STATS	1	0.8	0	0.75	-12.8
850.000	STATS	1	0.8	0	1	-12.8
850.000	STATS	1	0.8	0	1.3	-12.8
850.000	STATS	1	0.8	0.2	0	-0.547
850.000	STATS	1	0.8	0.2	0.25	-0.981
850.000	STATS	1	0.8	0.2	0.5	-1.62
850.000	STATS	1	0.8	0.2	0.75	-2.3
850.000	STATS	1	0.8	0.2	1	-2.89
850.000	STATS	1	0.8	0.2	1.3	-3.74

850.000	STATS	1	0.8	0.4	0	9.66
850.000	STATS	1	0.8	0.4	0.25	8.92
850.000	STATS	1	0.8	0.4	0.5	7.56
850.000	STATS	1	0.8	0.4	0.75	6.13
850.000	STATS	1	0.8	0.4	1	4.81
850.000	STATS	1	0.8	0.4	1.3	3.22
850.000	STATS	1	0.8	0.6	0	18.8
850.000	STATS	1	0.8	0.6	0.25	17.8
850.000	STATS	1	0.8	0.6	0.5	15.7
850.000	STATS	1	0.8	0.6	0.75	13.6
850.000	STATS	1	0.8	0.6	1	11.5
850.000	STATS	1	0.8	0.6	1.3	9.02
850.000	STATS	1	0.8	0.8	0	28.6
850.000	STATS	1	0.8	0.8	0.25	27.3
850.000	STATS	1	0.8	0.8	0.5	24.5
850.000	STATS	1	0.8	0.8	0.75	21.6
850.000	STATS	1	0.8	0.8	1	18.9
850.000	STATS	1	0.8	0.8	1.3	15.5
850.000	STATS	1	0.8	1	0	38.1
850.000	STATS	1	0.8	1	0.25	36.5
850.000	STATS	1	0.8	1	0.5	33.1
850.000	STATS	1	0.8	1	0.75	29.5
850.000	STATS	1	0.8	1	1	26
850.000	STATS	1	0.8	1	1.3	21.8
850.000	STATS	1	1	0	0	-13.2
850.000	STATS	1	1	0	0.25	-13.2
850.000	STATS	1	1	0	0.5	-13.2
850.000	STATS	1	1	0	0.75	-13.2
850.000	STATS	1	1	0	1	-13.2
850.000	STATS	1	1	0	1.3	-13.2
850.000	STATS	1	1	0.2	0	0.38
850.000	STATS	1	1	0.2	0.25	-0.163
850.000	STATS	1	1	0.2	0.5	-1.02
850.000	STATS	1	1	0.2	0.75	-1.93
850.000	STATS	1	1	0.2	1	-2.87
850.000	STATS	1	1	0.2	1.3	-3.94
850.000	STATS	1	1	0.4	0	13.4
850.000	STATS	1	1	0.4	0.25	12.3
850.000	STATS	1	1	0.4	0.5	10.5
850.000	STATS	1	1	0.4	0.75	8.62
850.000	STATS	1	1	0.4	1	6.83
850.000	STATS	1	1	0.4	1.3	4.61

850.000	STATS	1	1	0.6	0	25.6
850.000	STATS	1	1	0.6	0.25	23.9
850.000	STATS	1	1	0.6	0.5	21.2
850.000	STATS	1	1	0.6	0.75	18.5
850.000	STATS	1	1	0.6	1	15.8
850.000	STATS	1	1	0.6	1.3	12.5
850.000	STATS	1	1	0.8	0	38.3
850.000	STATS	1	1	0.8	0.25	36
850.000	STATS	1	1	0.8	0.5	32.4
850.000	STATS	1	1	0.8	0.75	28.7
850.000	STATS	1	1	0.8	1	25.1
850.000	STATS	1	1	0.8	1.3	20.7
850.000	STATS	1	1	1	0	55
850.000	STATS	1	1	1	0.25	52.4
850.000	STATS	1	1	1	0.5	47.8
850.000	STATS	1	1	1	0.75	43.2
850.000	STATS	1	1	1	1	38.7
850.000	STATS	1	1	1	1.3	33.2

APPENDIX K

CRR	RSR	REP	MEAN	VARIANCE	ST-DEV
0.0	1.0	1.30	-6.924	83.4027	9.1325
0.2	0.0	0.00	-9.586	90.7717	9.5274
0.2	0.0	0.25	-9.586	90.7717	9.5274
0.2	0.0	0.50	-9.586	90.7717	9.5274
0.2	0.0	0.75	-9.586	90.7717	9.5274
0.2	0.0	1.00	-9.586	90.7717	9.5274
0.2	0.0	1.30	-9.586	90.7717	9.5274
0.2	0.2	0.00	-7.192	87.9998	9.3808
0.2	0.2	0.25	-7.363	87.8412	9.3724
0.2	0.2	0.50	-7.526	87.9361	9.3774
0.2	0.2	0.75	-7.692	88.0031	9.3810
0.2	0.2	1.00	-7.842	87.7895	9.3696
0.2	0.2	1.30	-8.047	87.9049	9.3758
0.2	0.4	0.00	-4.756	86.6598	9.3091
0.2	0.4	0.25	-5.160	86.3786	9.2940
0.2	0.4	0.50	-5.520	86.5386	9.3026
0.2	0.4	0.75	-5.870	86.6164	9.3068
0.2	0.4	1.00	-6.209	86.4455	9.2976
0.2	0.4	1.30	-6.626	86.4795	9.2994
0.2	0.6	0.00	-2.474	84.1084	9.1711
0.2	0.6	0.25	-2.989	83.8024	9.1544
0.2	0.6	0.50	-3.498	84.0638	9.1686
0.2	0.6	0.75	-4.018	84.0494	9.1678
0.2	0.6	1.00	-4.549	84.2198	9.1771
0.2	0.6	1.30	-5.193	84.5408	9.1946
0.2	0.8	0.00	0.036	78.9848	8.8873
0.2	0.8	0.25	-0.668	78.8546	8.8800
0.2	0.8	0.50	-1.382	79.0709	8.8922
0.2	0.8	0.75	-2.087	79.2589	8.9027
0.2	0.8	1.00	-2.804	78.9891	8.8876
0.2	0.8	1.30	-3.617	79.7442	8.9300
0.2	1.0	0.00	2.807	76.6830	8.7569
0.2	1.0	0.25	1.916	76.4364	8.7428
0.2	1.0	0.50	1.022	76.6377	8.7543
0.2	1.0	0.75	0.127	77.2647	8.7900
0.2	1.0	1.00	-0.768	77.9995	8.8317
0.2	1.0	1.30	-1.844	78.9419	8.8849
0.4	0.0	0.00	-9.227	78.9929	8.8878
0.4	0.0	0.25	-9.227	78.9929	8.8878
0.4	0.0	0.50	-9.227	78.9929	8.8878
0.4	0.0	0.75	-9.227	78.9929	8.8878

0.4	0.0	1.00	-9.227	78.9929	8.8878
0.4	0.0	1.30	-9.227	78.9929	8.8878
0.4	0.2	0.00	-4.568	73.4691	8.5714
0.4	0.2	0.25	-4.885	73.5367	8.5754
0.4	0.2	0.50	-5.190	73.2943	8.5612
0.4	0.2	0.75	-5.507	73.0319	8.5459
0.4	0.2	1.00	-5.837	73.4869	8.5725
0.4	0.2	1.30	-6.220	73.9867	8.6016
0.4	0.4	0.00	0.159	65.7100	8.1062
0.4	0.4	0.25	-0.520	65.6389	8.1018
0.4	0.4	0.50	-1.205	66.4721	8.1530
0.4	0.4	0.75	-1.924	66.6263	8.1625
0.4	0.4	1.00	-2.653	66.3979	8.1485
0.4	0.4	1.30	-3.424	66.4627	8.1525
0.4	0.6	0.00	5.076	66.3885	8.1479
0.4	0.6	0.25	4.053	65.5499	8.0963
0.4	0.6	0.50	3.035	64.2299	8.0144
0.4	0.6	0.75	1.955	63.9193	7.9950
0.4	0.6	1.00	0.944	64.1926	8.0120
0.4	0.6	1.30	-0.247	64.3586	8.0224
0.4	0.8	0.00	10.324	57.9158	7.6102
0.4	0.8	0.25	8.959	57.6615	7.5935
0.4	0.8	0.50	7.543	58.2234	7.6304
0.4	0.8	0.75	6.097	59.3593	7.7045
0.4	0.8	1.00	4.698	59.6415	7.7228
0.4	0.8	1.30	3.002	61.2233	7.8245
0.4	1.0	0.00	15.434	59.4418	7.7099
0.4	1.0	0.25	13.654	58.6721	7.6598
0.4	1.0	0.50	11.873	58.9733	7.6794
0.4	1.0	0.75	10.092	59.4929	7.7132
0.4	1.0	1.00	8.310	60.0643	7.7501
0.4	1.0	1.30	6.170	60.8938	7.8034
0.6	0.0	0.00	-9.508	74.4691	8.6295
0.6	0.0	0.25	-9.508	74.4691	8.6295
0.6	0.0	0.50	-9.508	74.4691	8.6295
0.6	0.0	0.75	-9.508	74.4691	8.6295
0.6	0.0	1.00	-9.508	74.4691	8.6295
0.6	0.0	1.30	-9.508	74.4691	8.6295
0.6	0.2	0.00	-2.393	69.6120	8.3434
0.6	0.2	0.25	-2.871	69.3930	8.3302
0.6	0.2	0.50	-3.373	69.7952	8.3543
0.6	0.2	0.75	-3.891	70.2010	8.3786
0.6	0.2	1.00	-4.442	70.5033	8.3966
0.6	0.2	1.30	-5.079	70.4697	8.3946
0.6	0.4	0.00	4.918	64.8864	8.0552
0.6	0.4	0.25	3.900	65.2406	8.0772
0.6	0.4	0.50	2.924	66.4273	8.1503
0.6	0.4	0.75	1.900	66.4847	8.1538
0.6	0.4	1.00	0.847	65.9220	8.1192
0.6	0.4	1.30	-0.371	65.5250	8.0948
0.6	0.6	0.00	12.312	55.4951	7.4495

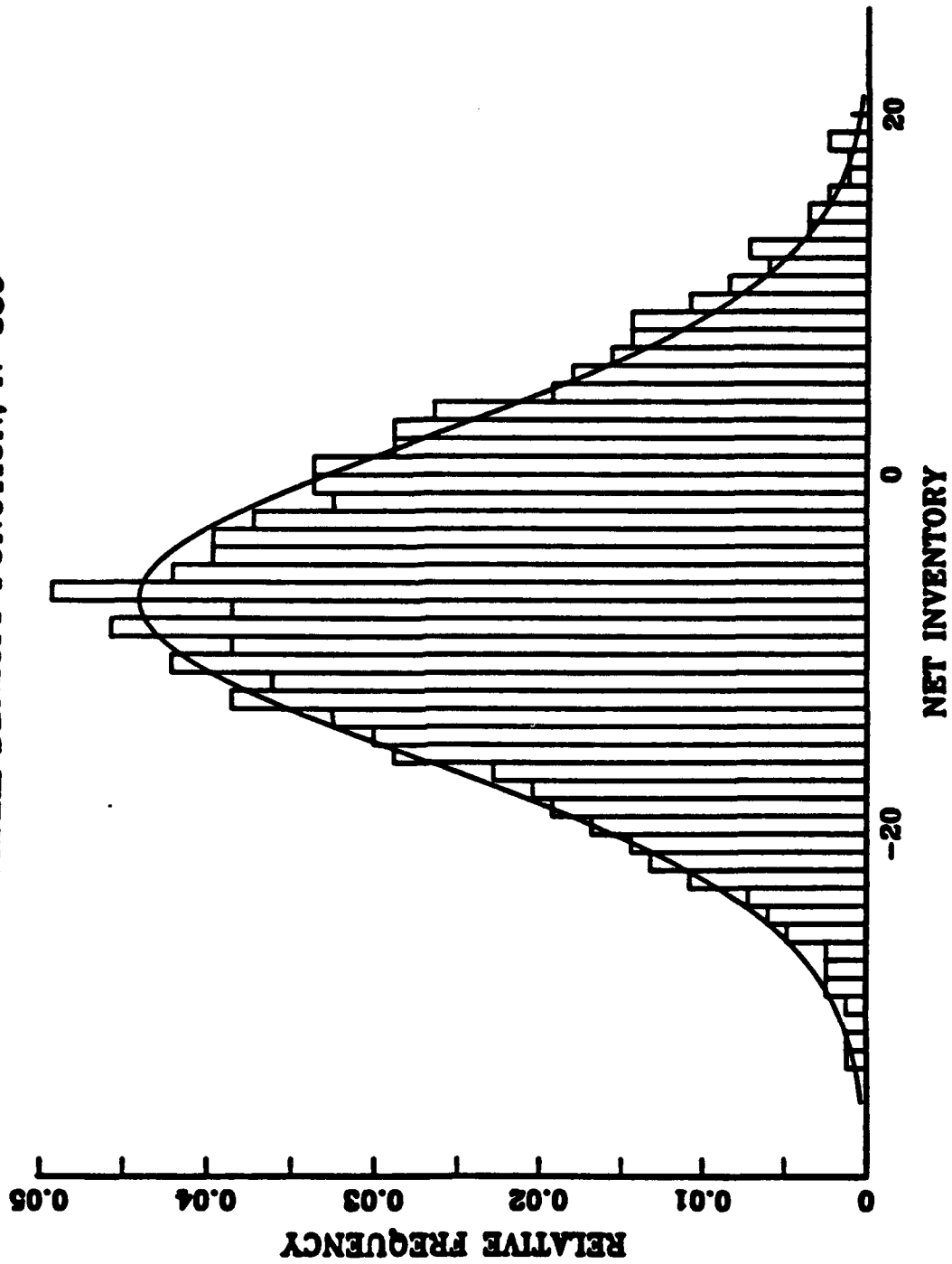
0.6	0.6	0.25	10.712	55.2881	7.4356
0.6	0.6	0.50	9.156	56.0255	7.4850
0.6	0.6	0.75	7.529	57.3637	7.5739
0.6	0.6	1.00	5.932	57.9952	7.6155
0.6	0.6	1.30	4.077	59.4761	7.7121
0.6	0.8	0.00	19.779	49.8901	7.0633
0.6	0.8	0.25	17.642	50.8193	7.1288
0.6	0.8	0.50	15.528	52.4471	7.2420
0.6	0.8	0.75	13.430	54.0038	7.3487
0.6	0.8	1.00	11.281	55.1402	7.4256
0.6	0.8	1.30	8.660	57.0515	7.5532
0.6	1.0	0.00	27.649	44.9441	6.7040
0.6	1.0	0.25	24.963	45.0078	6.7088
0.6	1.0	0.50	22.278	46.5564	6.8232
0.6	1.0	0.75	19.594	48.0005	6.9282
0.6	1.0	1.00	16.908	49.3433	7.0245
0.6	1.0	1.30	13.683	50.7103	7.1211
0.8	0.0	0.00	-7.236	92.4231	9.6137
0.8	0.0	0.25	-7.236	92.4231	9.6137
0.8	0.0	0.50	-7.236	92.4231	9.6137
0.8	0.0	0.75	-7.236	92.4231	9.6137
0.8	0.0	1.00	-7.236	92.4231	9.6137
0.8	0.0	1.30	-7.236	92.4231	9.6137
0.8	0.2	0.00	2.382	79.8981	8.9386
0.8	0.2	0.25	1.622	80.6850	8.9825
0.8	0.2	0.50	0.921	81.4997	9.0277
0.8	0.2	0.75	0.280	82.1106	9.0615
0.8	0.2	1.00	-0.332	82.7030	9.0941
0.8	0.2	1.30	-1.116	83.8587	9.1574
0.8	0.4	0.00	12.141	67.3923	8.2093
0.8	0.4	0.25	10.742	68.3175	8.2654
0.8	0.4	0.50	9.350	69.5608	8.3403
0.8	0.4	0.75	7.957	71.4155	8.4508
0.8	0.4	1.00	6.607	73.2690	8.5597
0.8	0.4	1.30	4.970	74.6654	8.6409
0.8	0.6	0.00	21.376	56.2820	7.5021
0.8	0.6	0.25	19.326	57.7322	7.5982
0.8	0.6	0.50	17.197	59.5051	7.7140
0.8	0.6	0.75	15.111	59.5713	7.7182
0.8	0.6	1.00	12.983	59.7227	7.7280
0.8	0.6	1.30	10.522	62.4625	7.9033
0.8	0.8	0.00	31.312	41.2927	6.4259
0.8	0.8	0.25	28.528	41.6101	6.4506
0.8	0.8	0.50	25.711	43.2976	6.5801
0.8	0.8	0.75	22.923	44.8130	6.6943
0.8	0.8	1.00	20.139	46.1667	6.7946
0.8	0.8	1.30	16.777	48.5269	6.9661
0.8	1.0	0.00	41.216	33.3640	5.7762
0.8	1.0	0.25	37.704	33.9749	5.8288
0.8	1.0	0.50	34.192	36.1890	6.0157
0.8	1.0	0.75	30.680	38.4534	6.2011

0.8	1.0	1.00	27.170	41.3674	6.4318
0.8	1.0	1.30	22.960	44.8398	6.6963
1.0	0.0	0.00	-7.705	87.5252	9.3555
1.0	0.0	0.25	-7.705	87.5252	9.3555
1.0	0.0	0.50	-7.705	87.5252	9.3555
1.0	0.0	0.75	-7.705	87.5252	9.3555
1.0	0.0	1.00	-7.705	87.5252	9.3555
1.0	0.0	1.30	-7.705	87.5252	9.3555
1.0	0.2	0.00	3.102	69.0518	8.3097
1.0	0.2	0.25	2.222	70.4545	8.3937
1.0	0.2	0.50	1.350	72.1900	8.4965
1.0	0.2	0.75	0.456	73.6136	8.5798
1.0	0.2	1.00	-0.497	75.5410	8.6914
1.0	0.2	1.30	-1.555	76.7505	8.7607
1.0	0.4	0.00	15.668	59.2519	7.6975
1.0	0.4	0.25	13.836	60.0704	7.7505
1.0	0.4	0.50	11.991	61.2048	7.8233
1.0	0.4	0.75	10.170	61.3440	7.8322
1.0	0.4	1.00	8.387	62.4435	7.9021
1.0	0.4	1.30	6.186	64.6141	8.0383
1.0	0.6	0.00	27.848	51.4302	7.1715
1.0	0.6	0.25	25.138	51.9700	7.2090
1.0	0.6	0.50	22.496	53.5713	7.3192
1.0	0.6	0.75	19.834	54.8789	7.4080
1.0	0.6	1.00	17.081	56.3733	7.5082
1.0	0.6	1.30	13.749	58.7922	7.6676
1.0	0.8	0.00	40.888	33.4163	5.7807
1.0	0.8	0.25	37.156	35.1384	5.9278
1.0	0.8	0.50	33.508	37.2102	6.1000
1.0	0.8	0.75	29.814	39.3533	6.2732
1.0	0.8	1.00	26.169	41.7881	6.4644
1.0	0.8	1.30	21.809	45.6746	6.7583
1.0	1.0	0.00	58.057	14.5254	3.8112
1.0	1.0	0.25	53.467	16.0579	4.0072
1.0	1.0	0.50	48.877	19.4832	4.4140
1.0	1.0	0.75	44.287	22.4841	4.7417
1.0	1.0	1.00	39.696	25.1406	5.0140
1.0	1.0	1.30	34.190	28.3186	5.3215

APPENDIX L

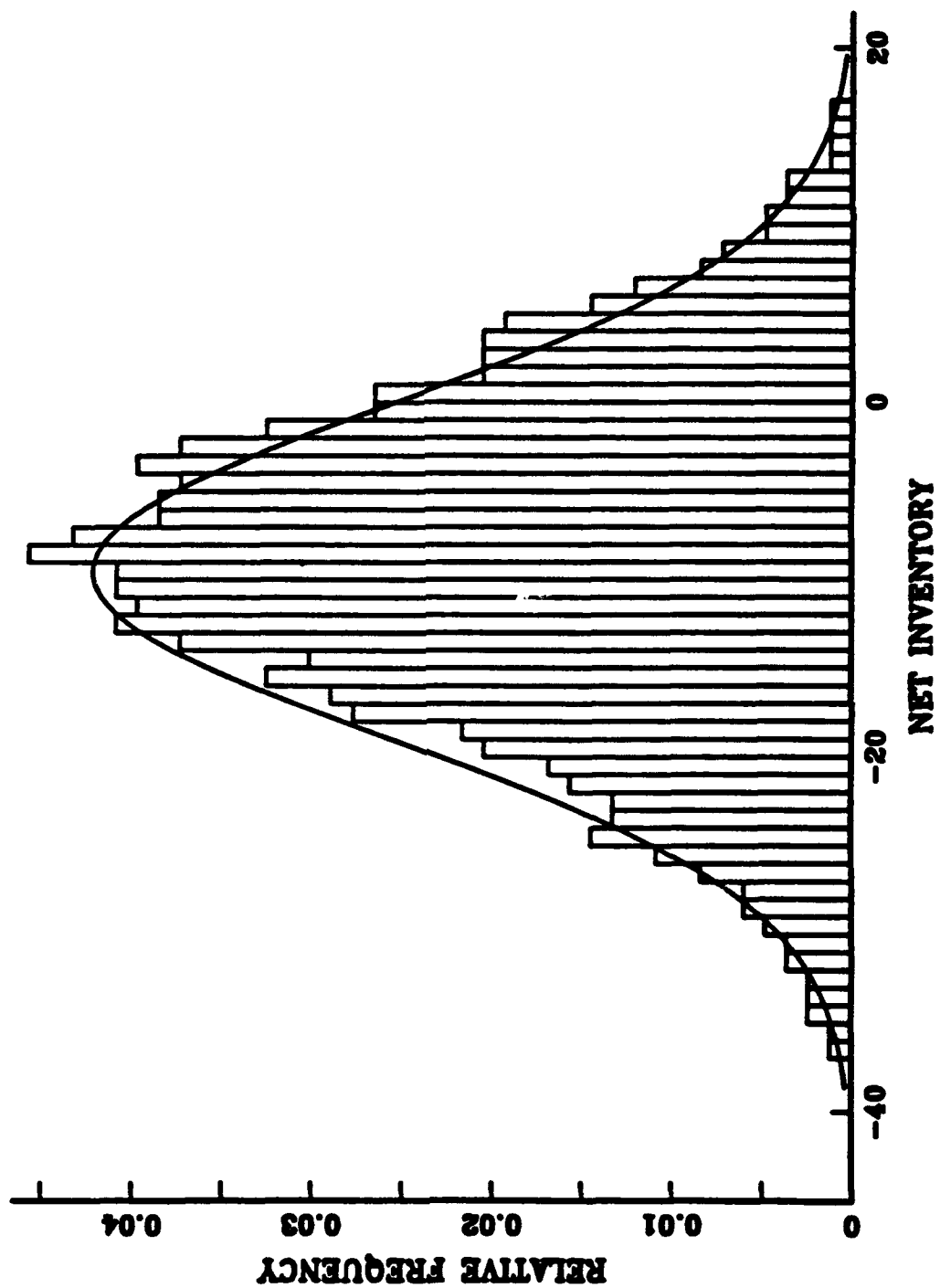
CRR = 0.0; RSR = 1.0; REP = 1.0

NORMAL DENSITY FUNCTION, N=830



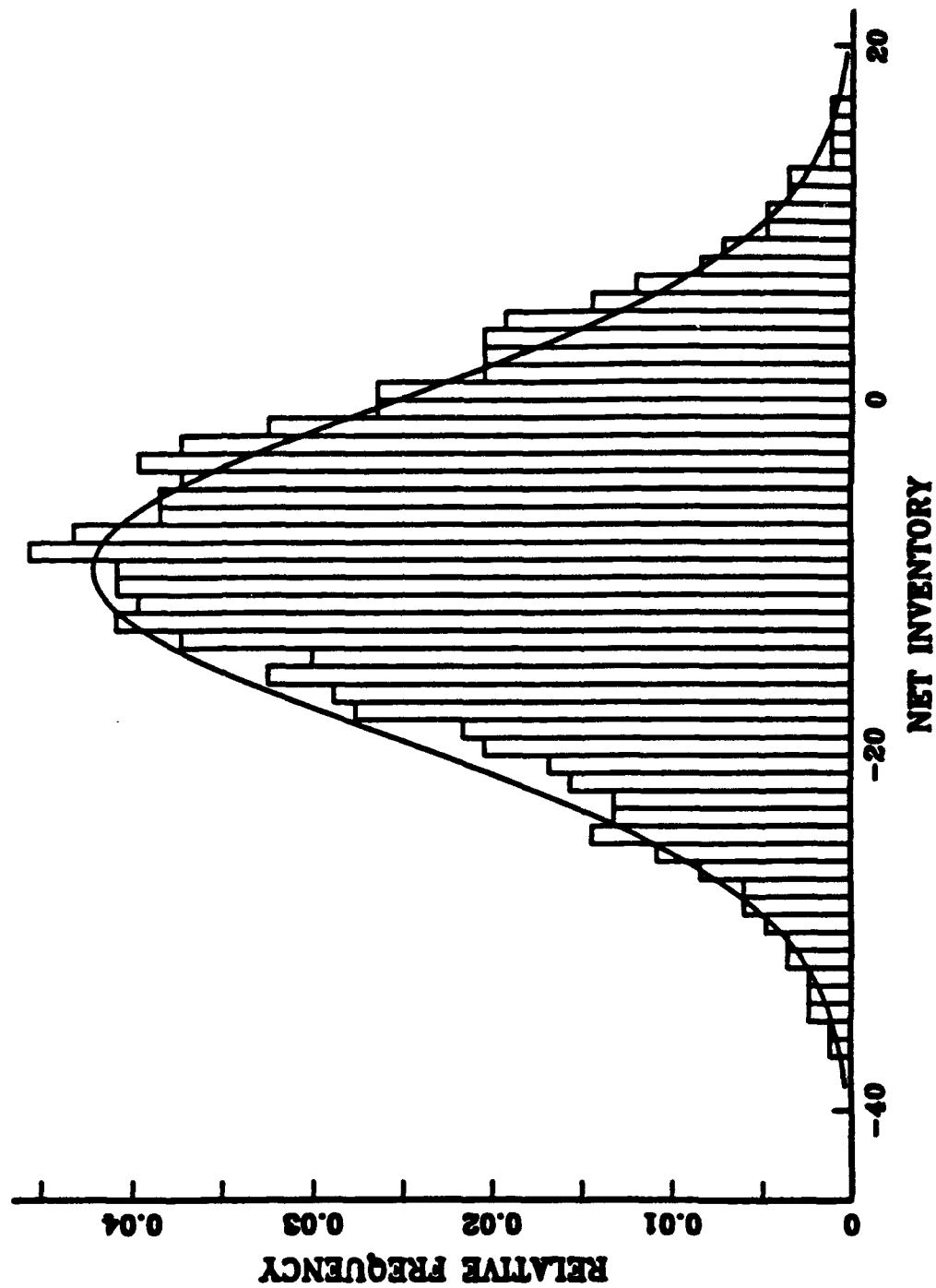
CRR = 0.2; RSR = 0.0; REP = 0.0

NORMAL DENSITY FUNCTION, N=829



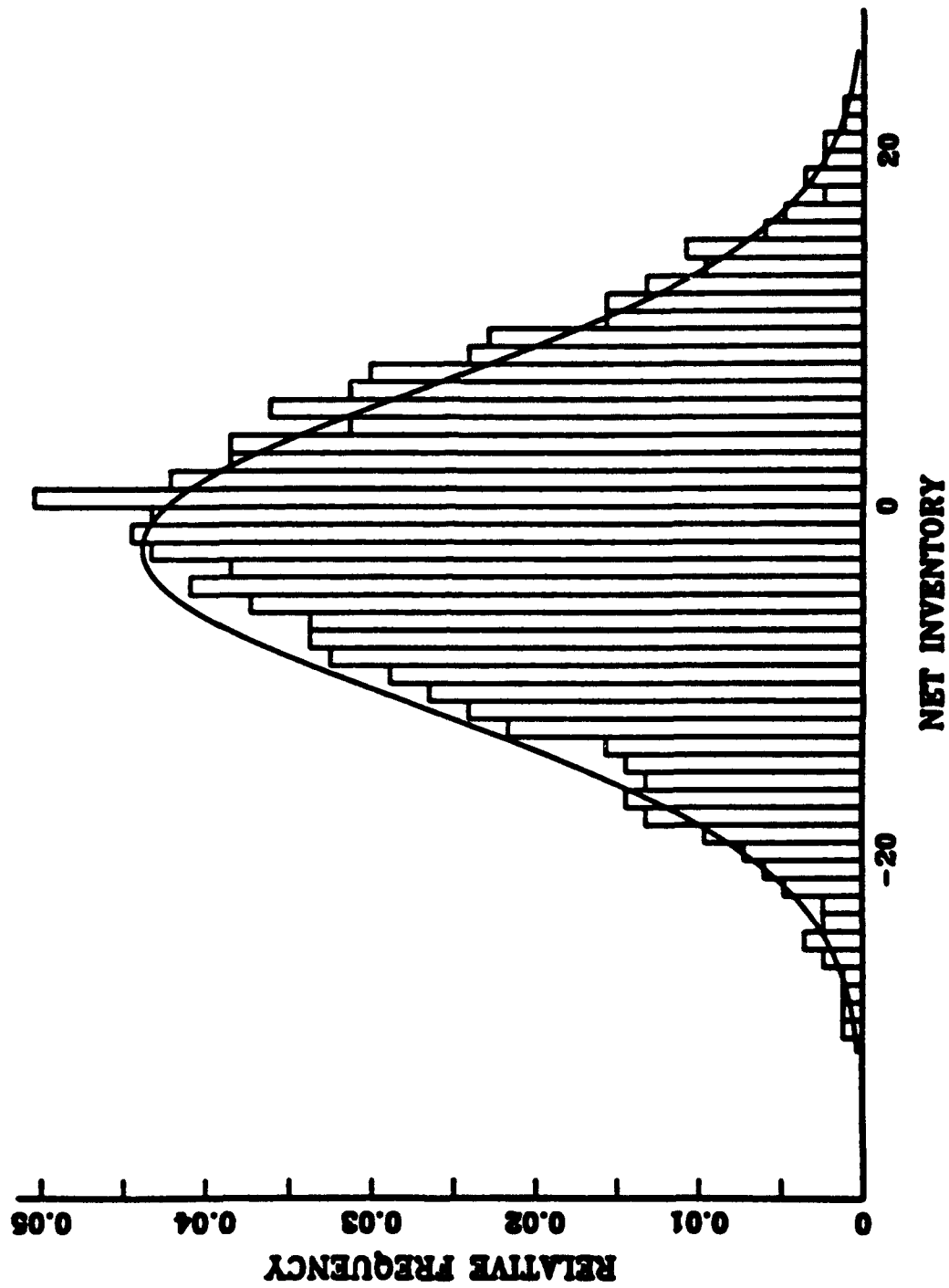
CRR = 0.2; RSR = 0.0; REP = 0.25

NORMAL DENSITY FUNCTION, N=829



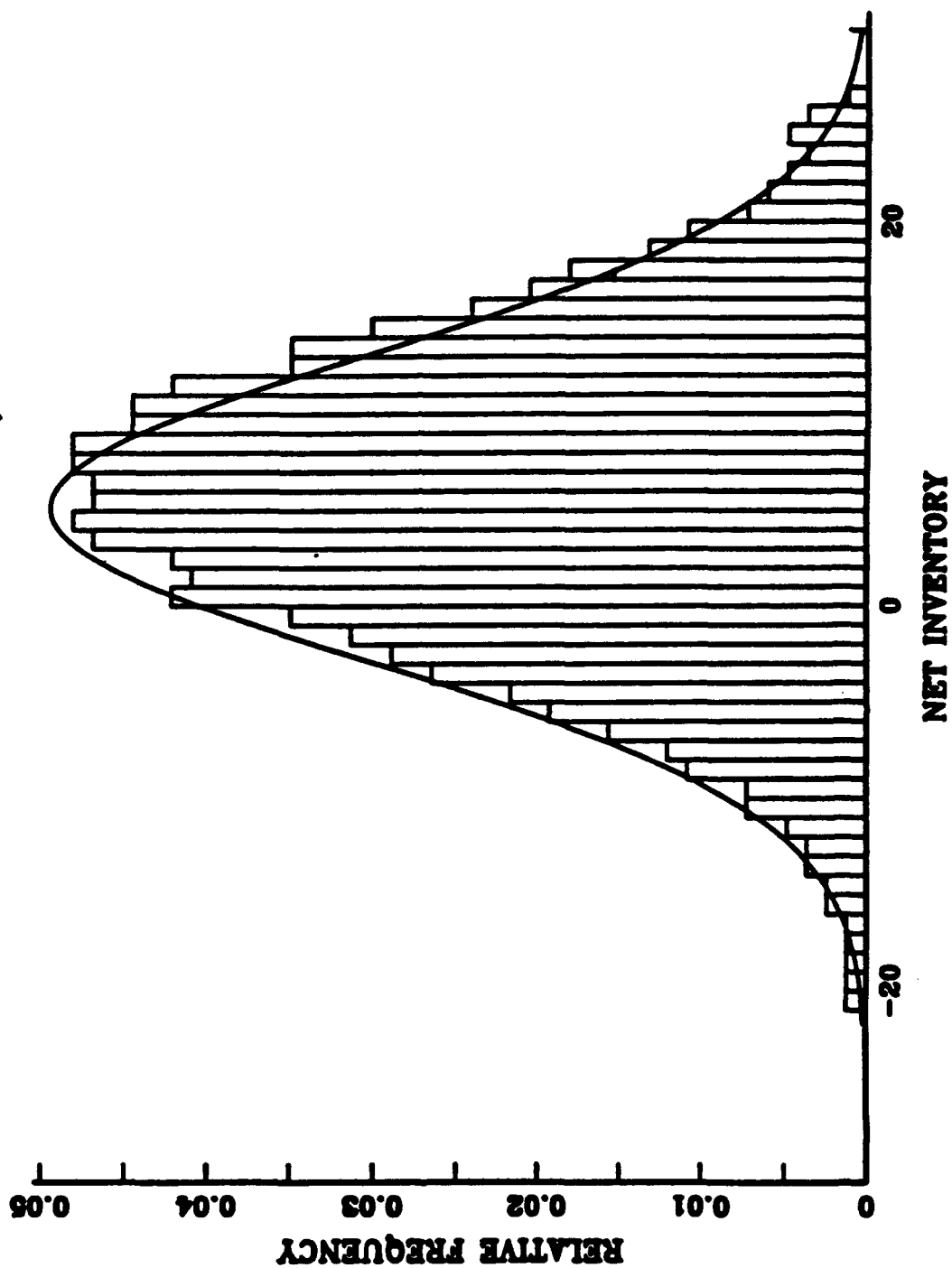
CRR = 0.2; RSR = 0.6; REP = 0.0

NORMAL DENSITY FUNCTION, N=829



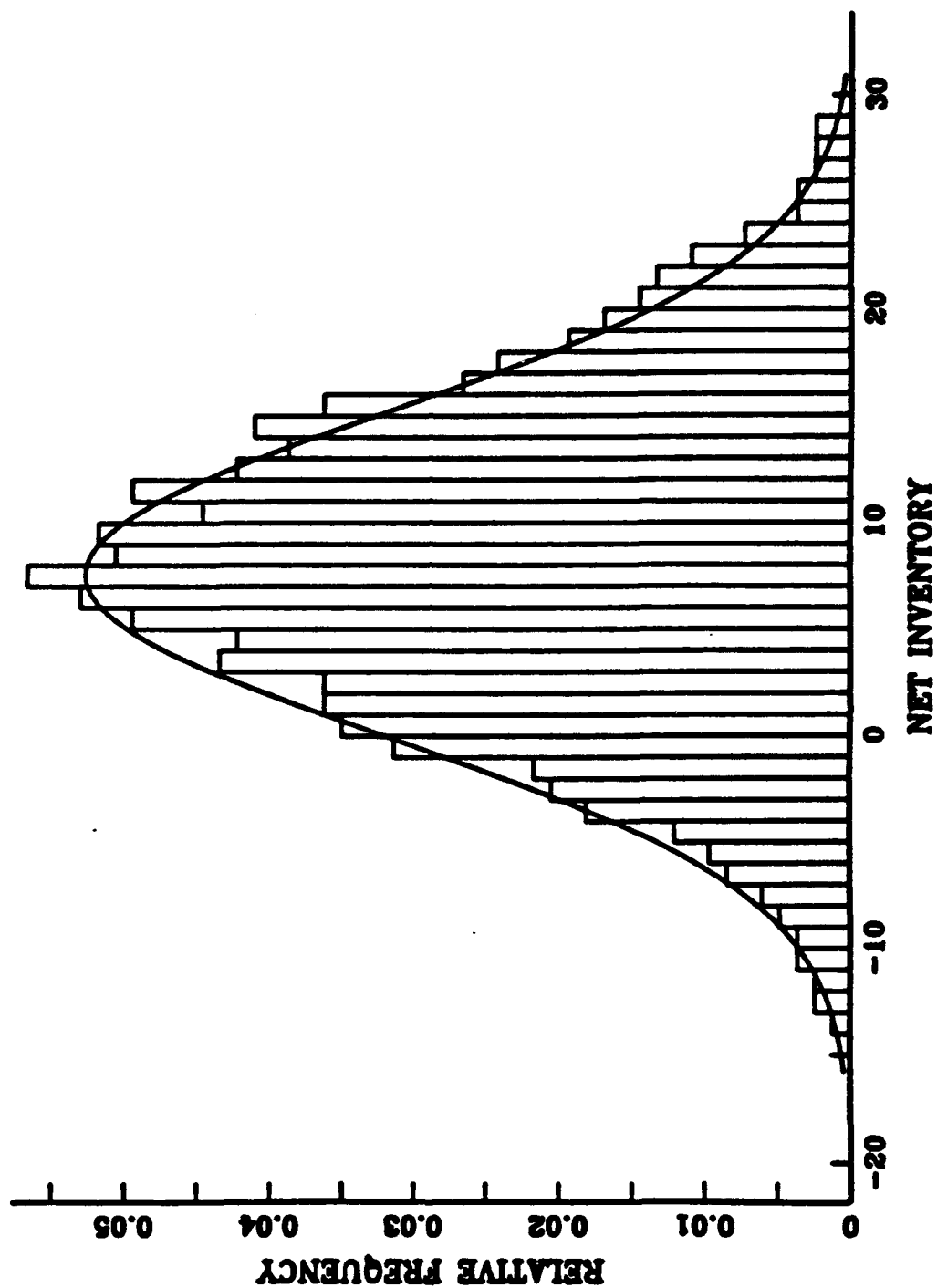
CRR = 0.4; RSR = 0.6; REP = 0.0

NORMAL DENSITY FUNCTION, N=829



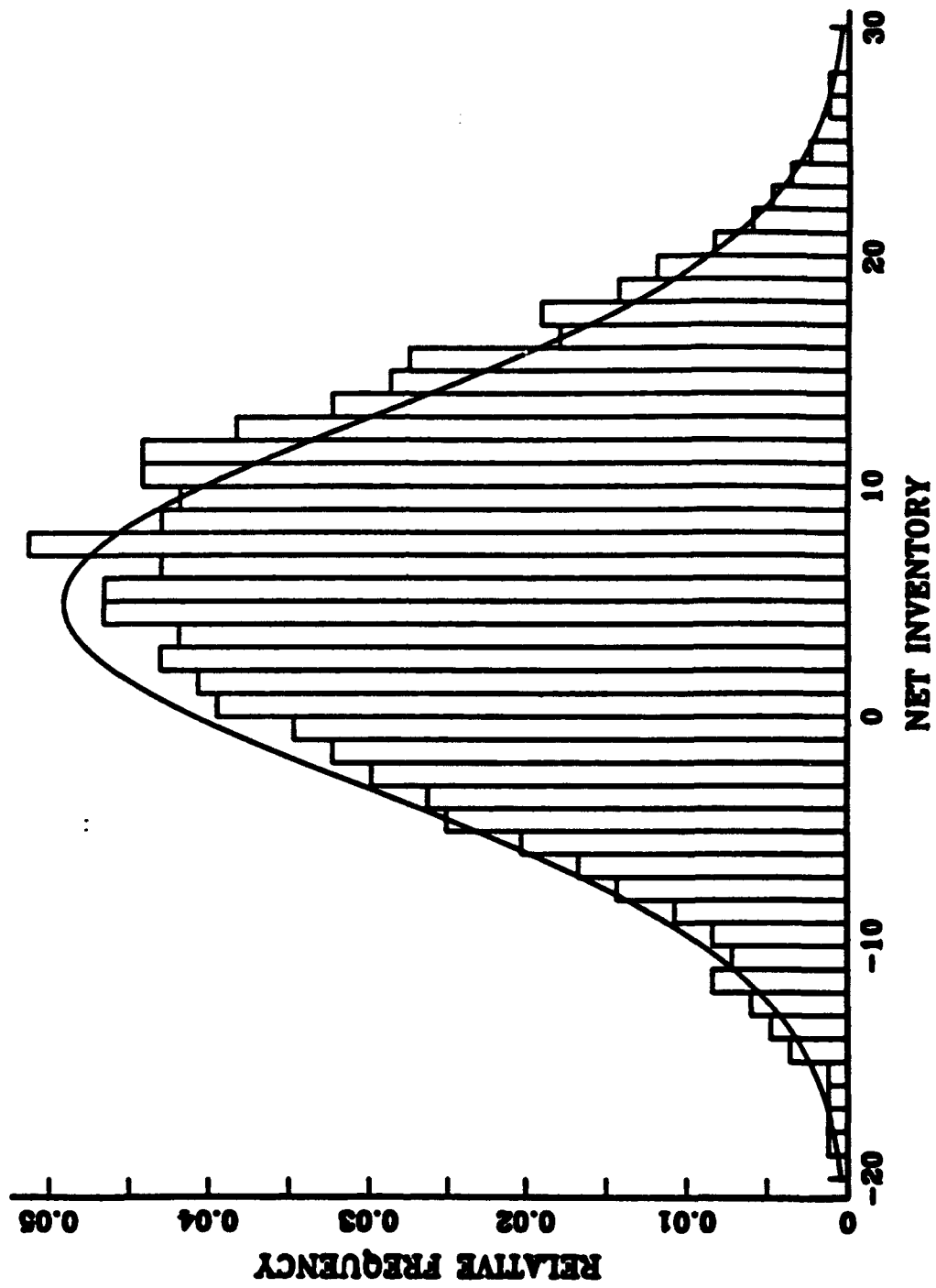
CRR = 0.4; RSR = 0.8; REP = 0.5

NORMAL DENSITY FUNCTION, N=829



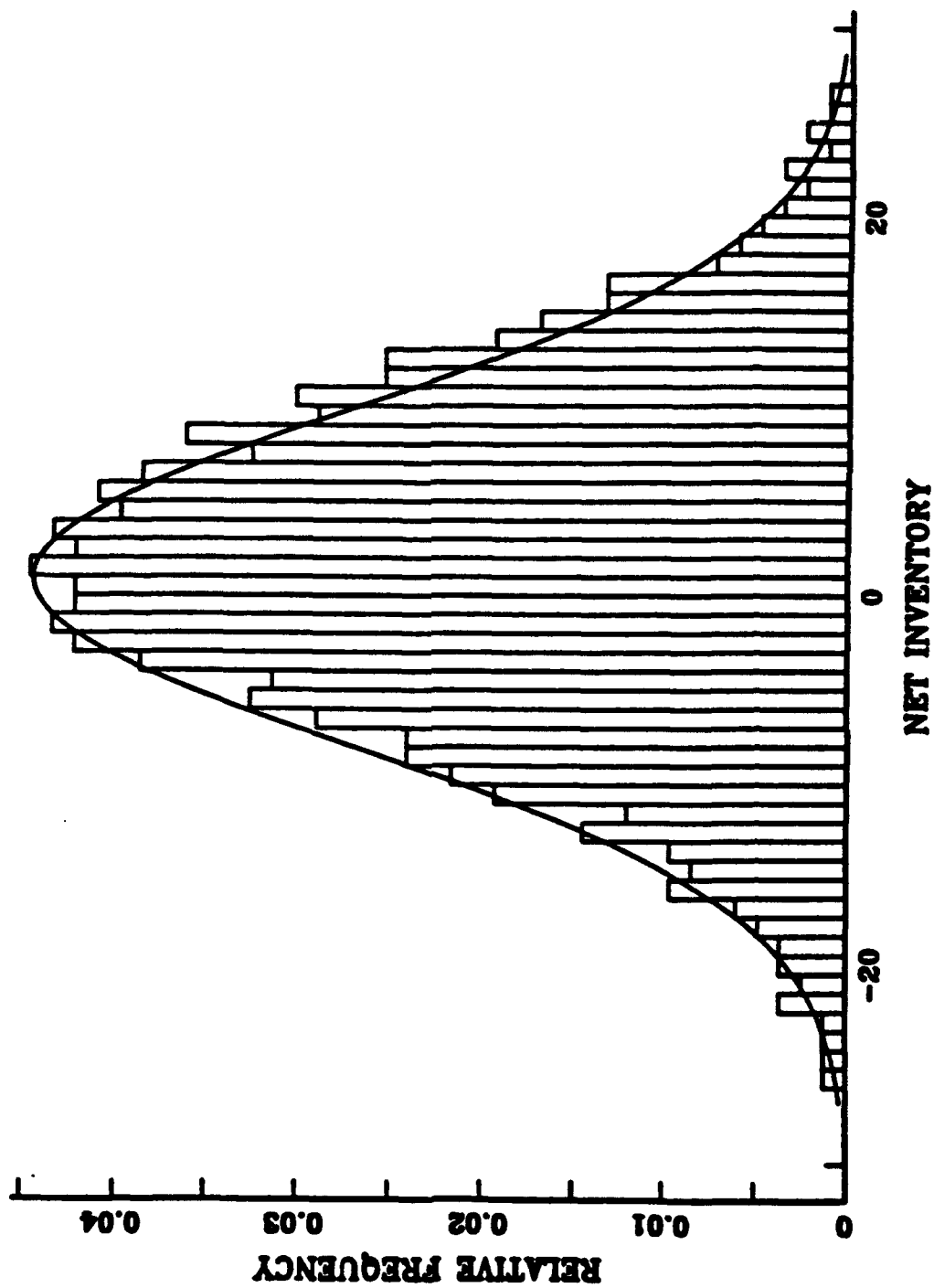
CRR = 0.6; RSR = 0.4; REP = 0.0

NORMAL DENSITY FUNCTION, N=834



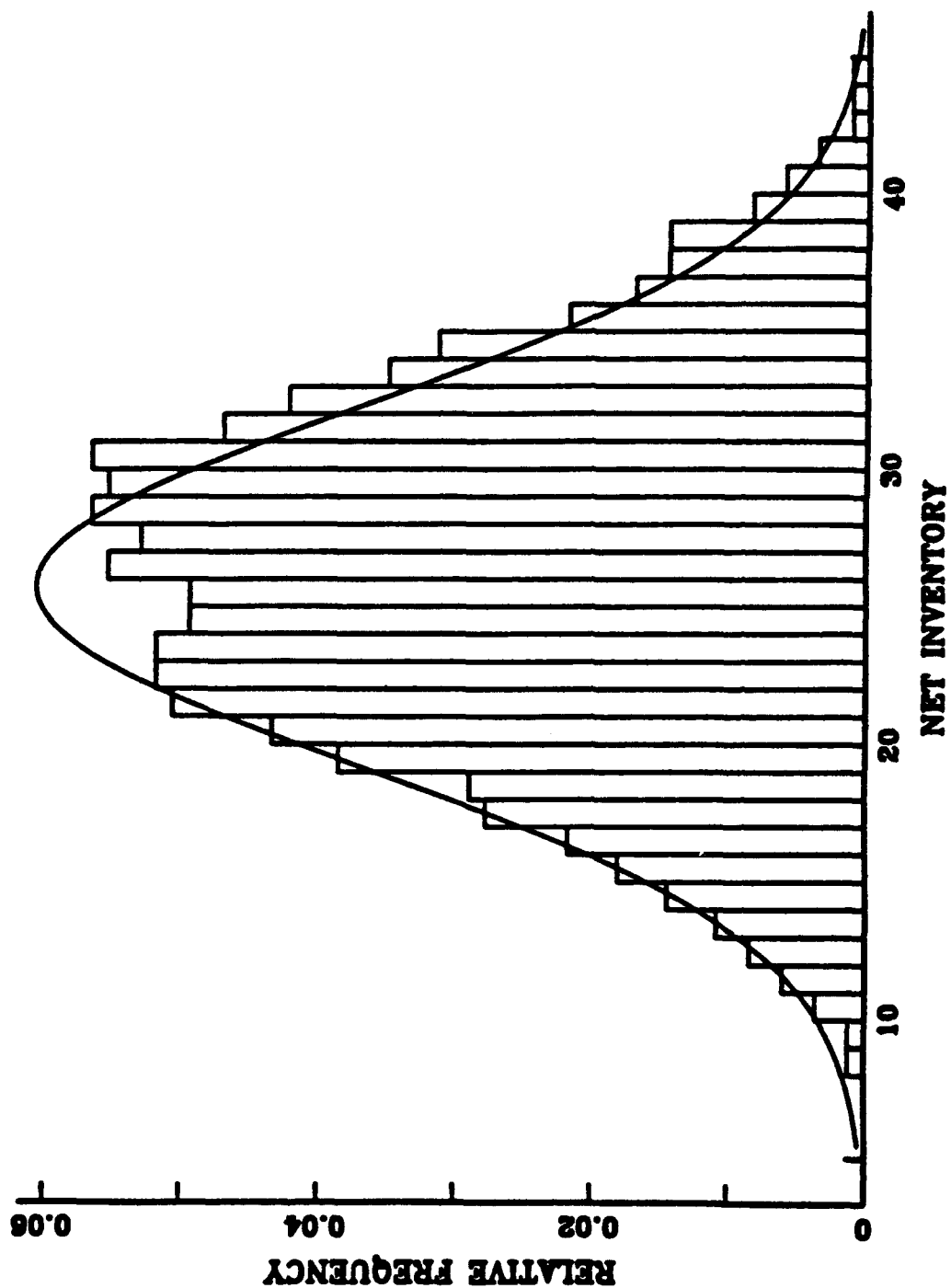
CRR = 0.8; RSR = 0.2; REP = 0.5

NORMAL DENSITY FUNCTION, N=828



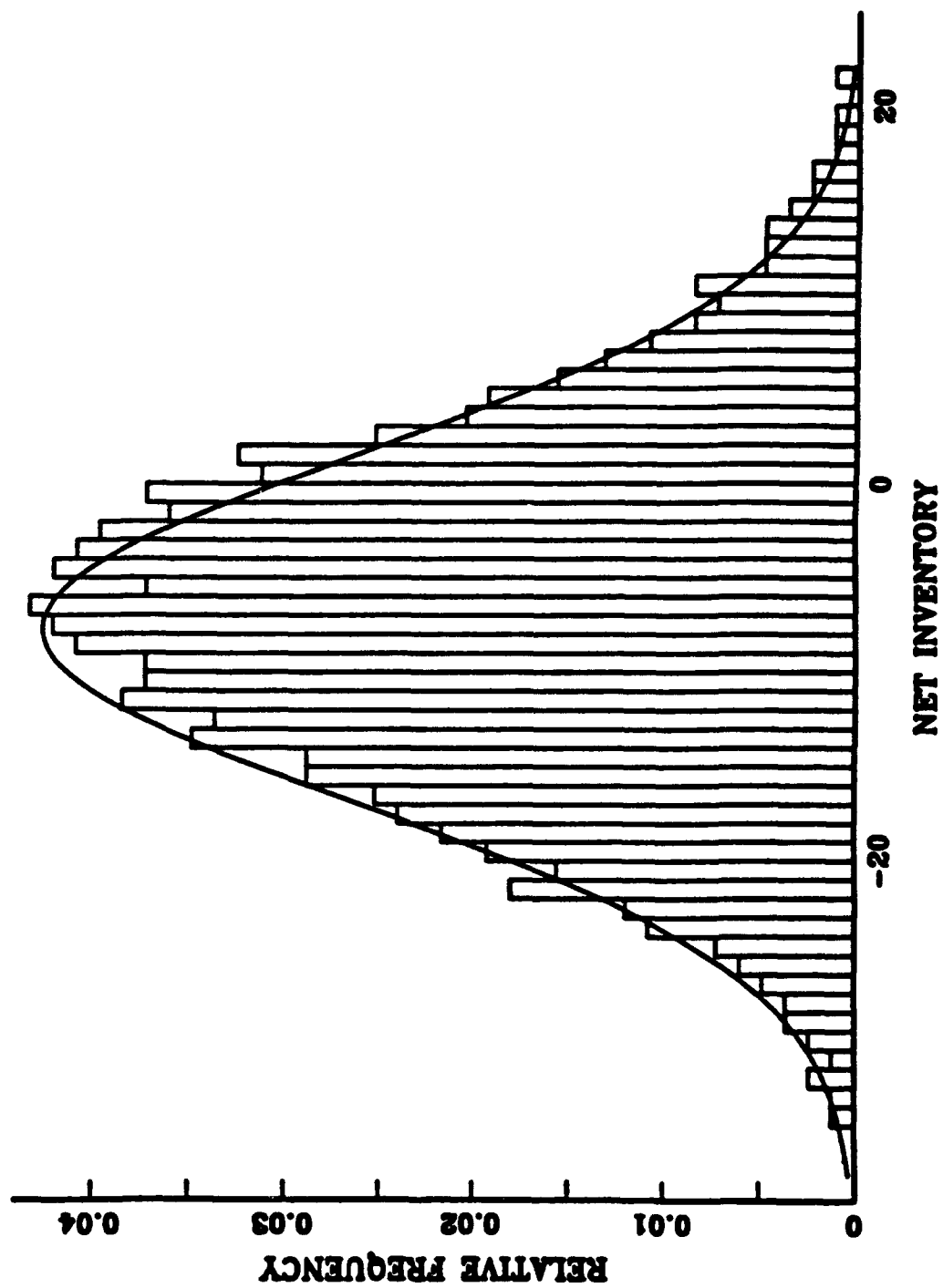
CRR = 0.8; RSR = 0.8; REP = 0.5

NORMAL DENSITY FUNCTION, N=829



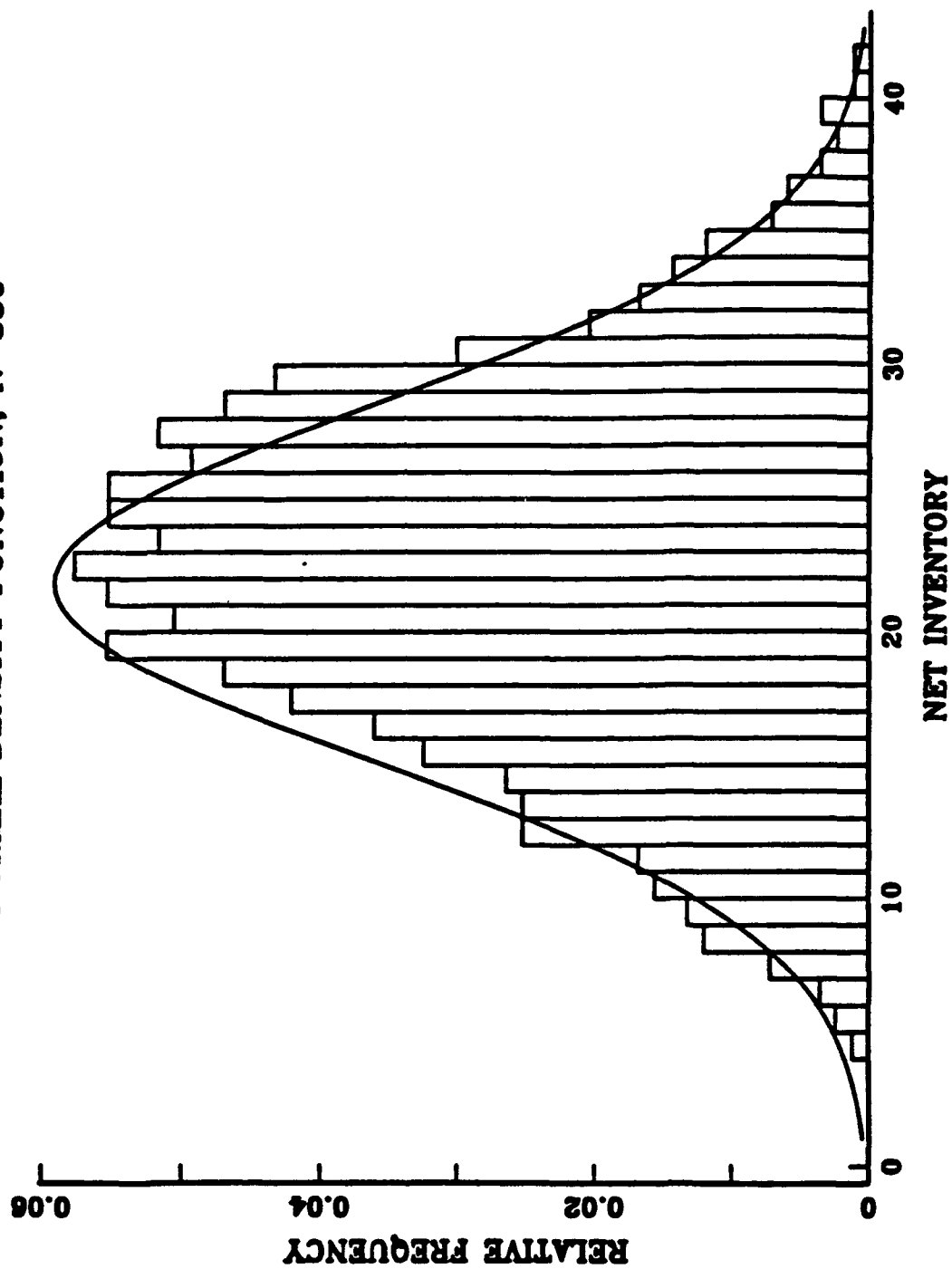
CRR = 1.0; RSR = 0.0; REP = 0.0

NORMAL DENSITY FUNCTION, N=831



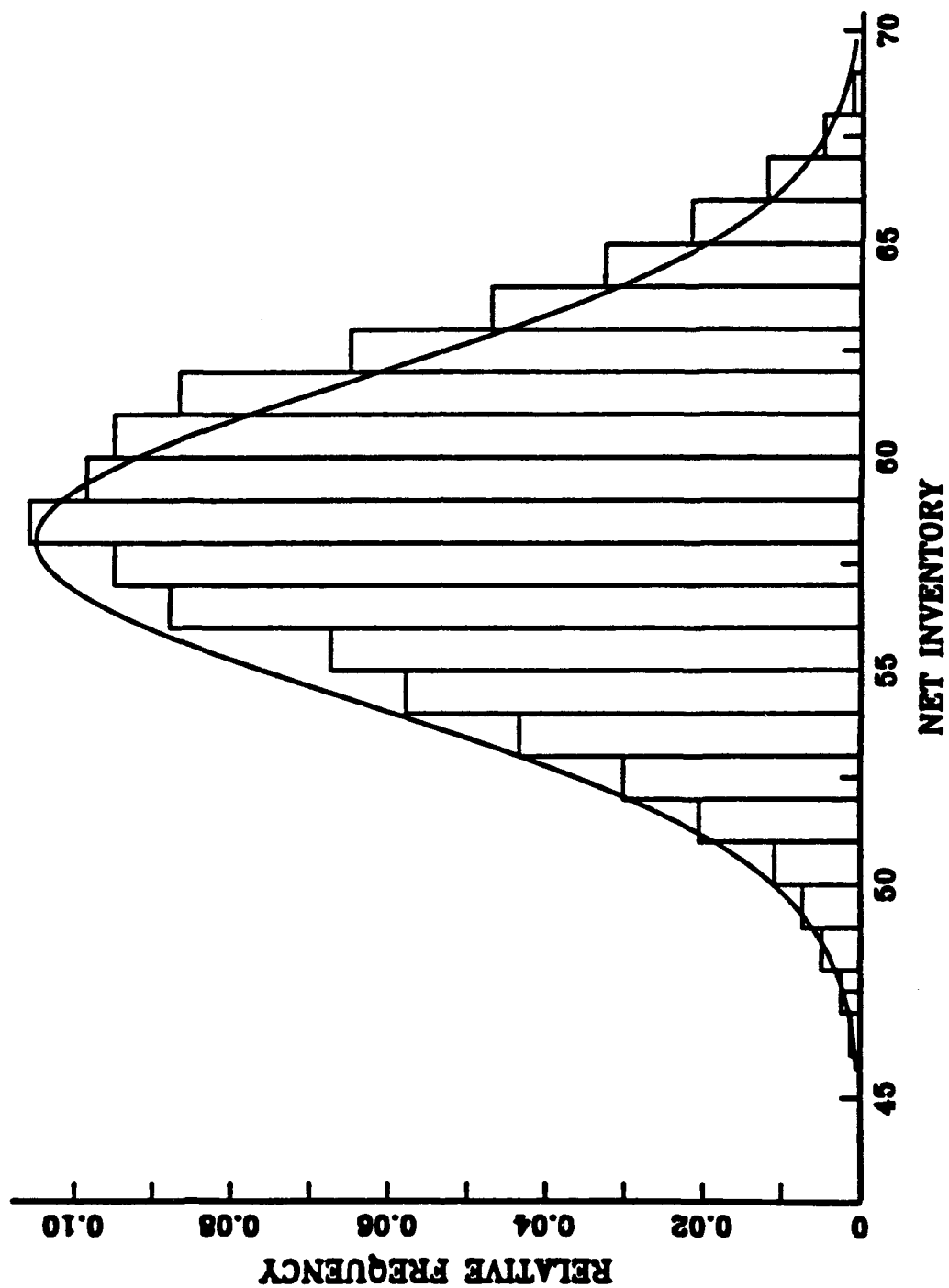
CRR = 1.0; RSR = 0.8; REP = 1.3

NORMAL DENSITY FUNCTION, N=830

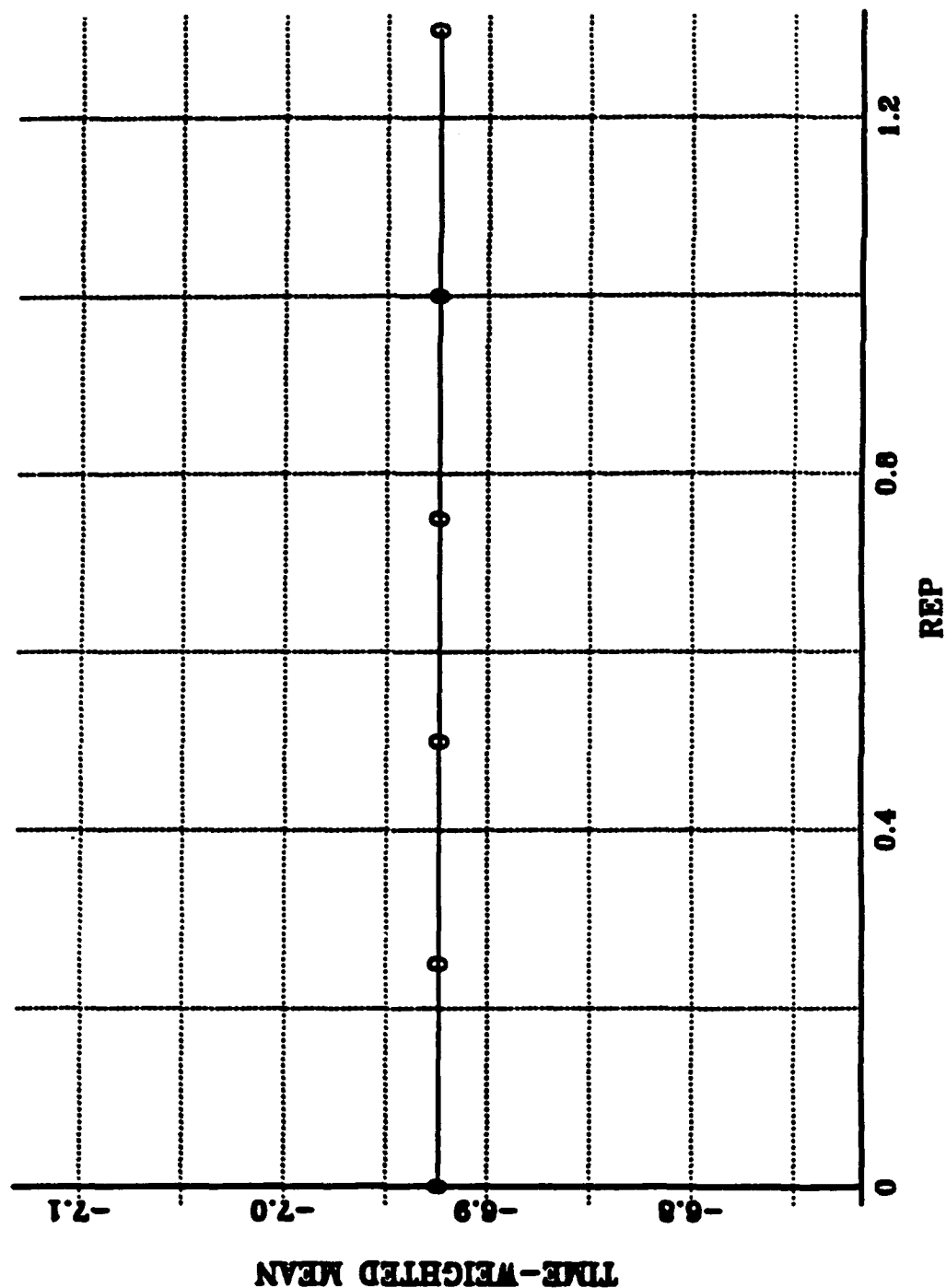


CRR = 1.0; RSR = 1.0; REP = 0.0

NORMAL DENSITY FUNCTION, N=830

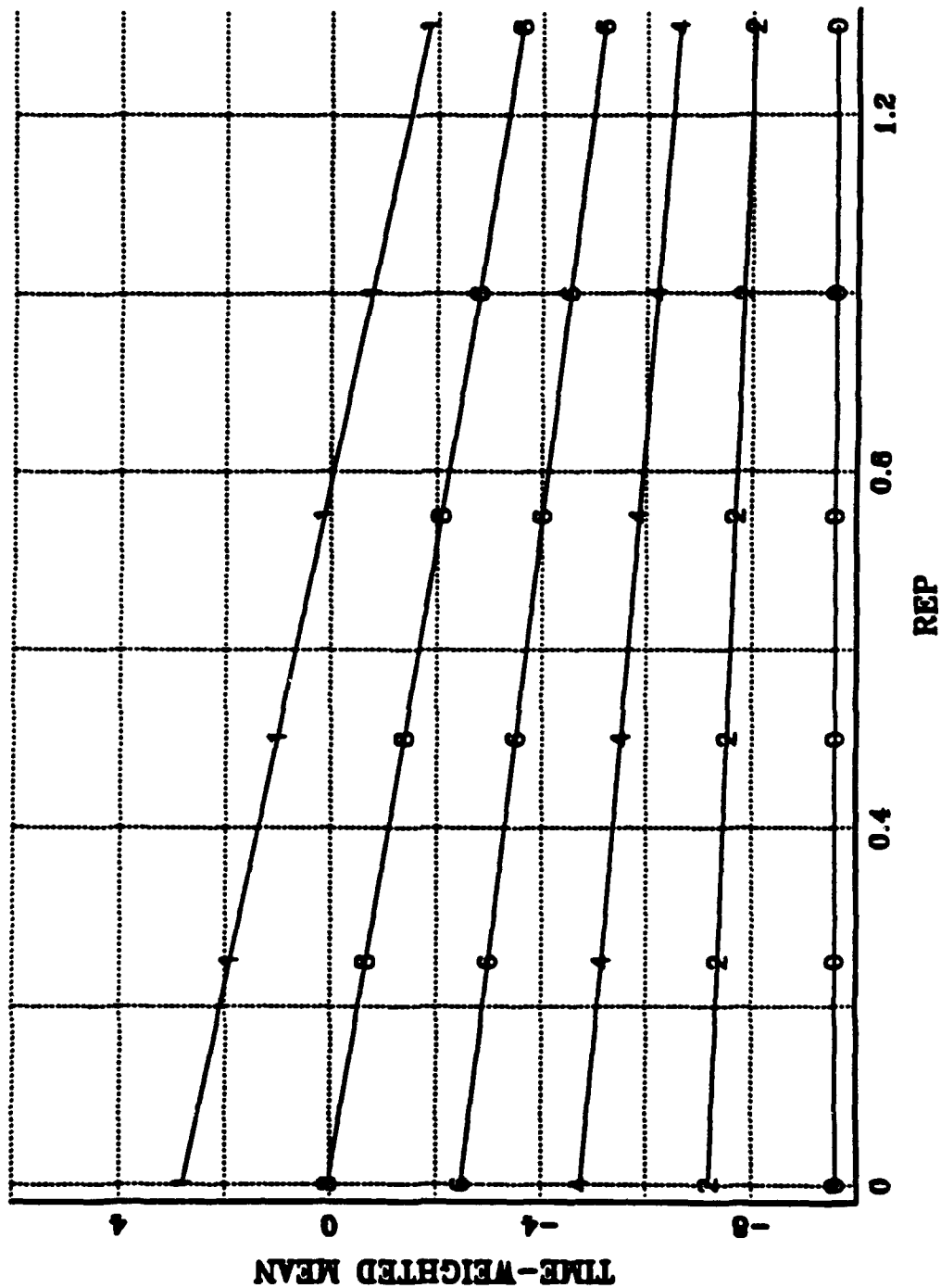


RSR IS VARIED FOR EACH LINE
CRR FIXED AT 0.0



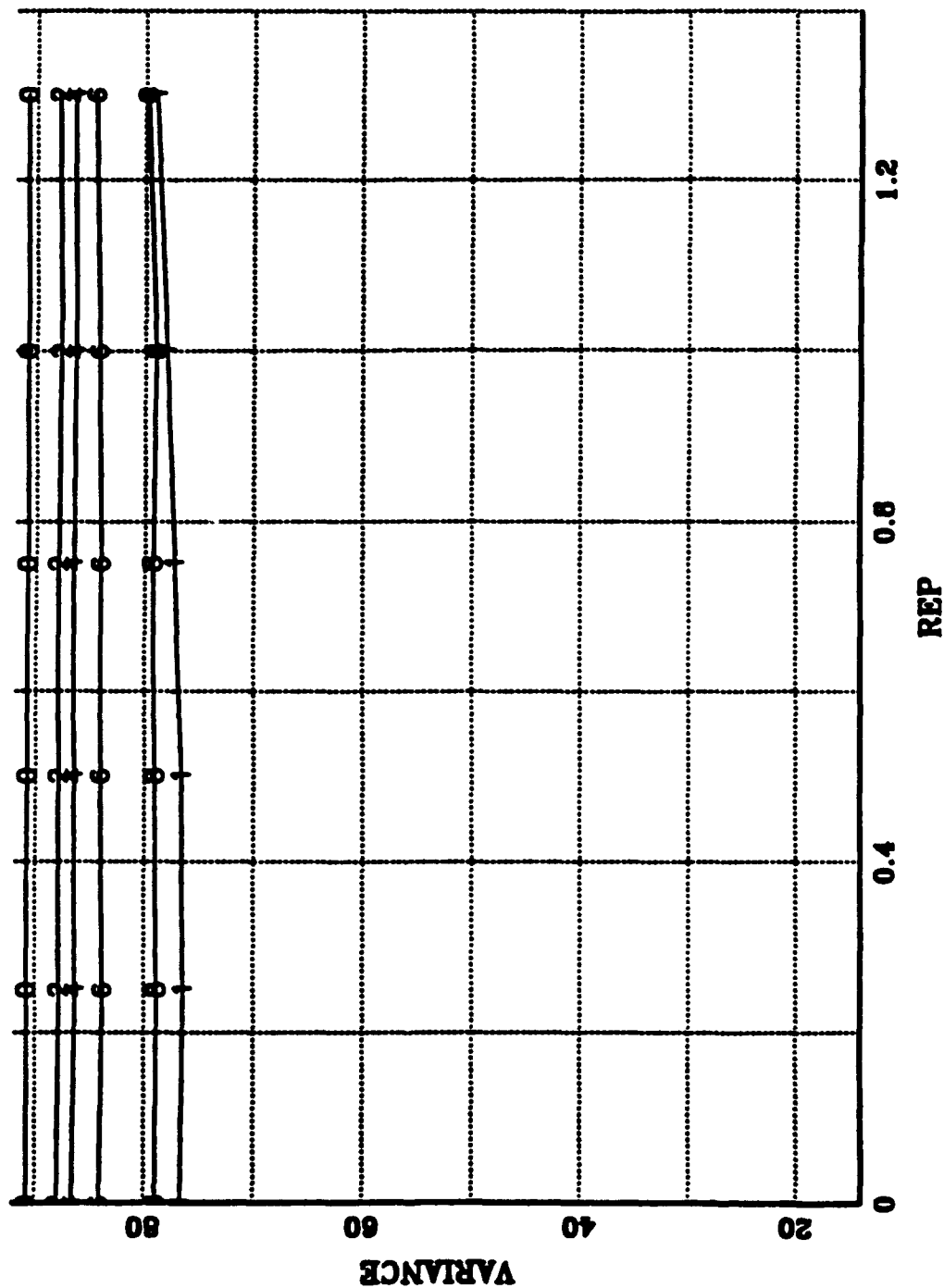
RSR IS VARIED FOR EACH LINE

CRR FIXED AT 0.2



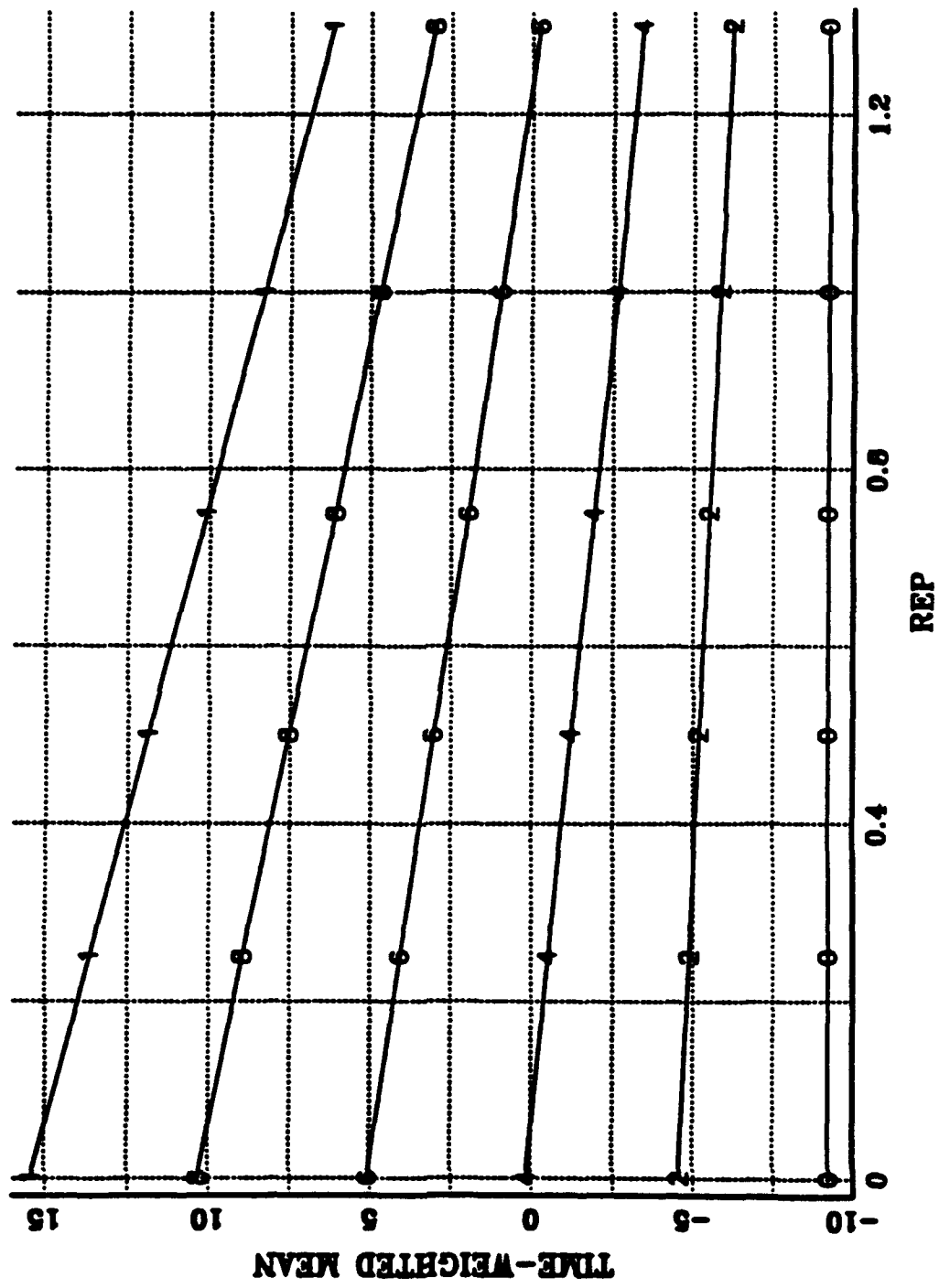
REP VS VARIANCE

CRR IS FIXED AT 0.2; DIFFERENT VALUES OF RSR



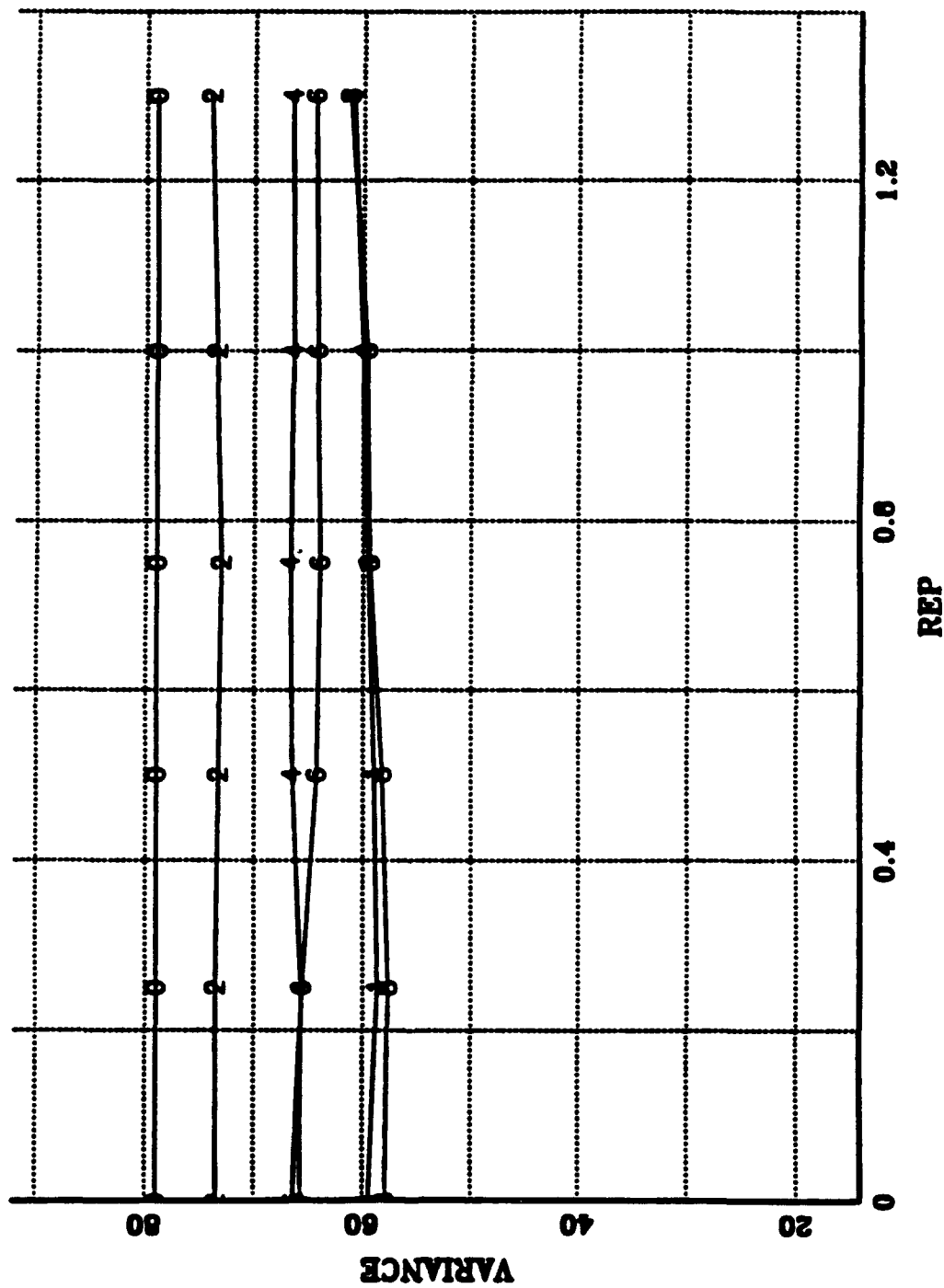
RSR IS VARIED FOR EACH LINE

CRR FIXED AT 0.4



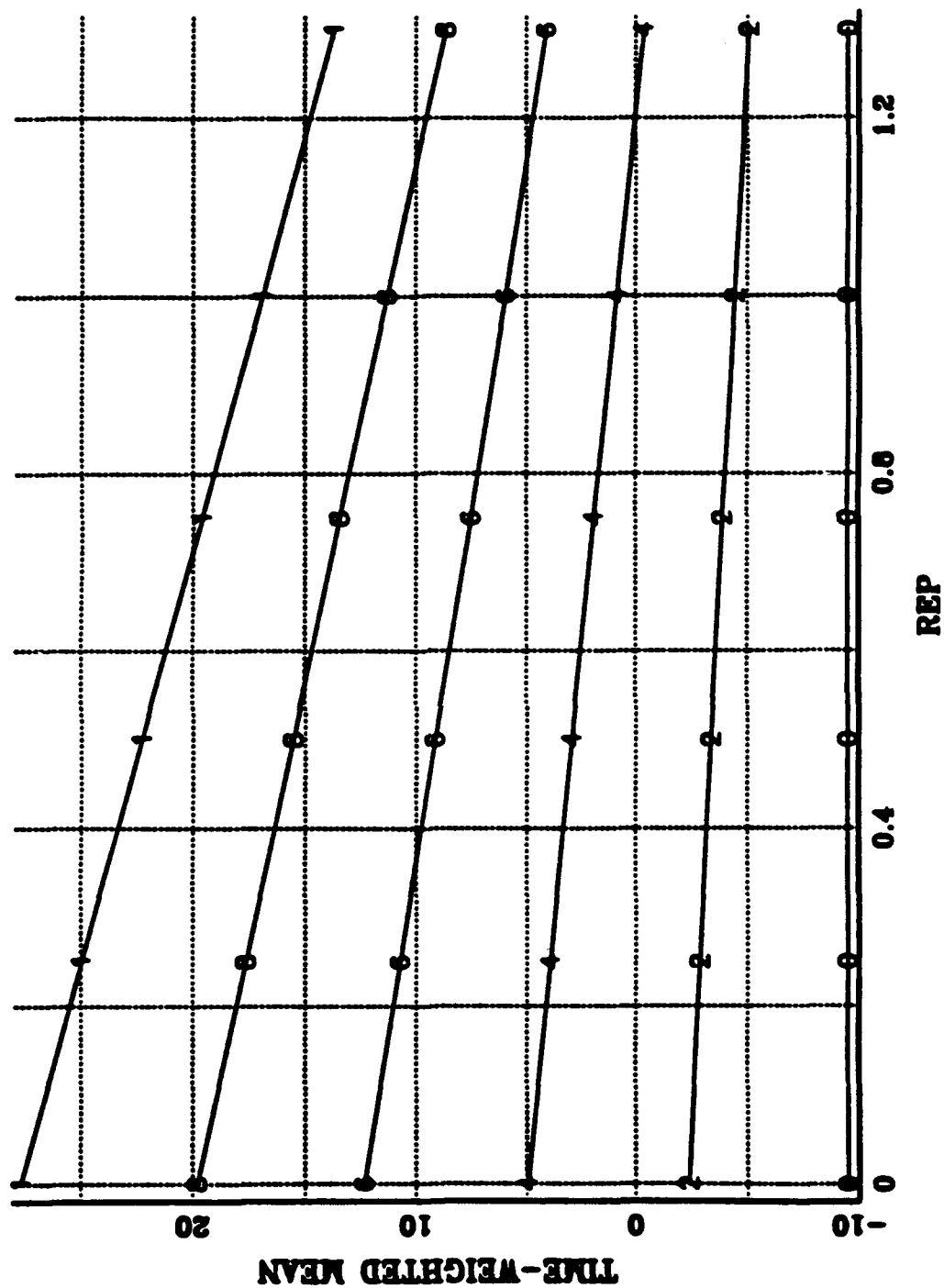
REP VS VARIANCE

CRR IS FIXED AT 0.4; DIFFERENT VALUES OF RSR



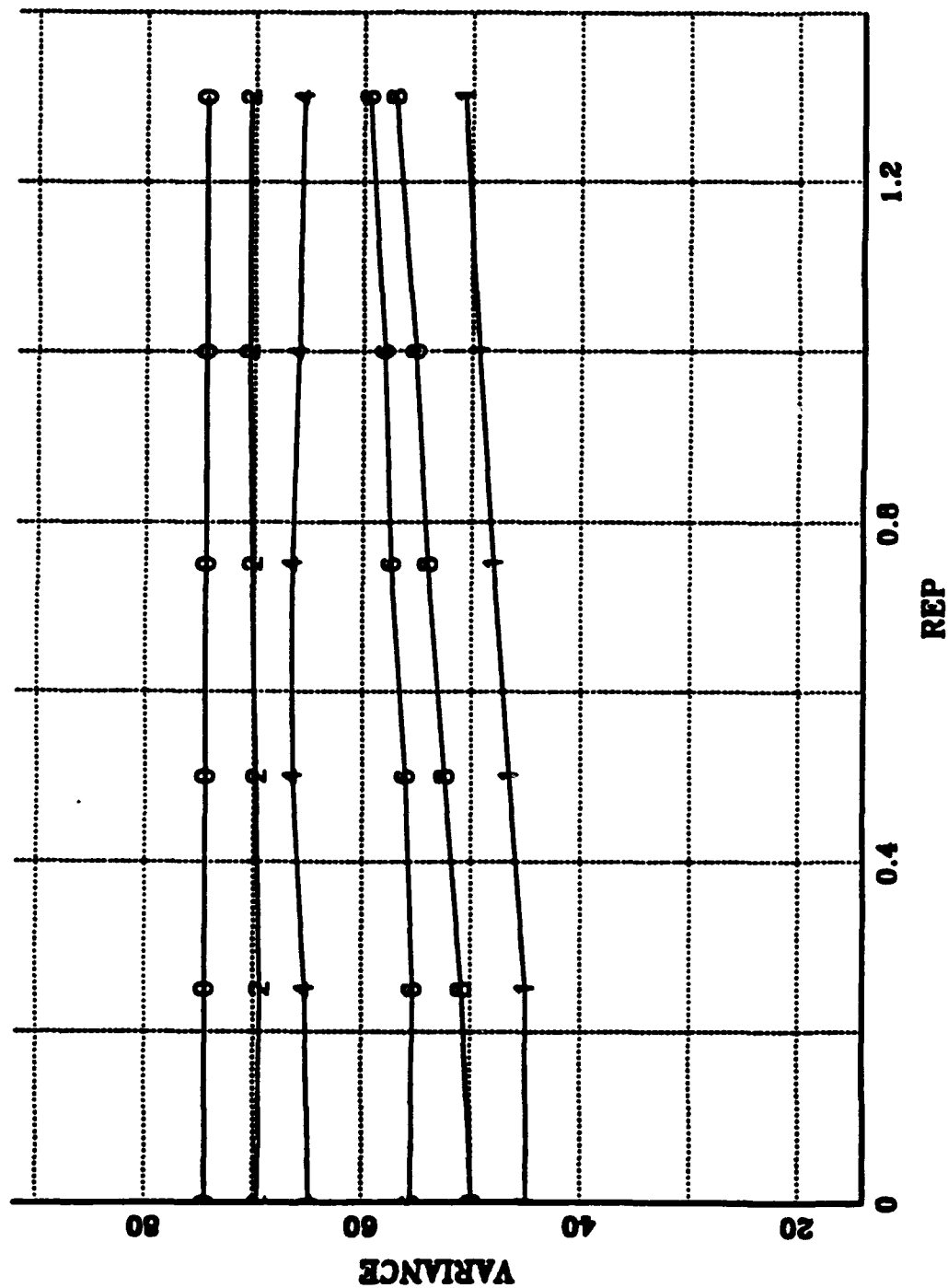
RSR IS VARIED FOR EACH LINE

CRR FIXED AT 0.6



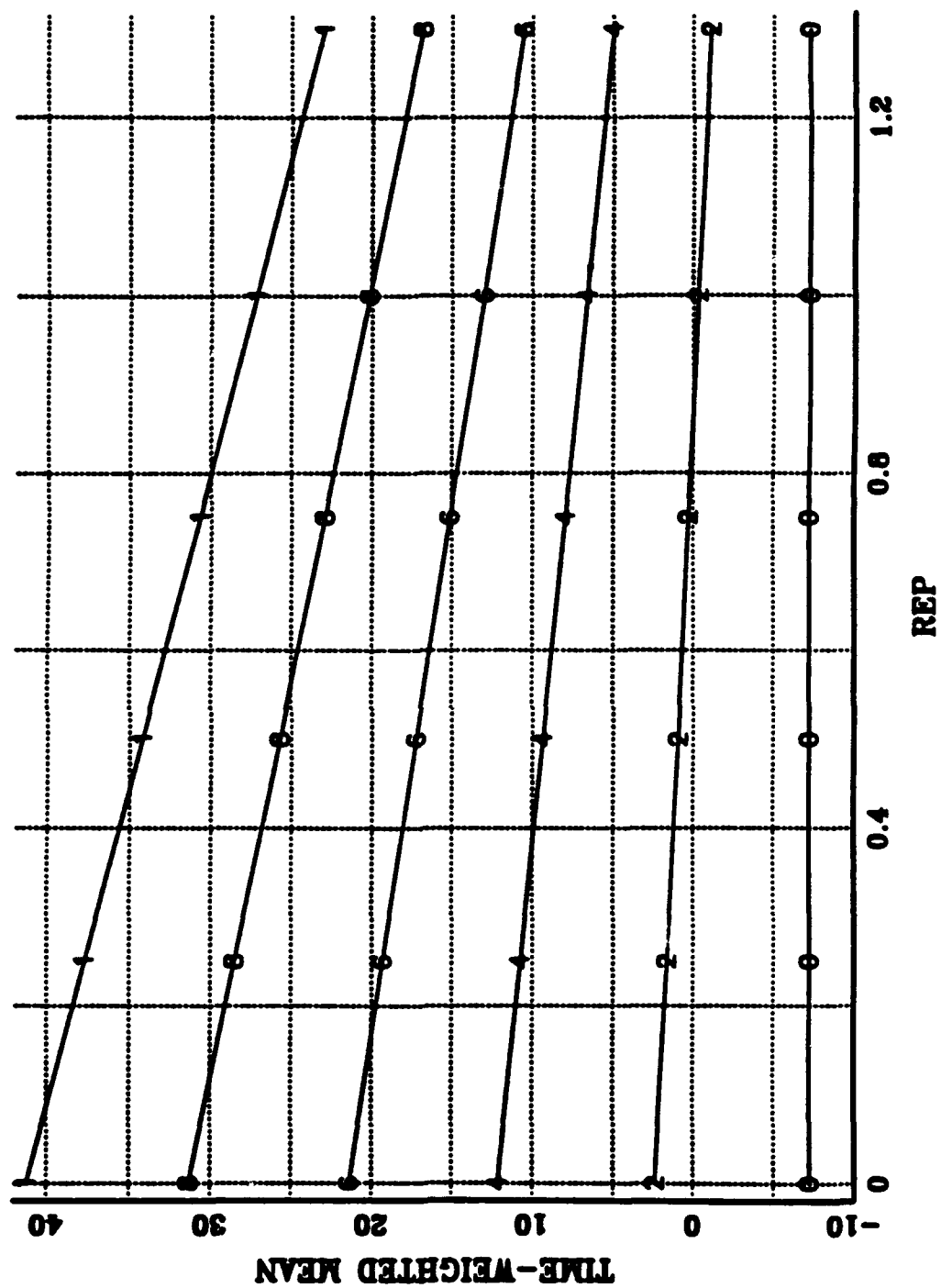
REP VS VARIANCE

CRR IS FIXED AT 0.6; DIFFERENT VALUES OF RSR



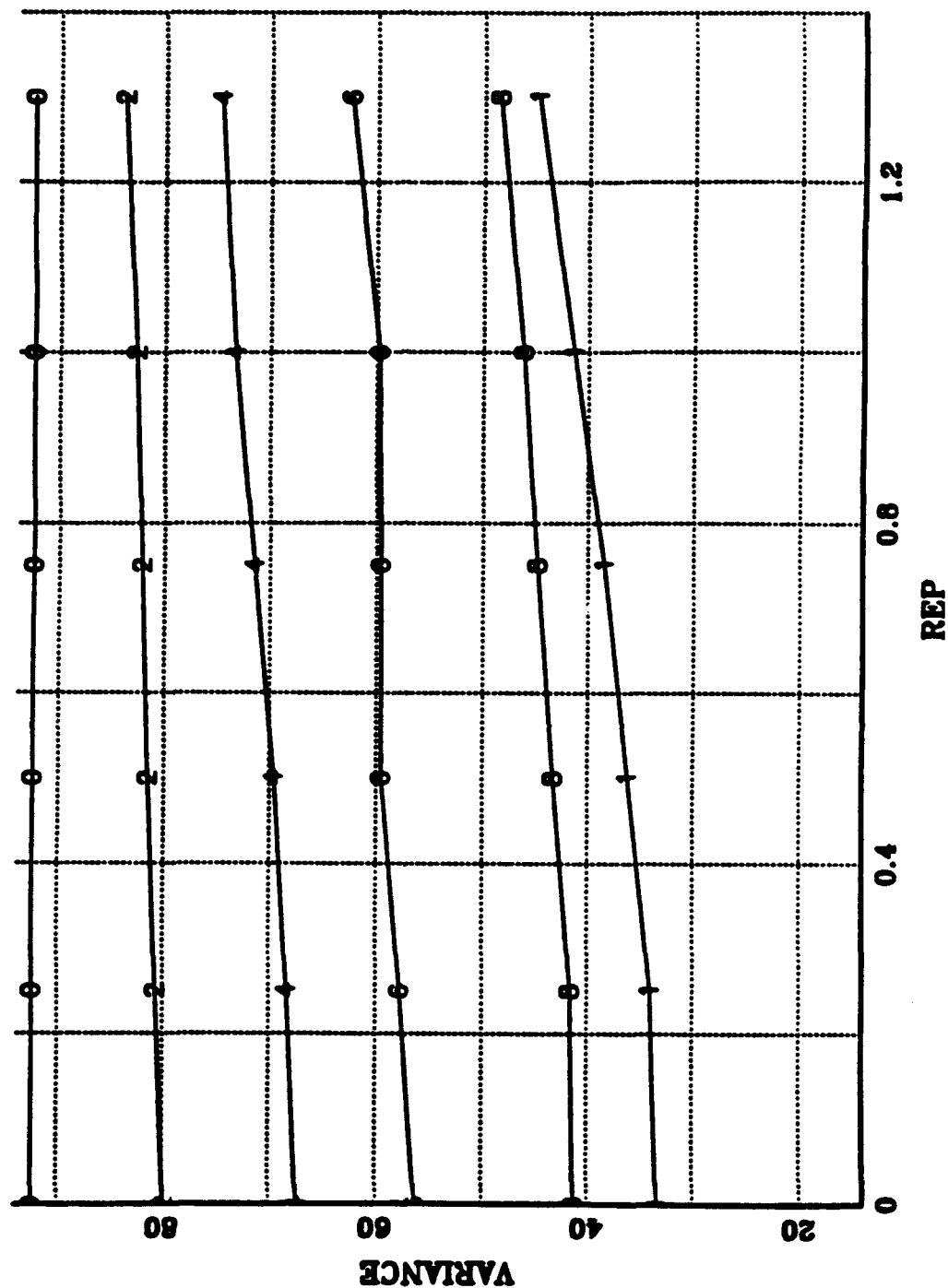
RSR IS VARIED FOR EACH LINE

CRR FIXED AT 0.8



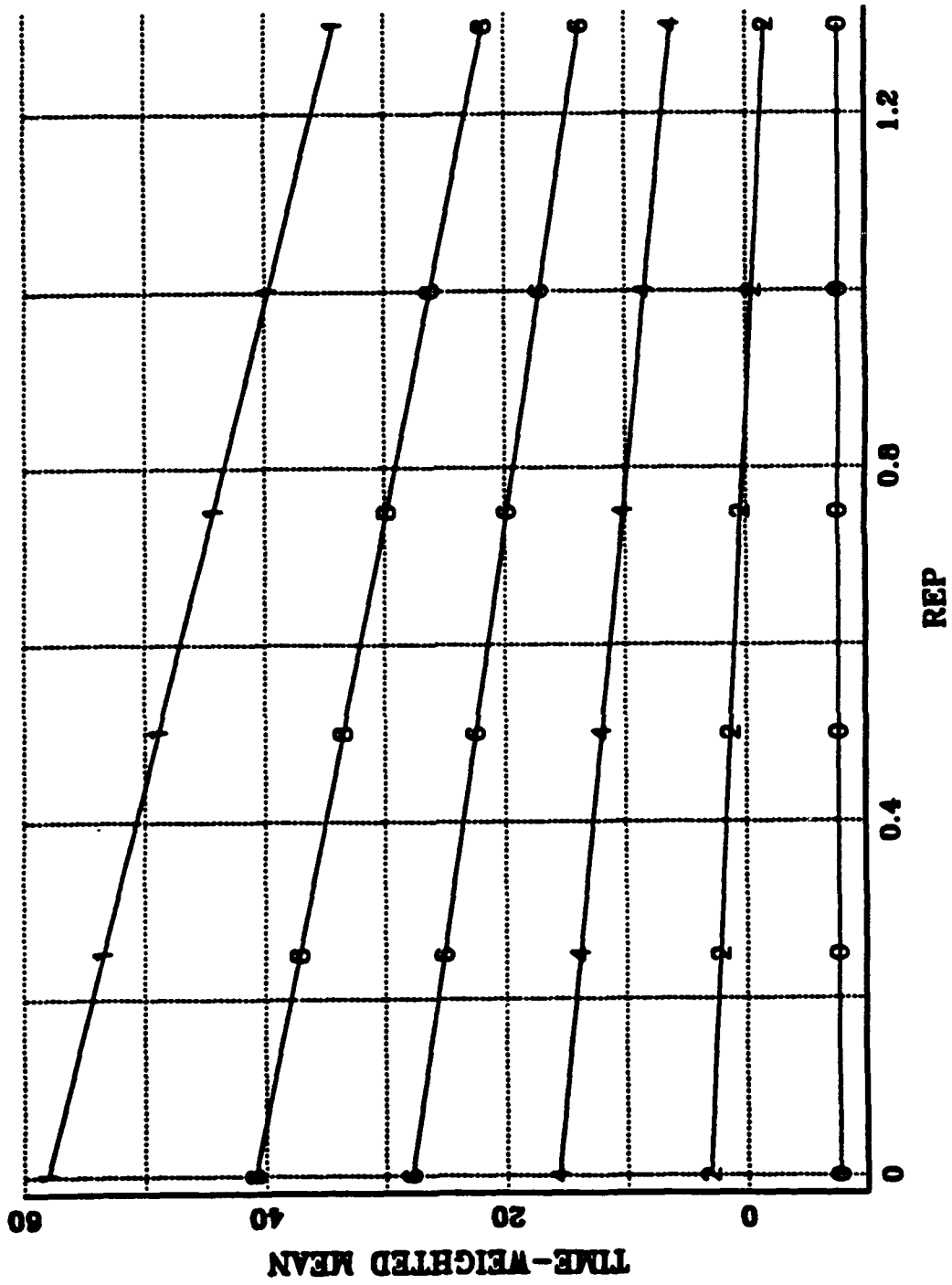
REP VS VARIANCE

CRR IS FIXED AT 0.8; DIFFERENT VALUES OF RSR



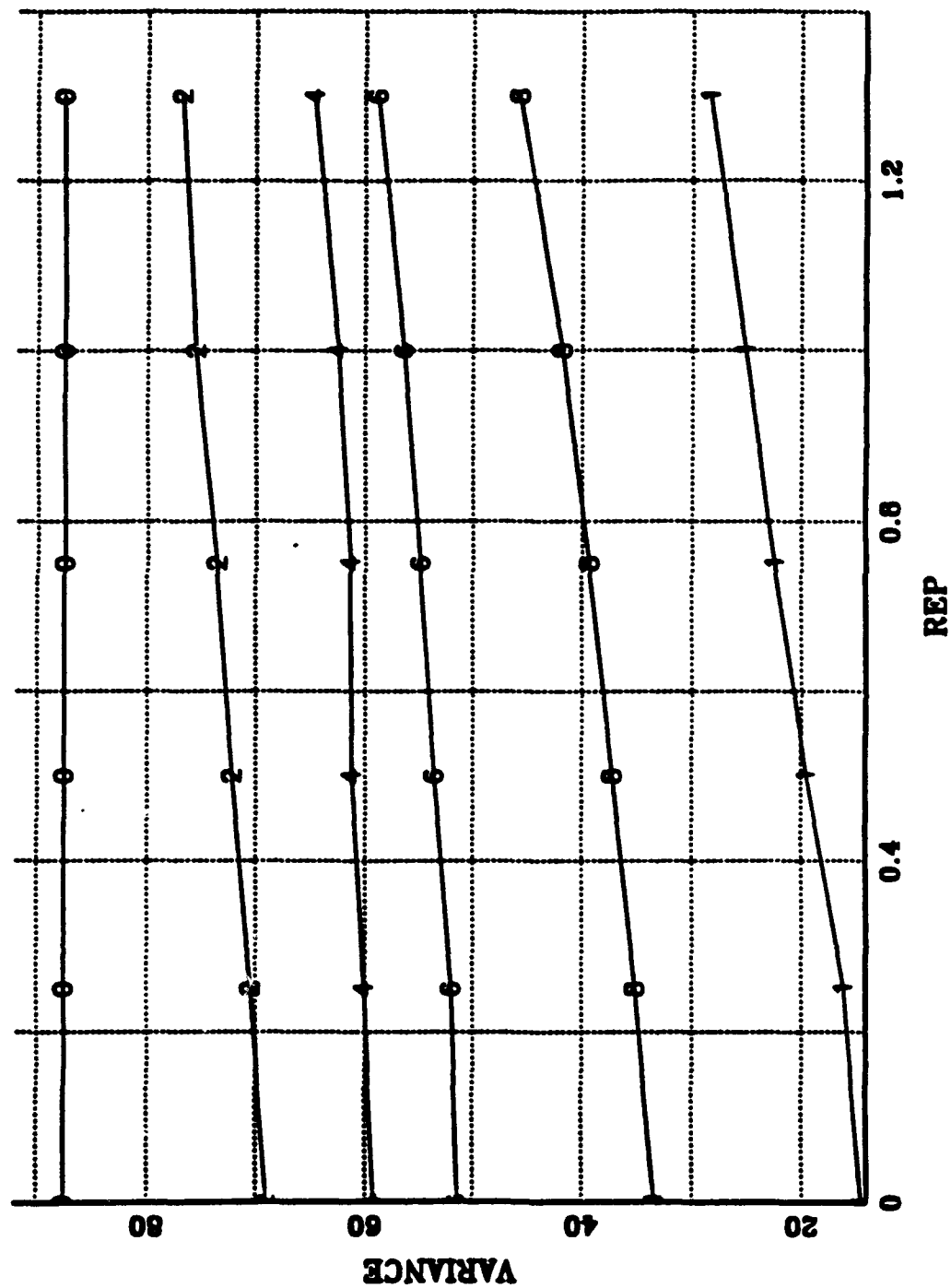
RSR IS VARIED FOR EACH LINE

CRR FIXED AT 1.0



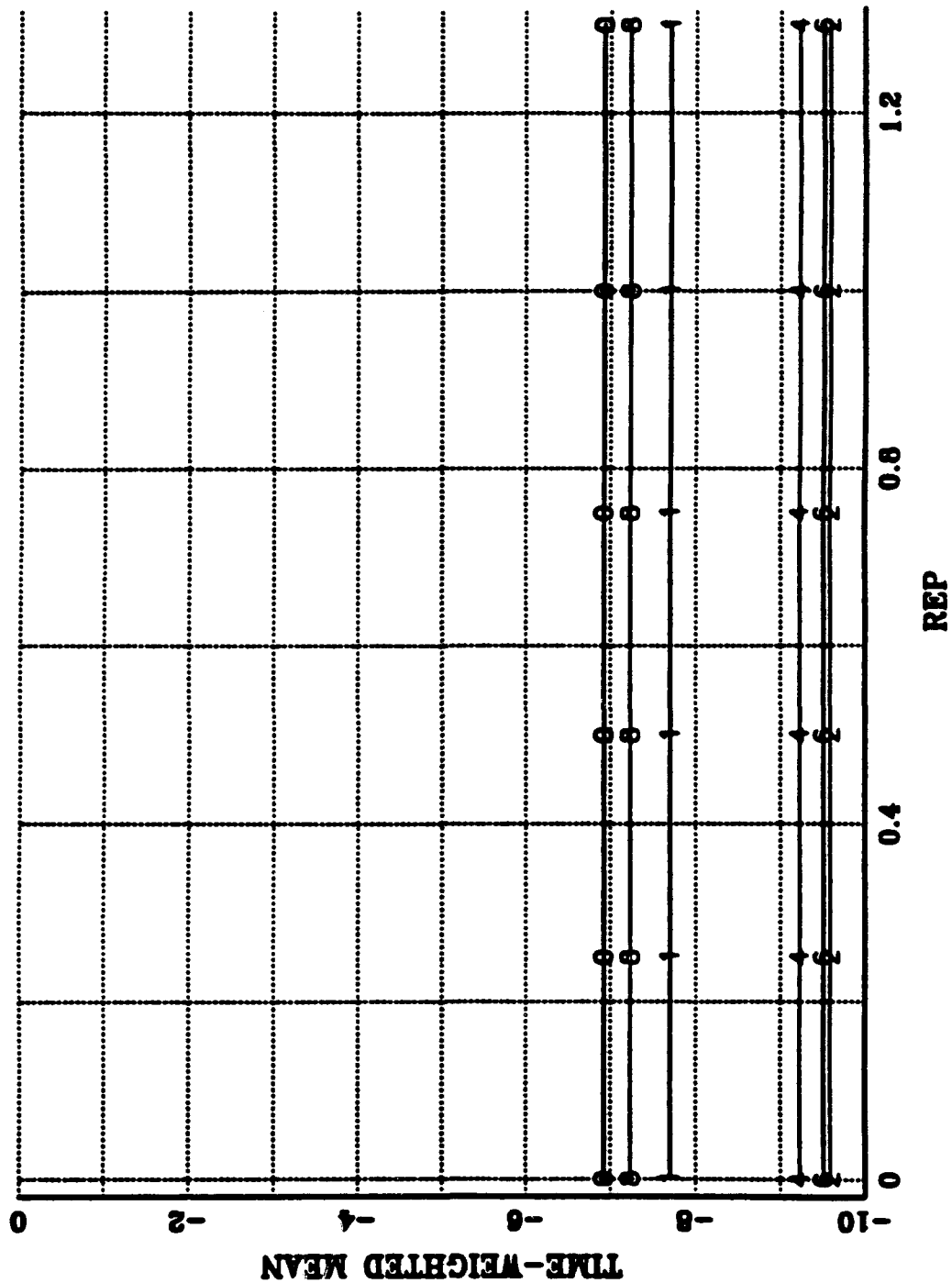
REP VS VARIANCE

CRR IS FIXED AT 1.0; DIFFERENT VALUES OF RSR



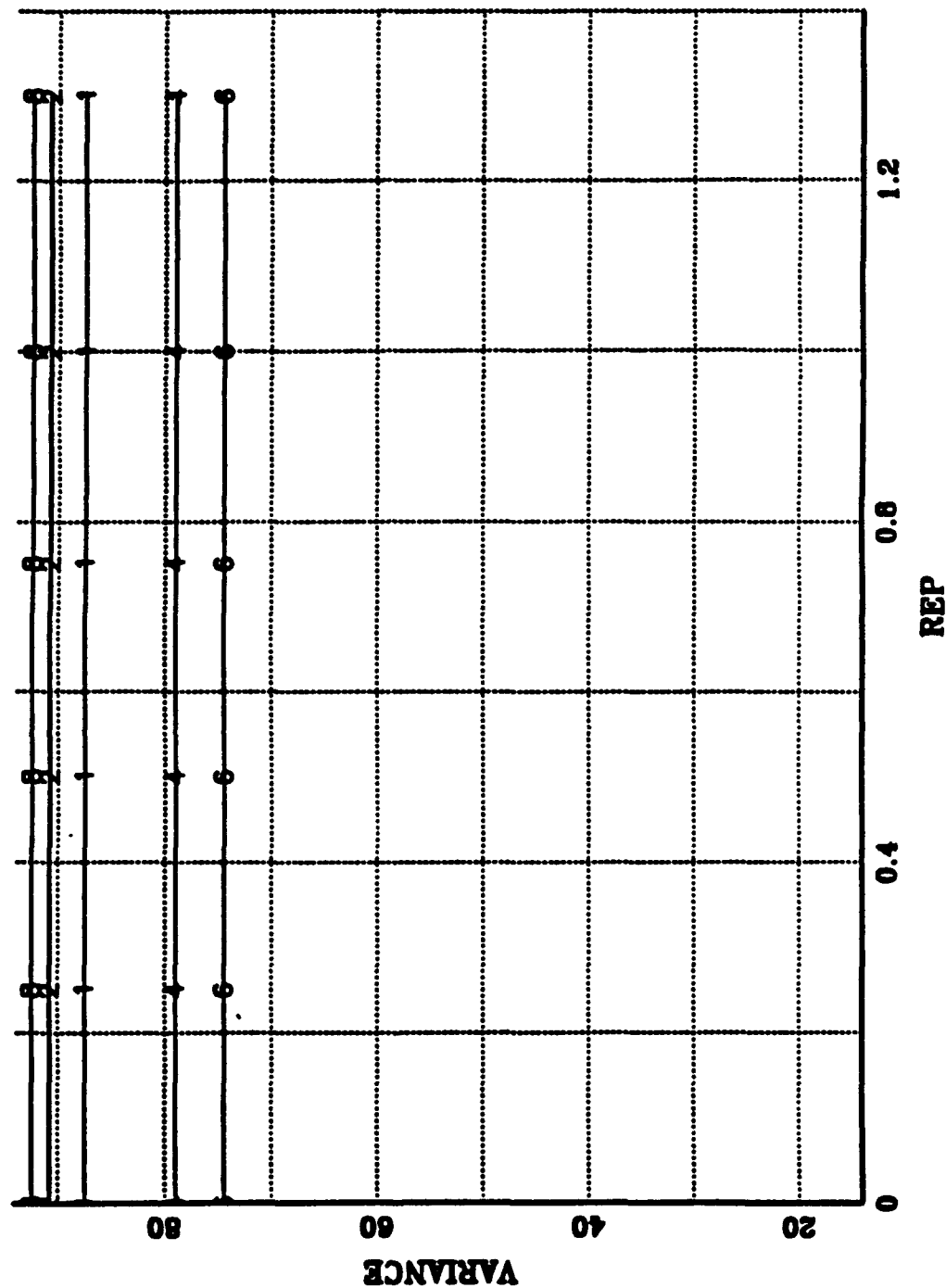
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 0.0



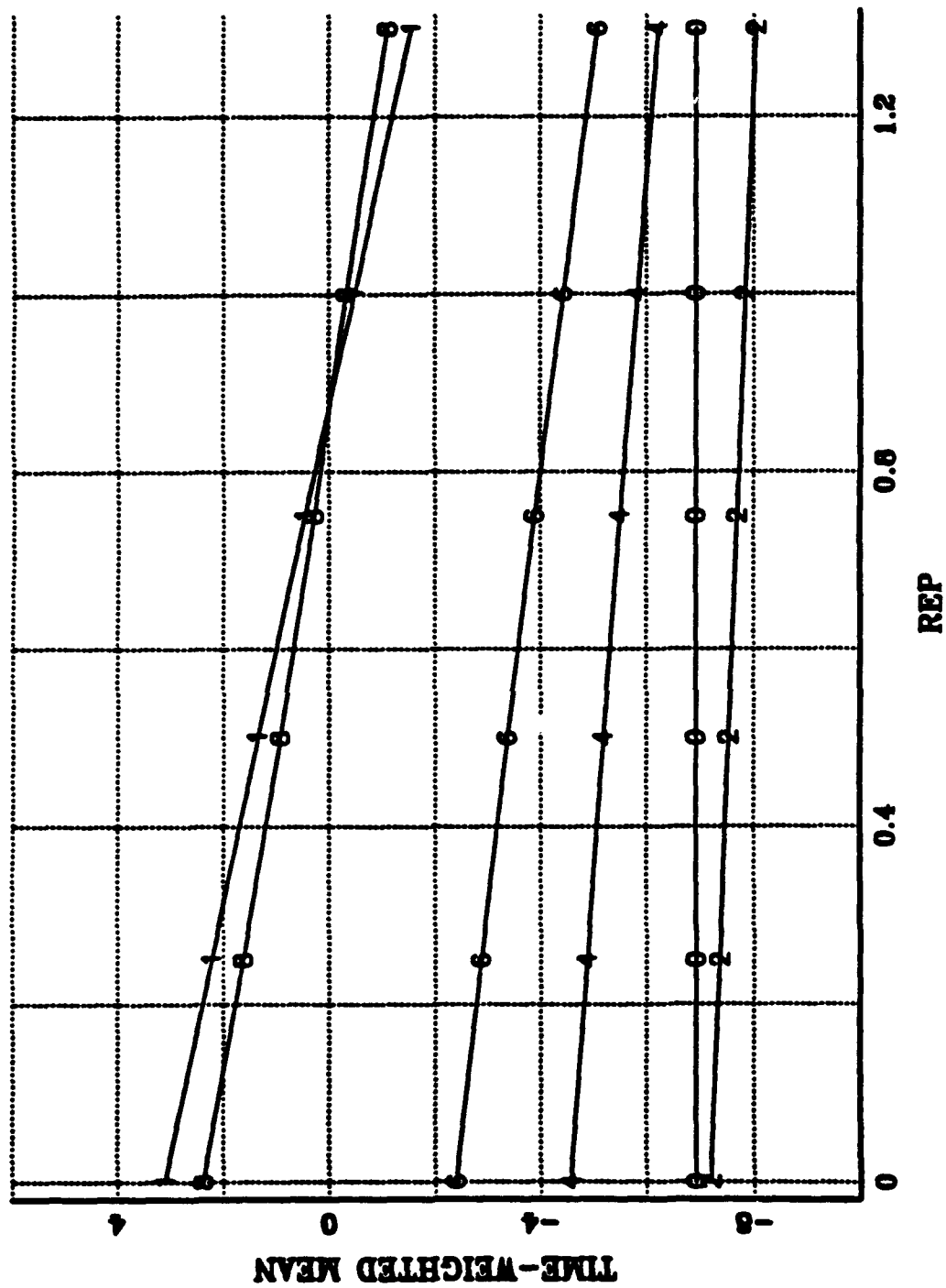
REP VS VARIANCE

RSR IS FIXED AT 0.0; DIFFERENT VALUES OF CRR



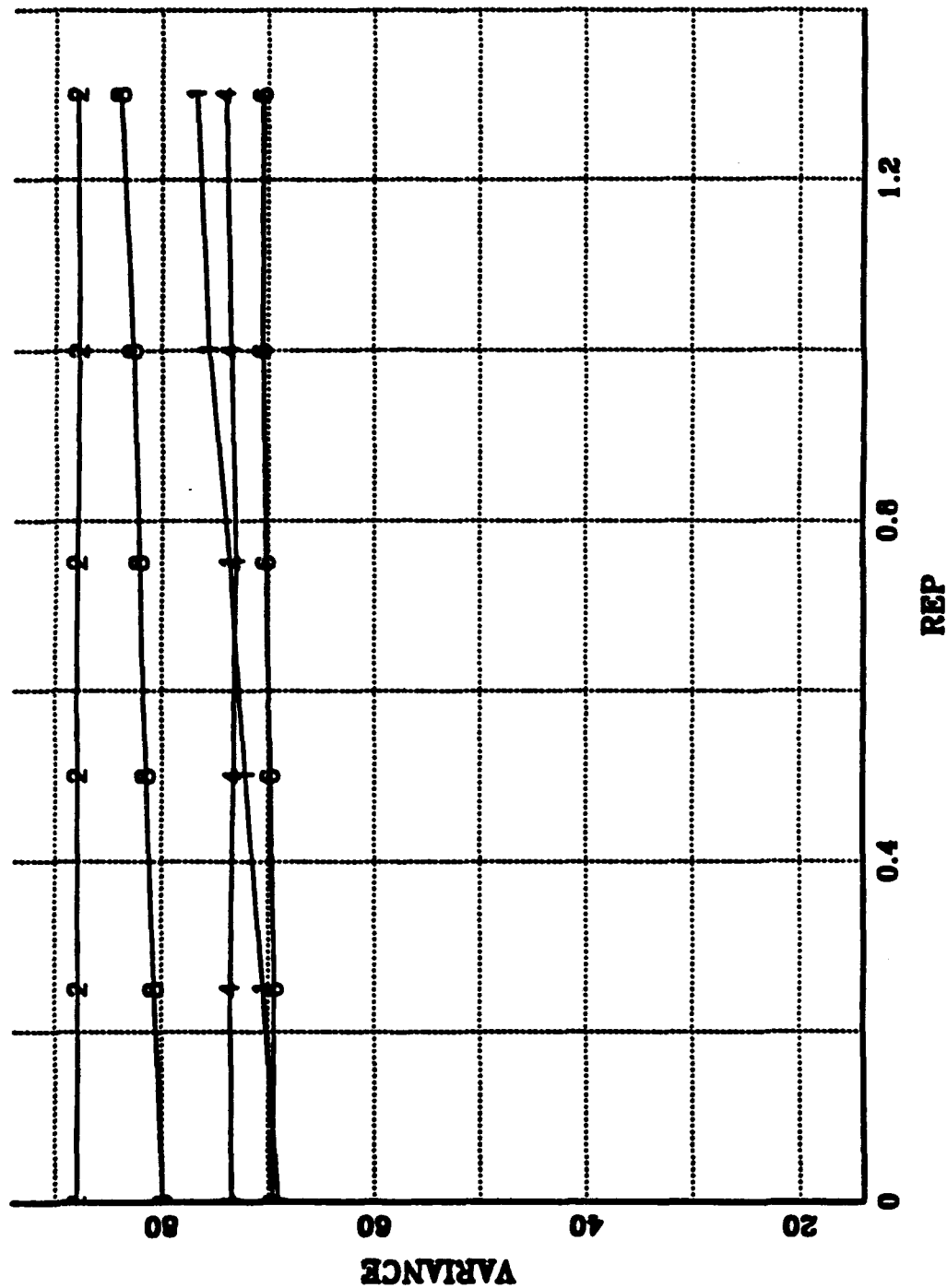
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 0.2



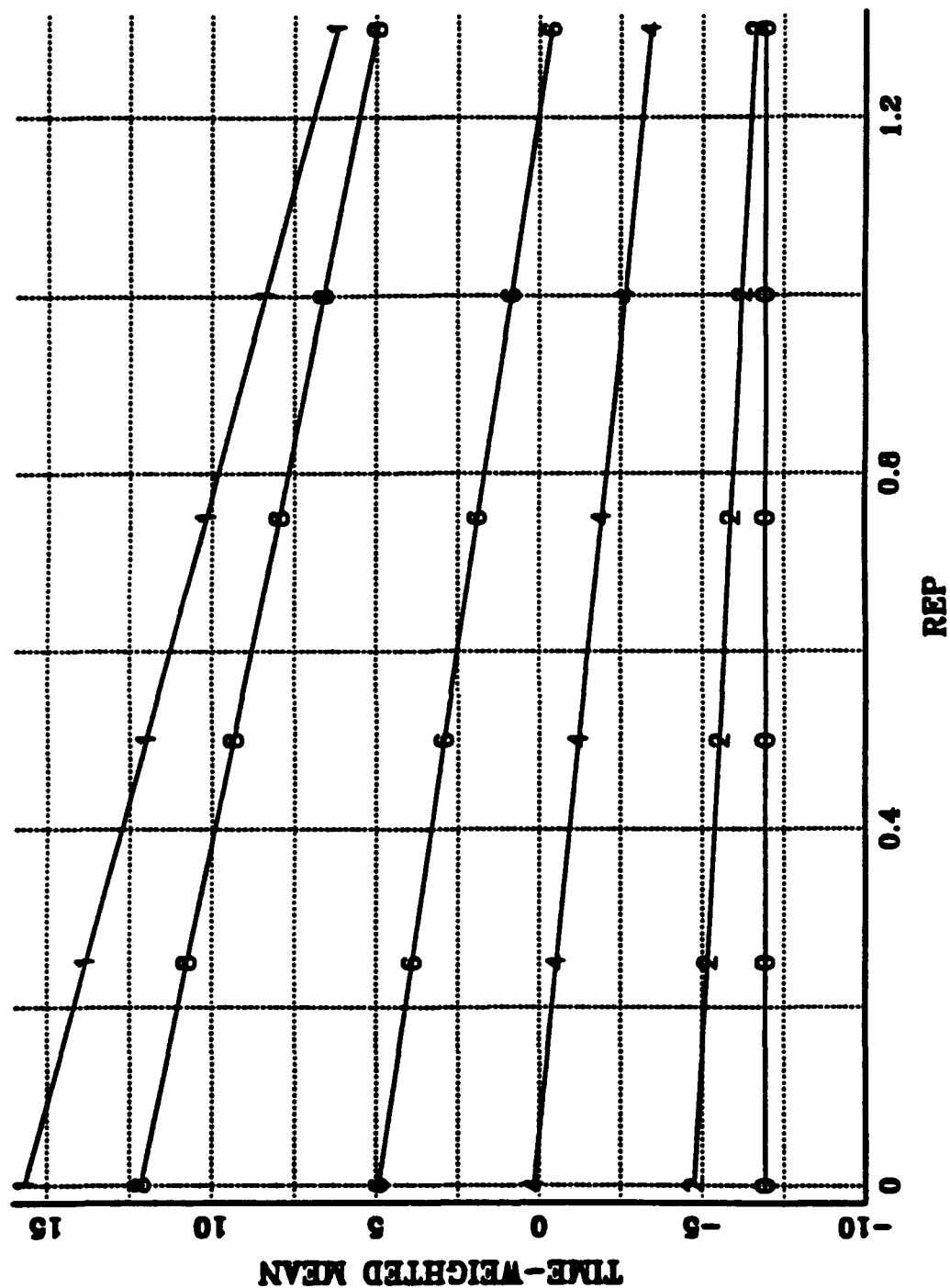
REP VS VARIANCE

RSR IS FIXED AT 0.2; DIFFERENT VALUES OF CRR



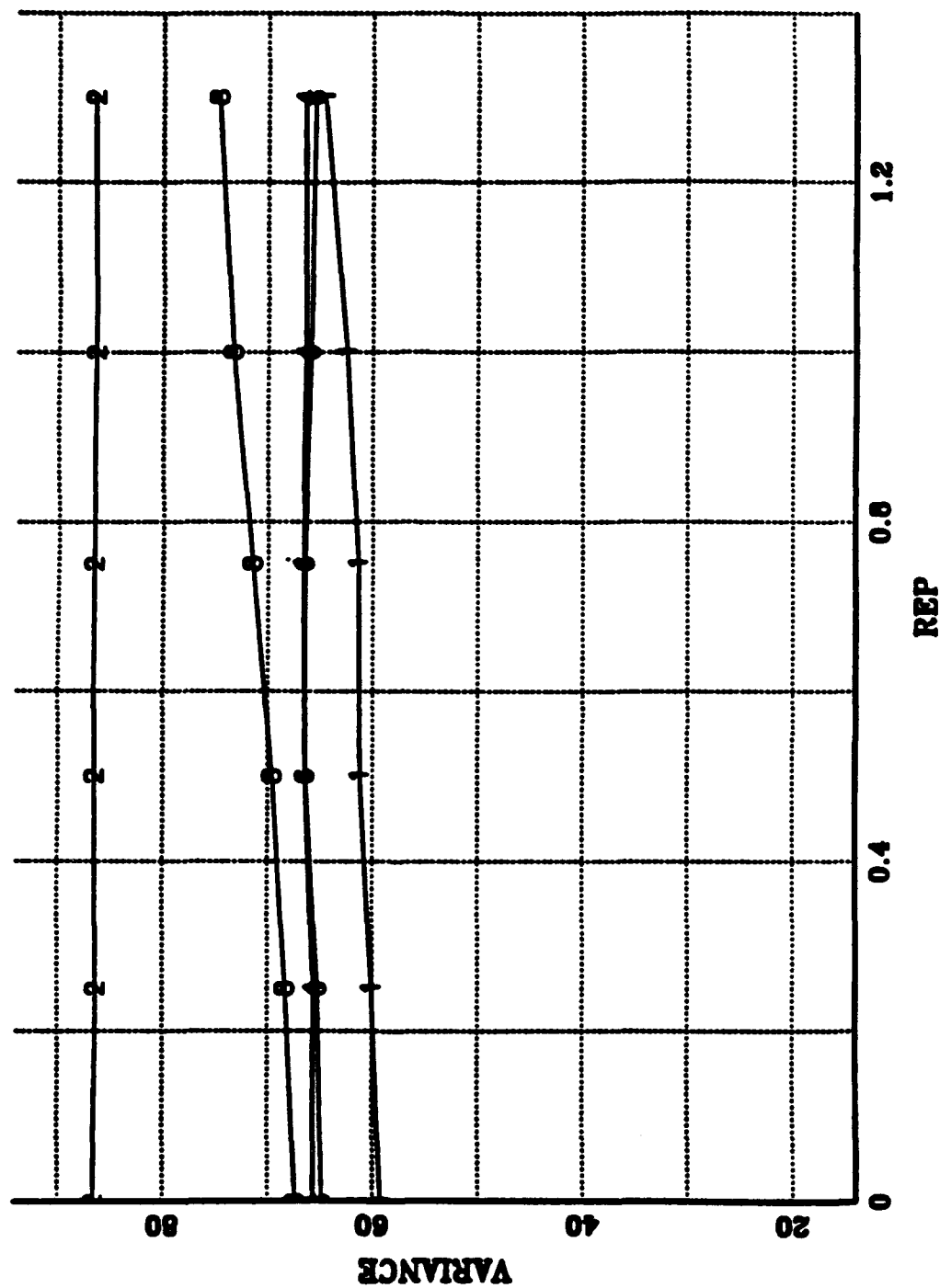
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 0.4



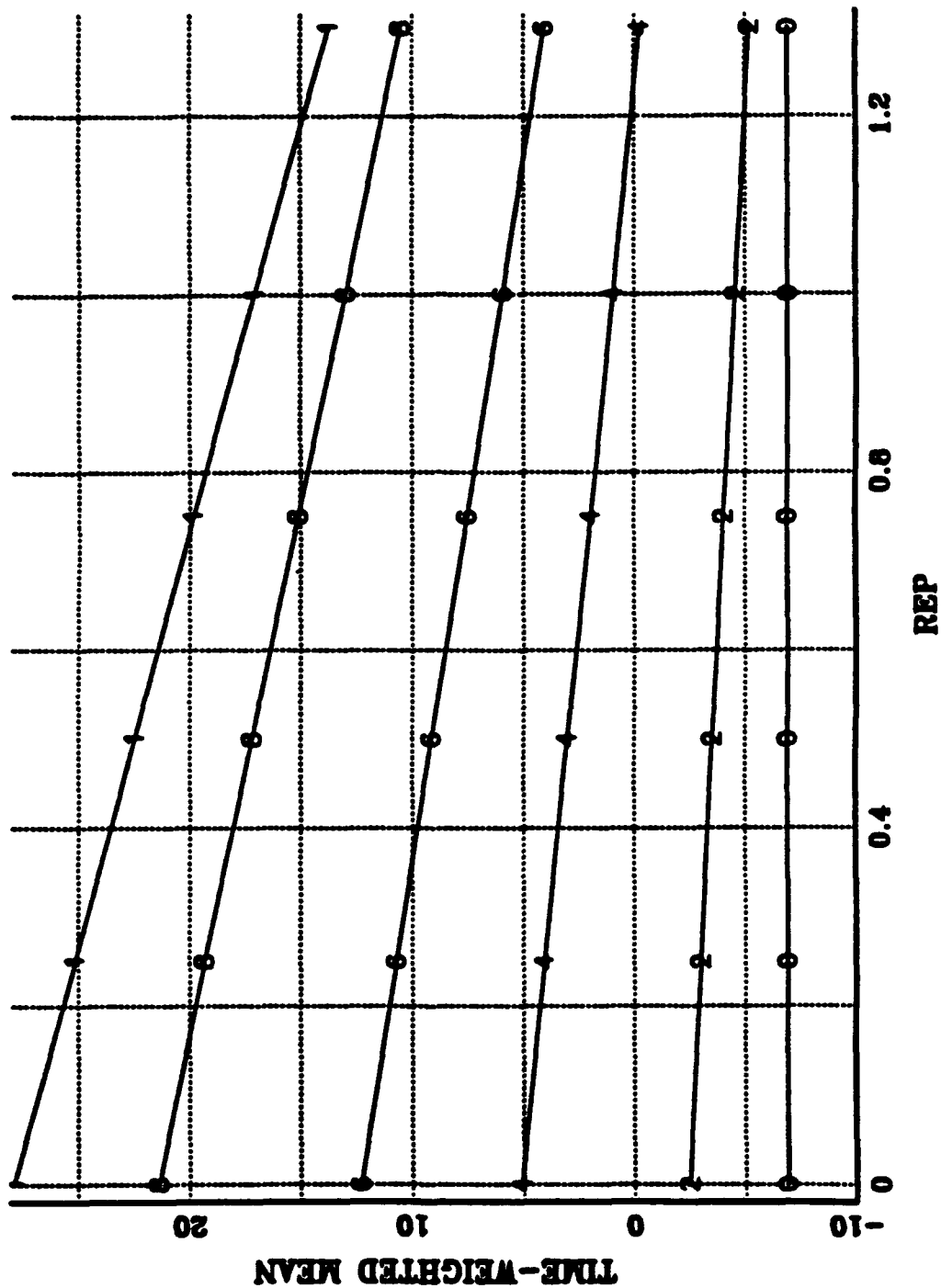
REP VS VARIANCE

RSR IS FIXED AT 0.4; DIFFERENT VALUES OF CRR



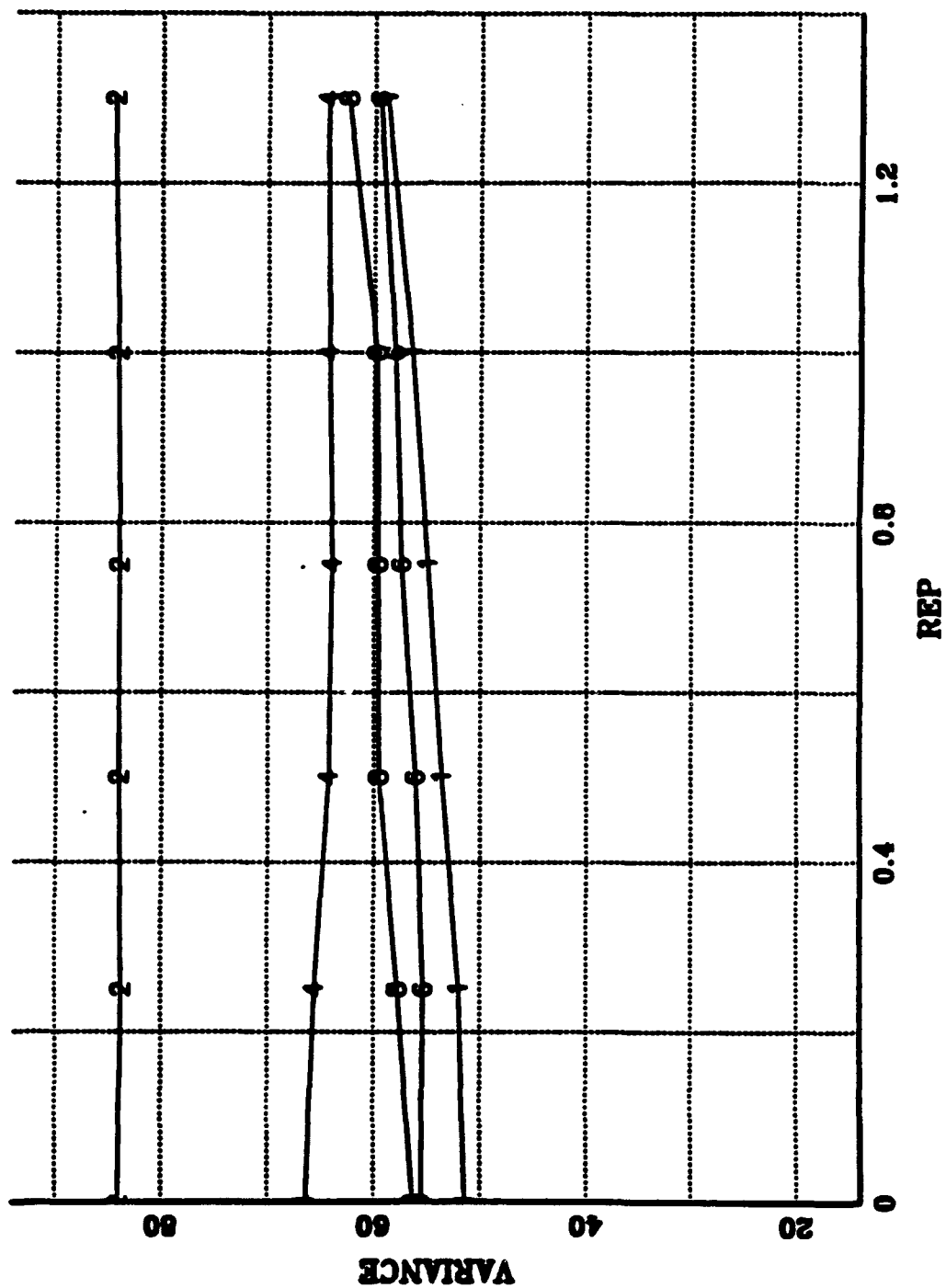
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 0.6



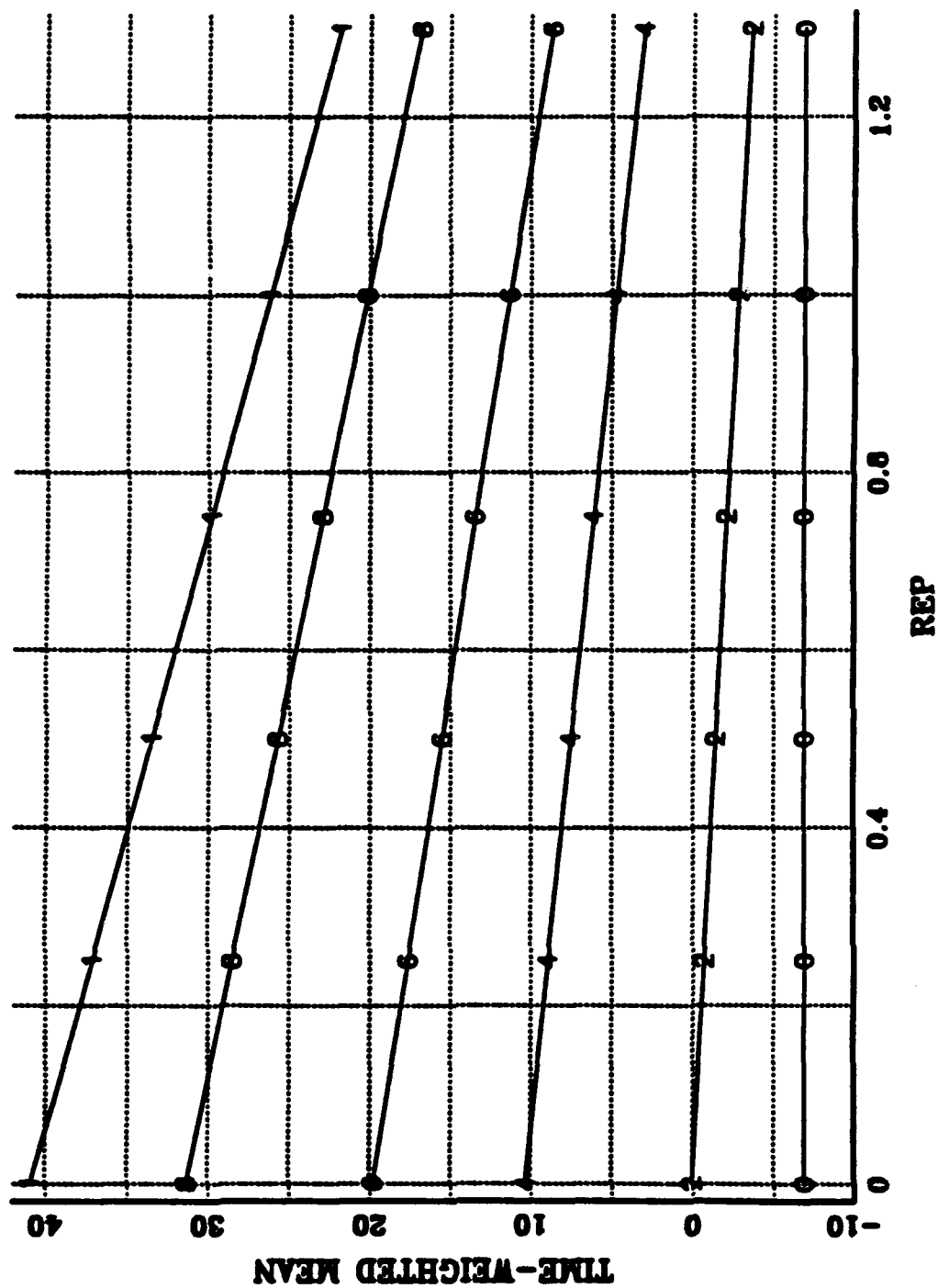
REP VS VARIANCE

RSR IS FIXED AT 0.6; DIFFERENT VALUES OF CRR



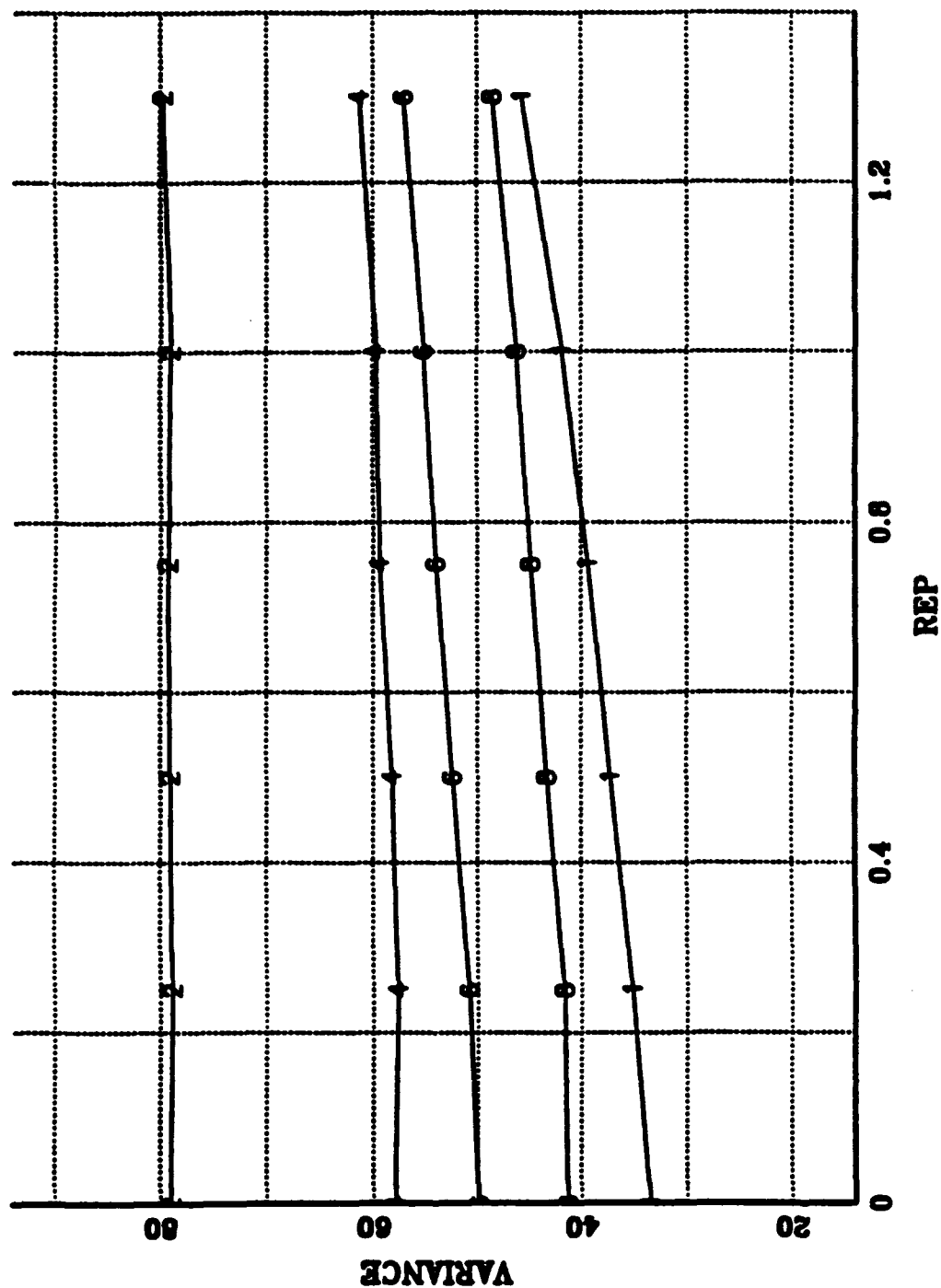
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 0.8



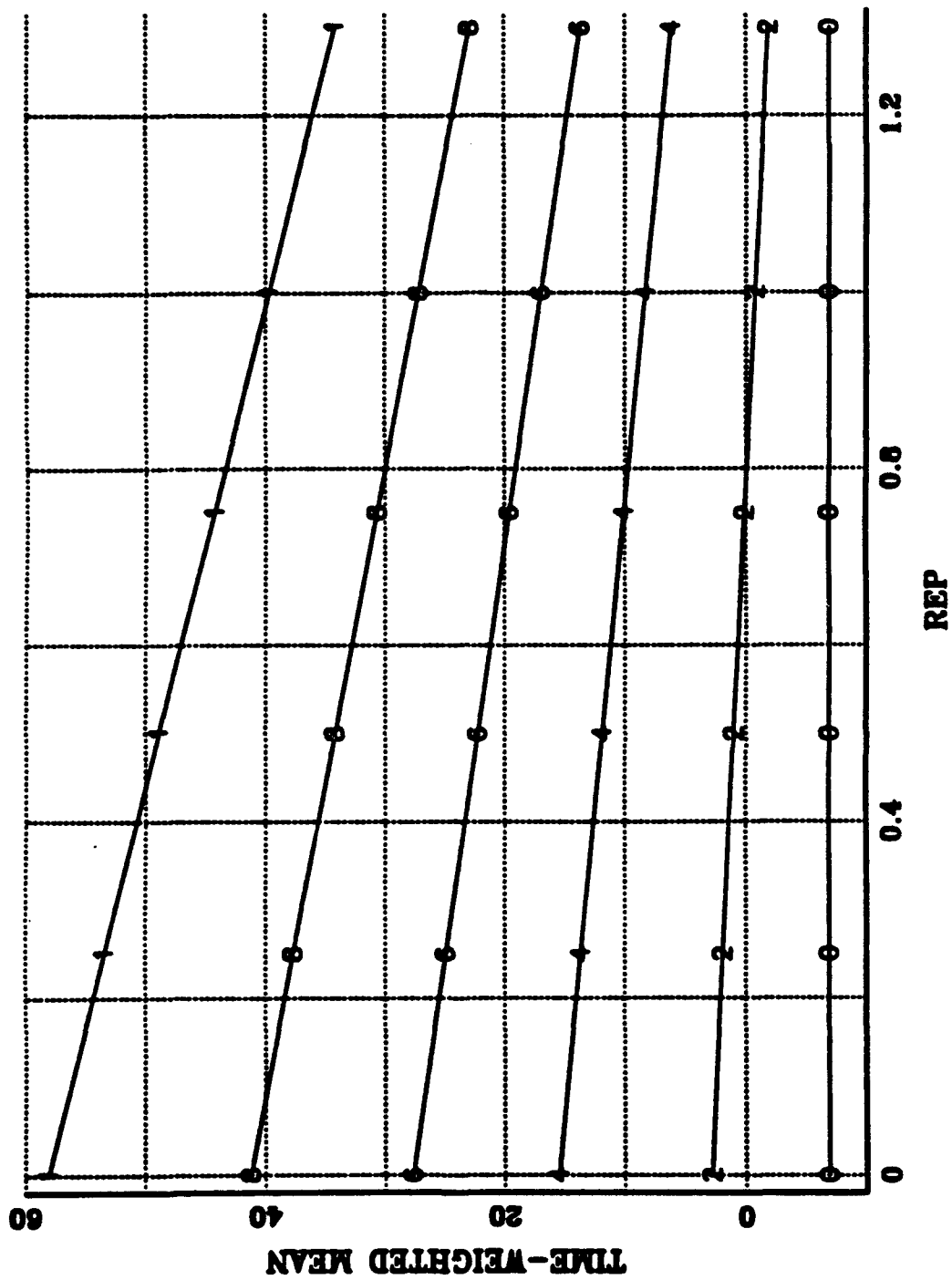
REP VS VARIANCE

RSR IS FIXED AT 0.8; DIFFERENT VALUES OF CRR



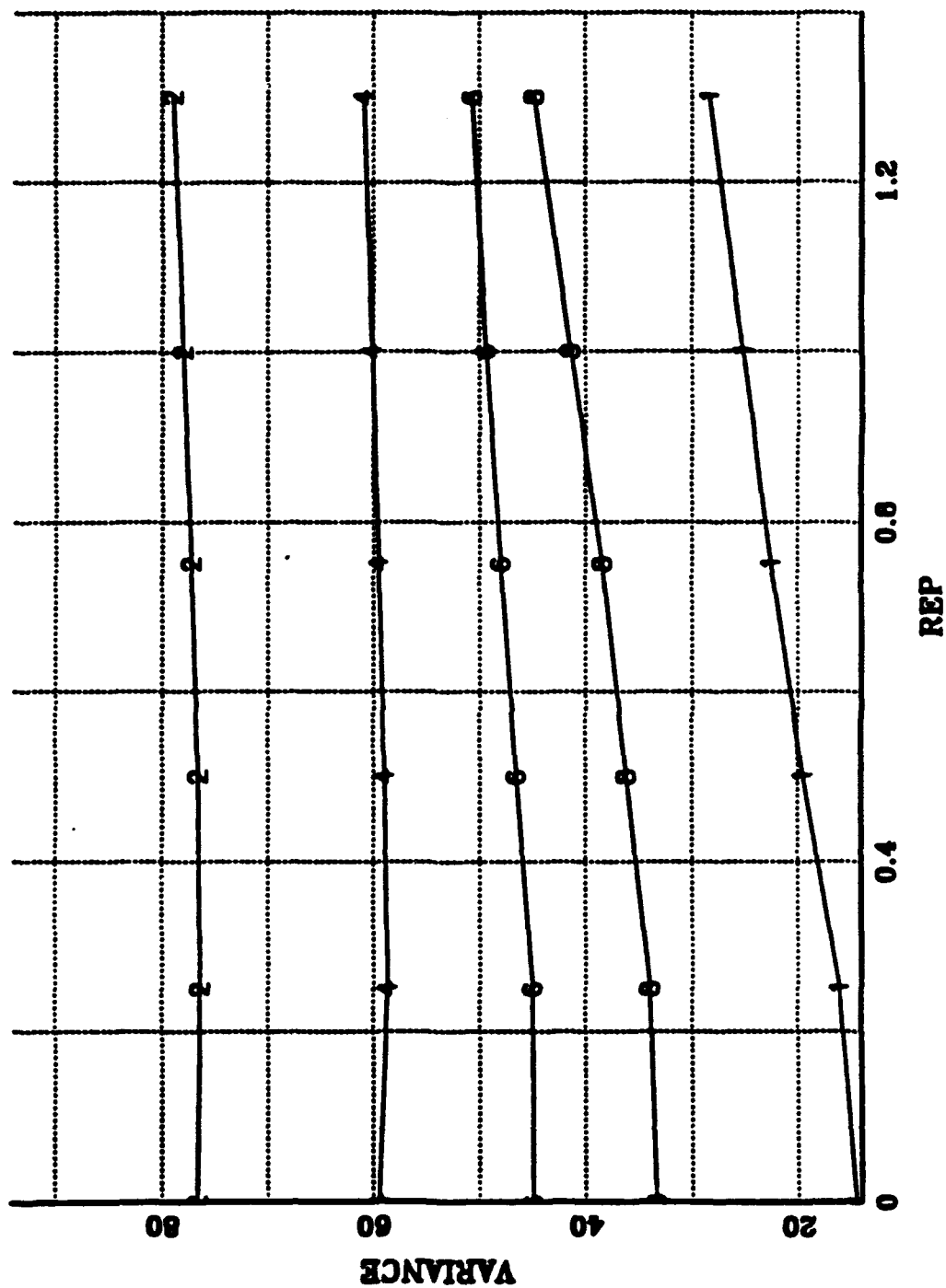
CRR IS VARIED FOR EACH LINE

RSR FIXED AT 1.0



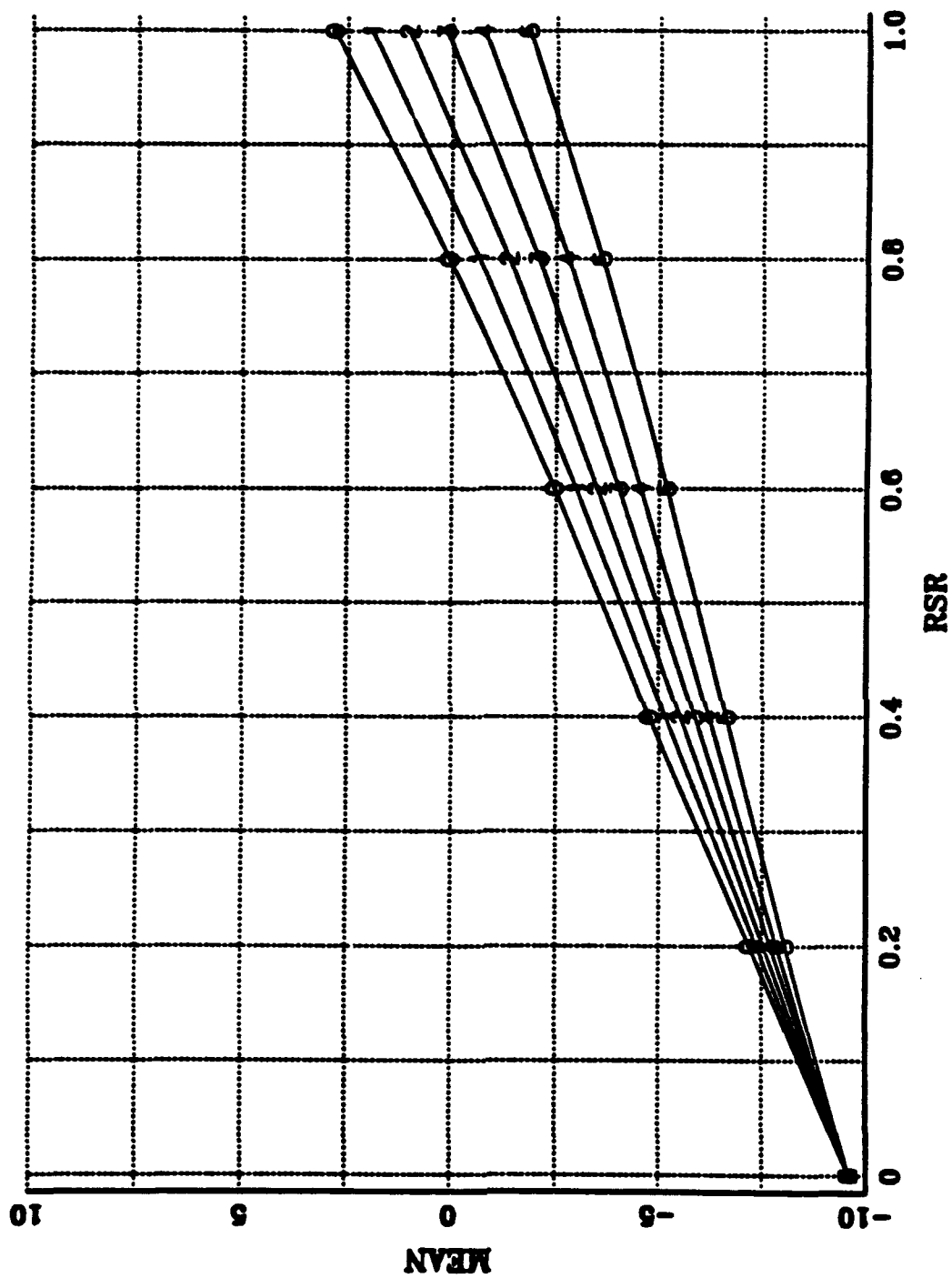
REP VS VARIANCE

RSR IS FIXED AT 1.0; DIFFERENT VALUES OF CRR

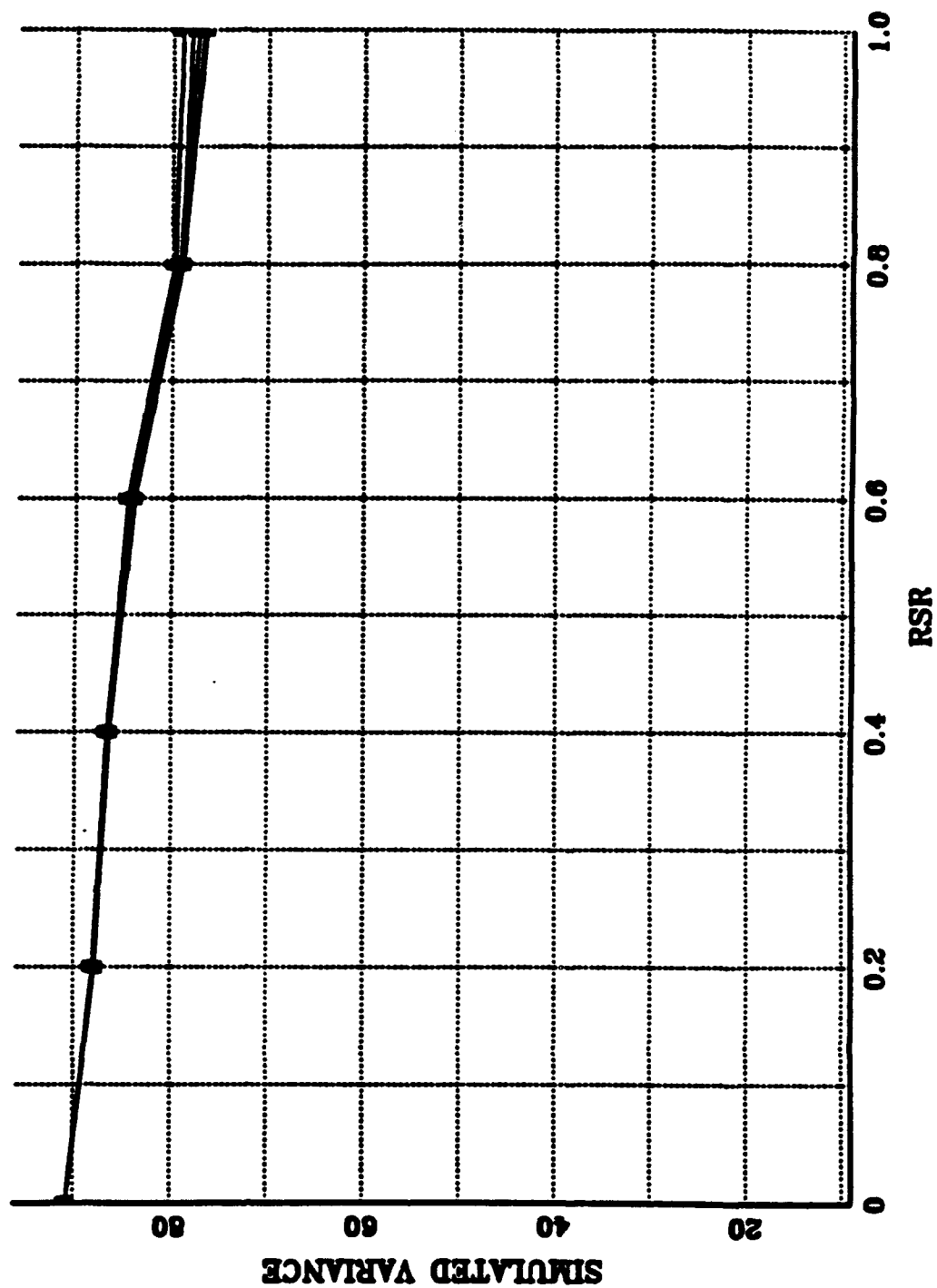


CRR IS FIXED AT 0.2

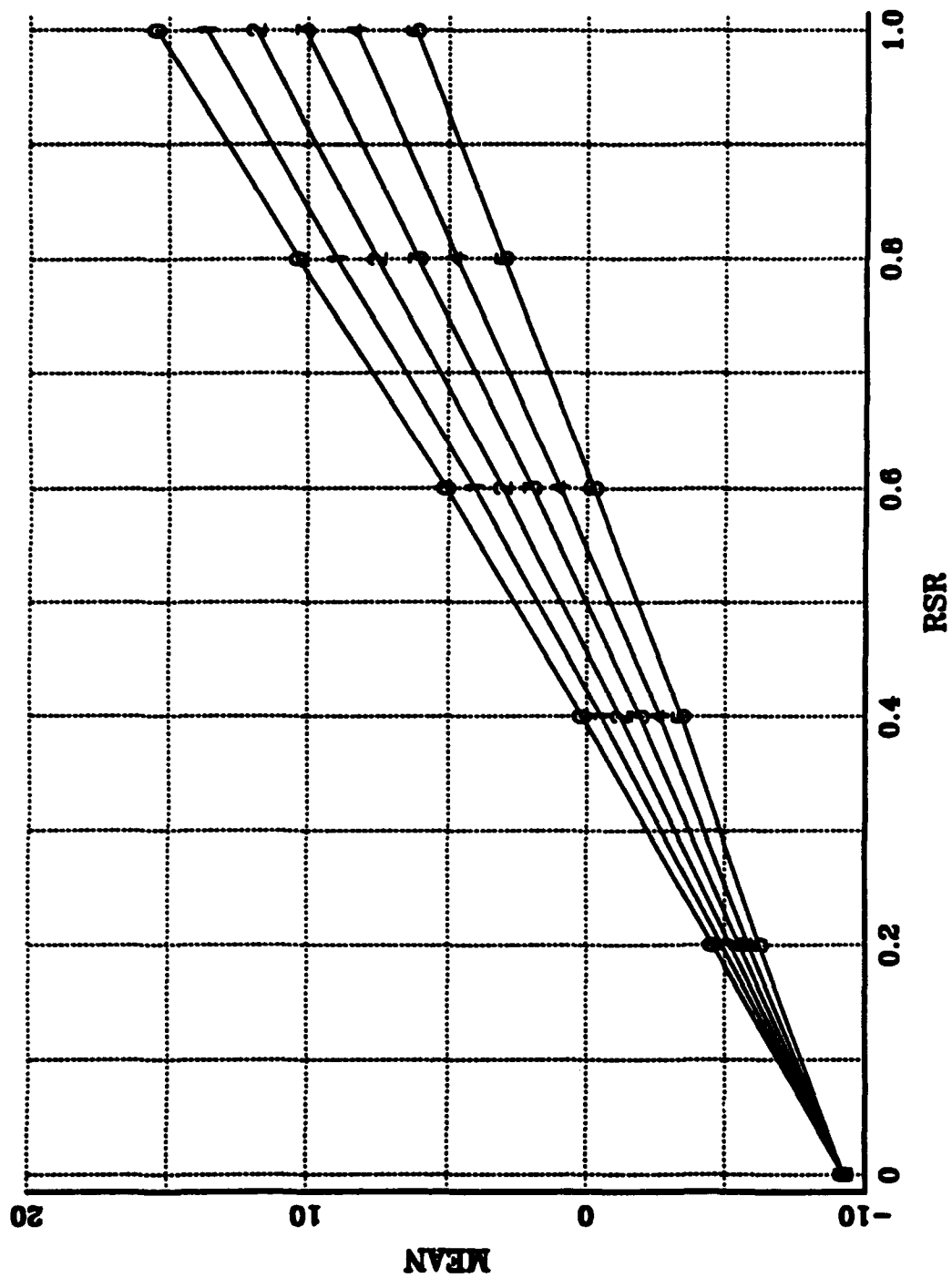
REP IS VARIED FOR EACH LINE



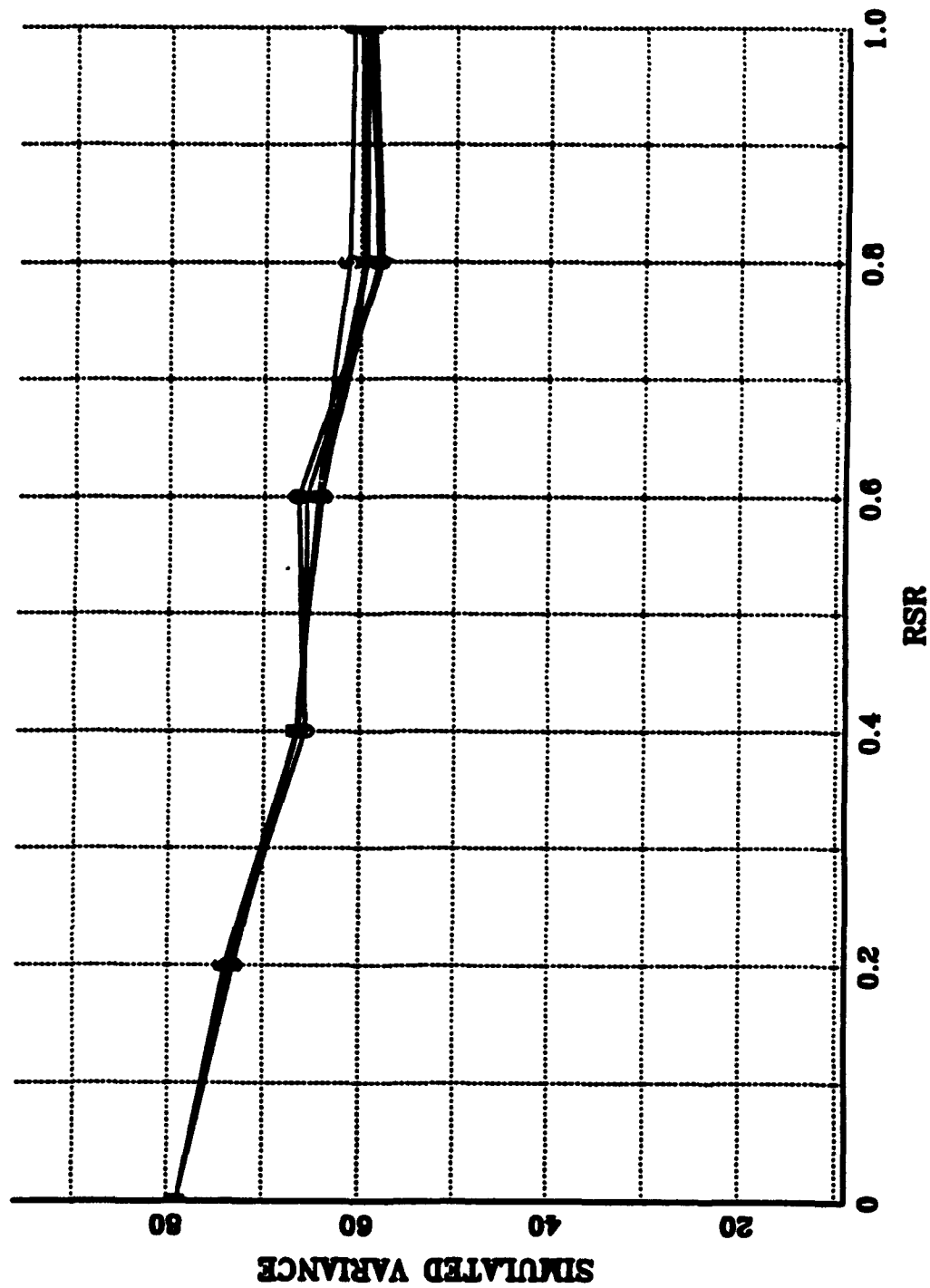
RSR VS SIMULATED VARIANCE
CRR IS FIXED AT 0.2; DIFFERENT VALUES OF REP



CRR IS FIXED AT 0.4
REP IS VARIED FOR EACH LINE

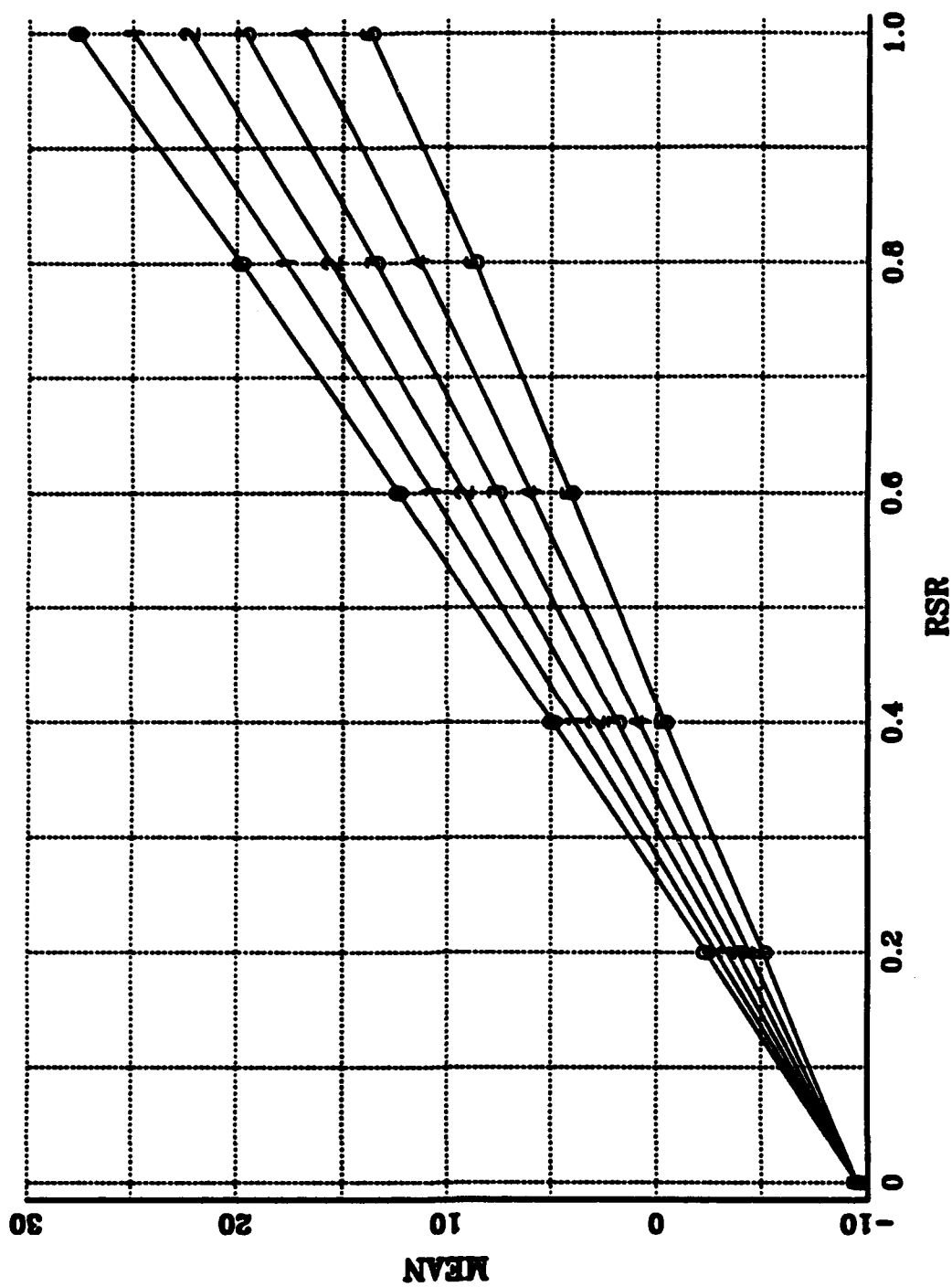


RSR VS SIMULATED VARIANCE
CRR IS FIXED AT 0.4; DIFFERENT VALUES OF REP

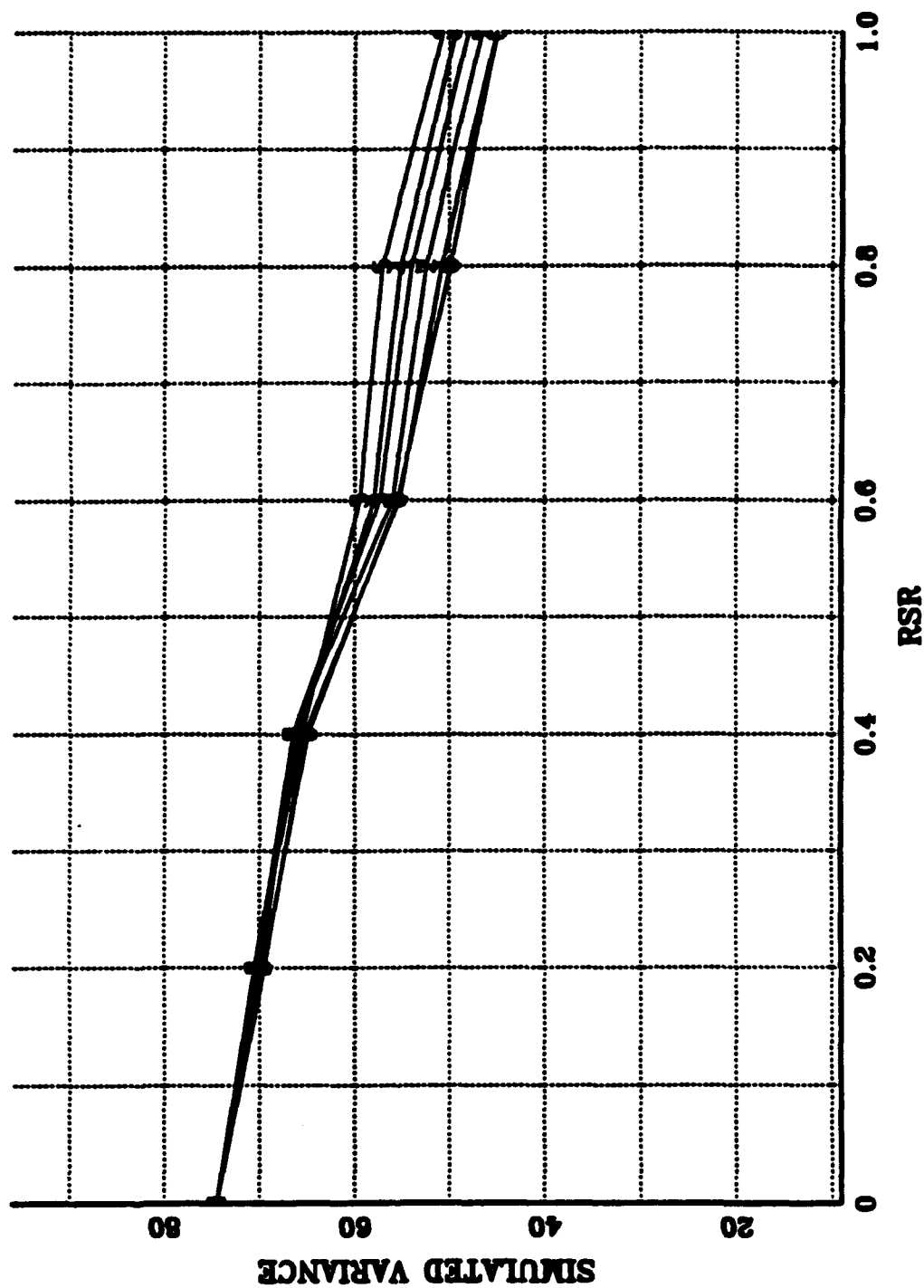


CRR IS FIXED AT 0.6

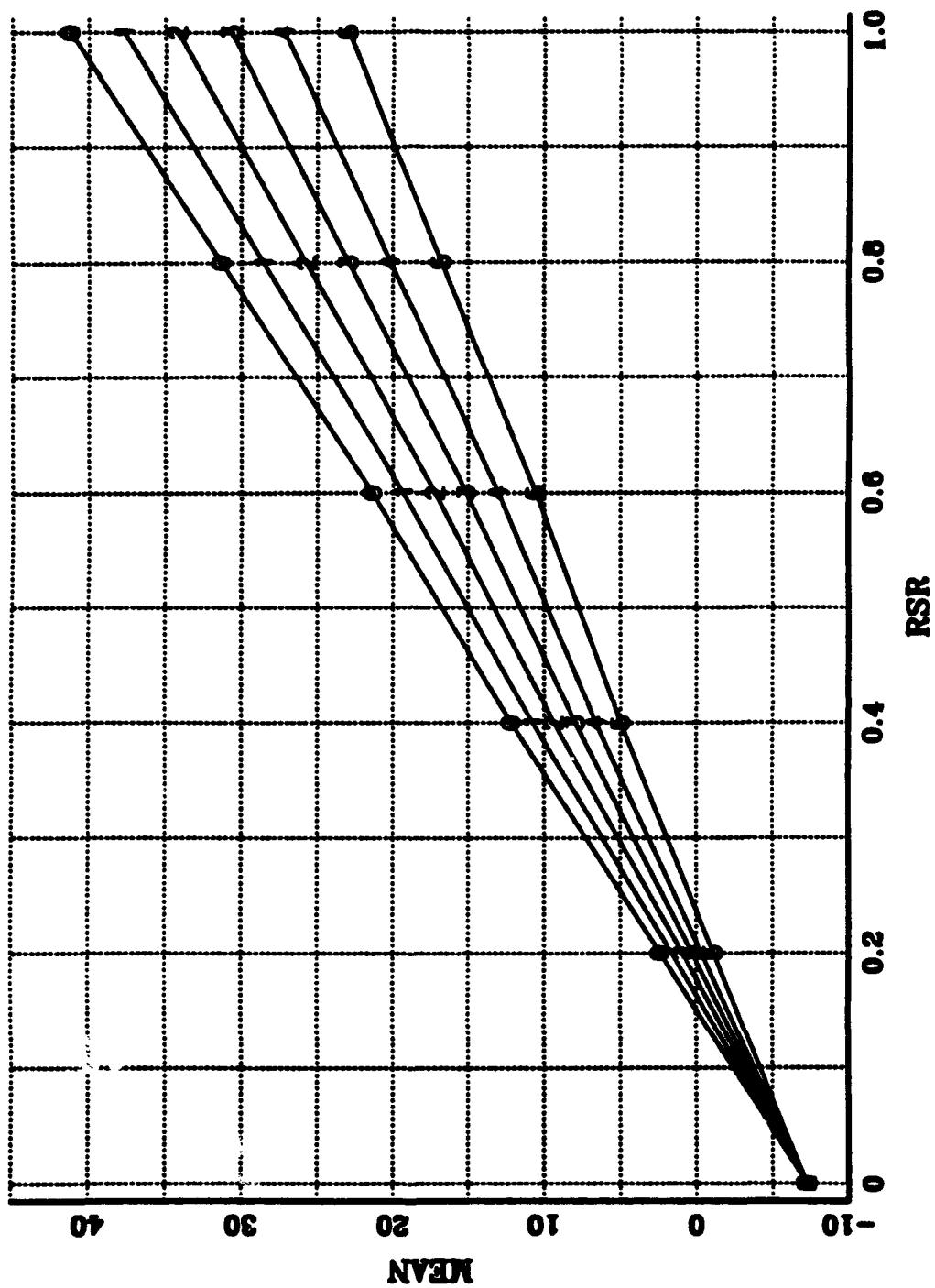
REP IS VARIED FOR EACH LINE



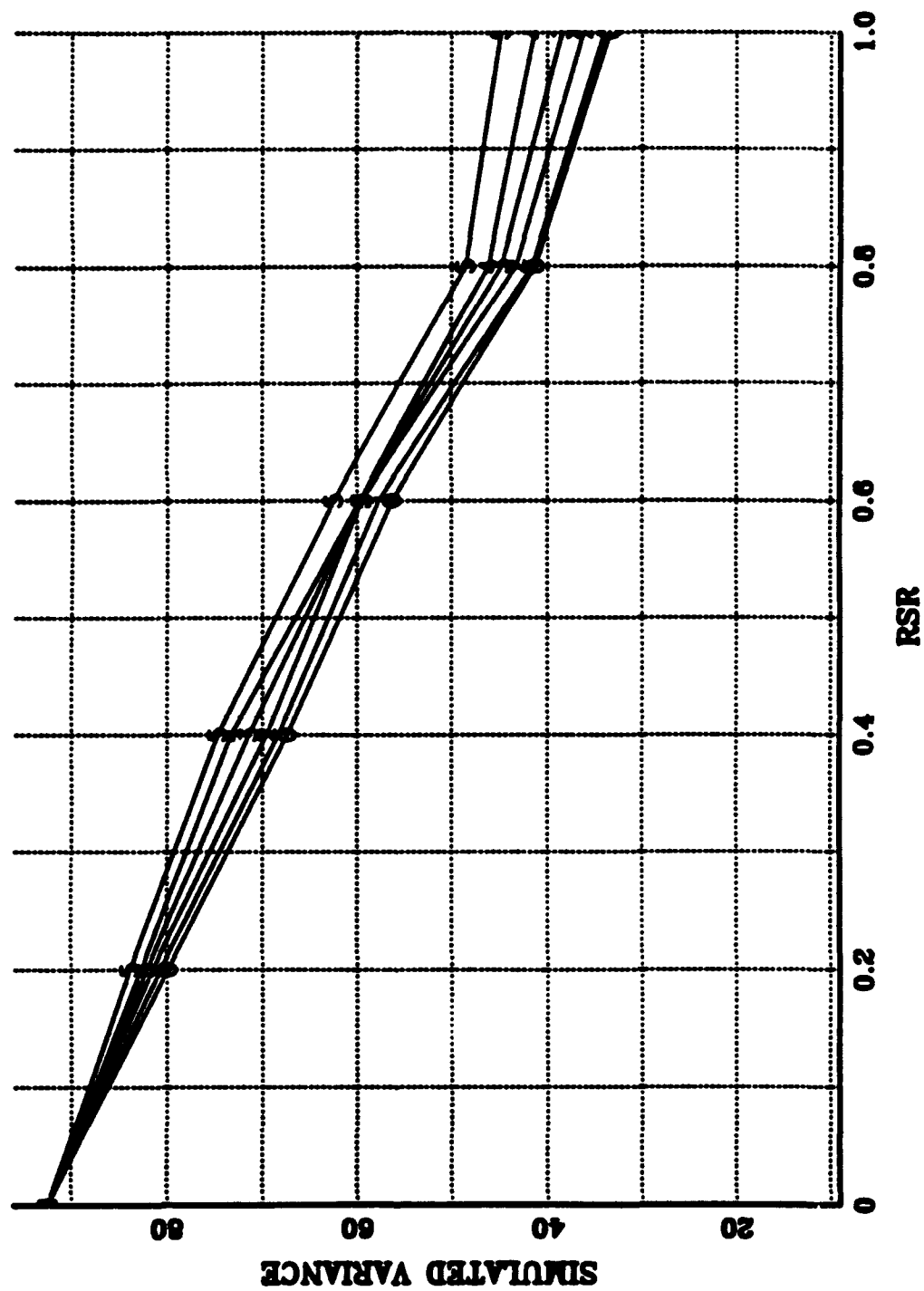
RSR VS SIMULATED VARIANCE
CRR IS FIXED AT 0.6; DIFFERENT VALUES OF REP



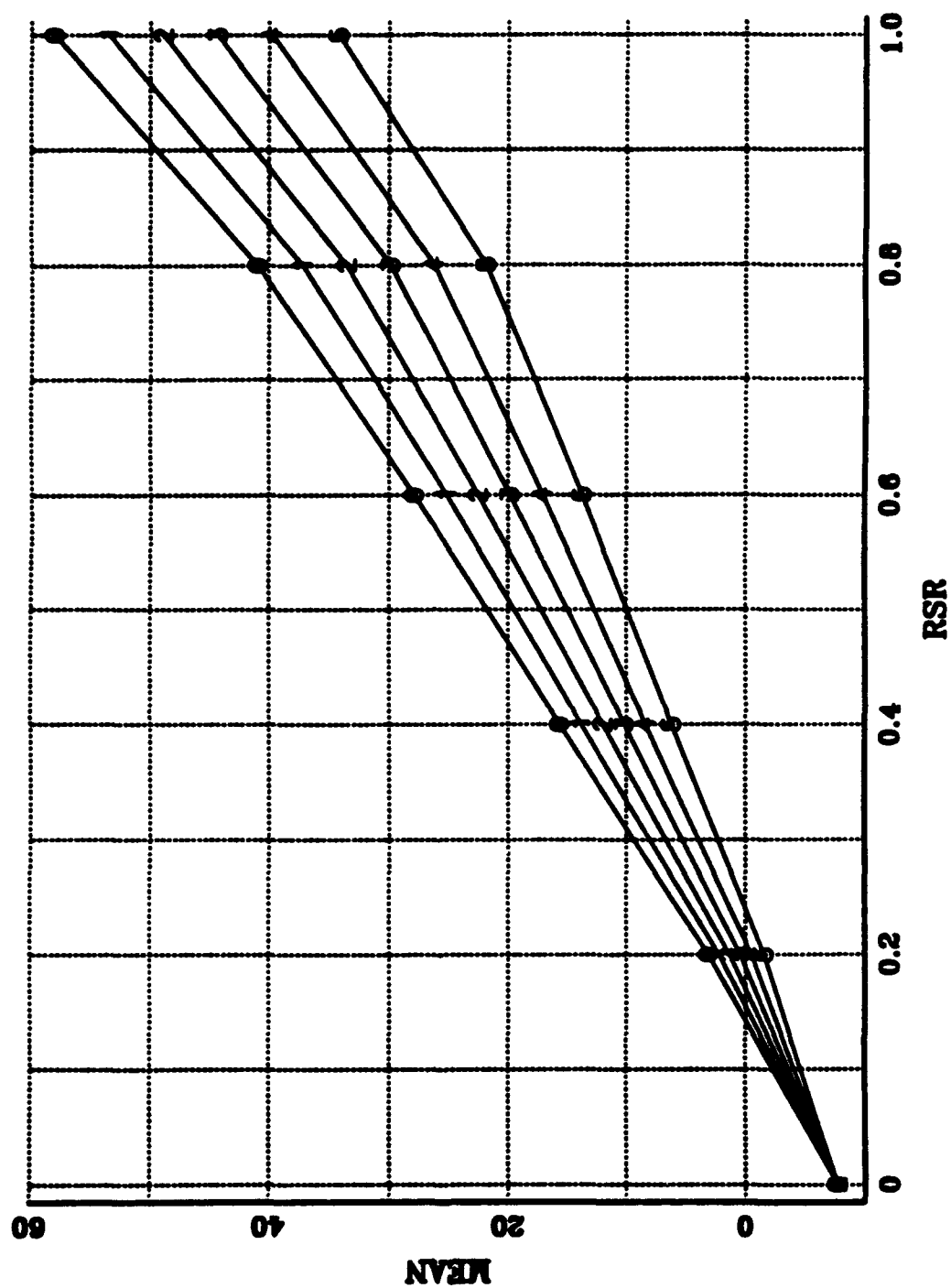
CRR IS FIXED AT 0.8
REP IS VARIED FOR EACH LINE



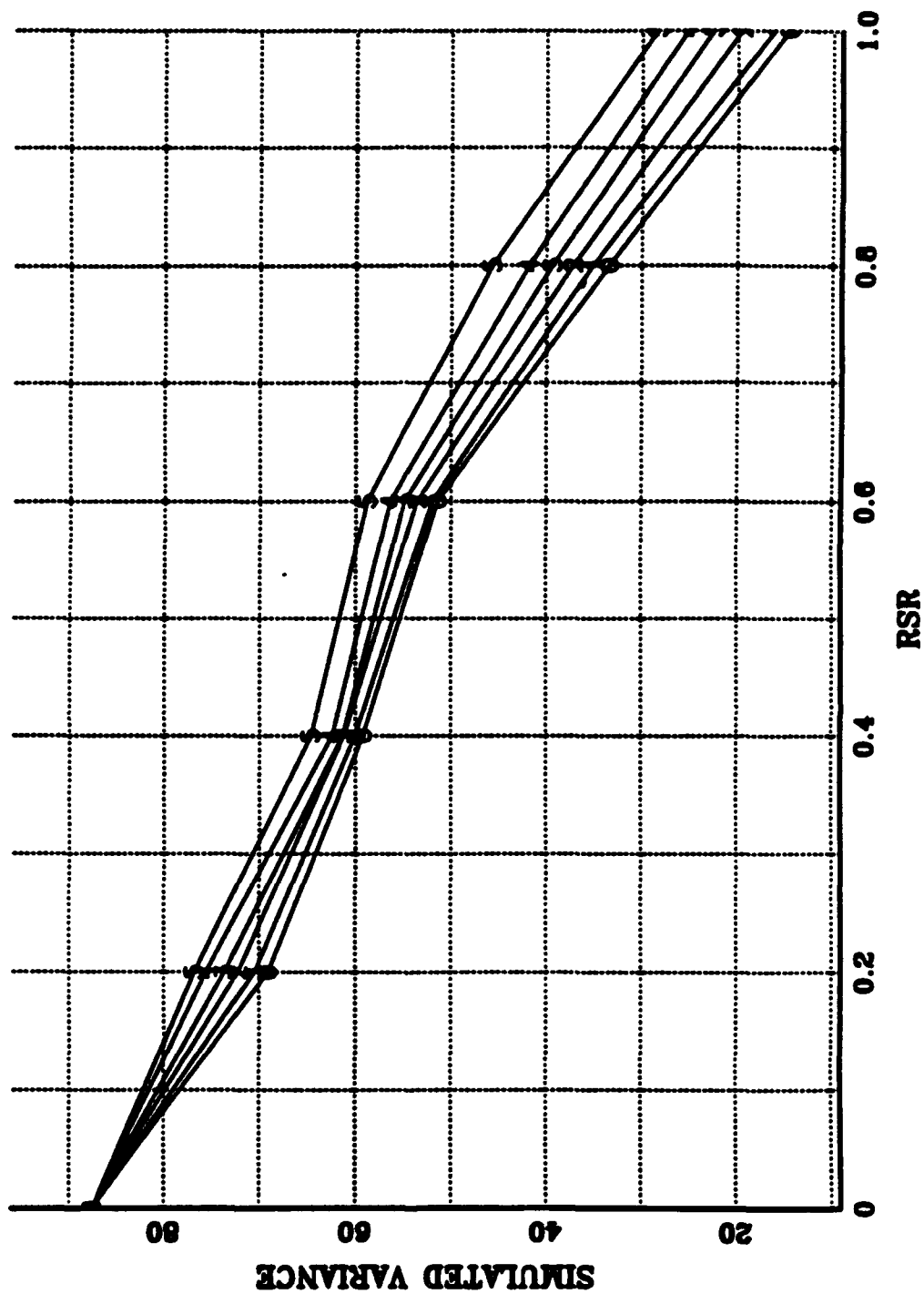
RSR VS SIMULATED VARIANCE
CRR IS FIXED AT 0.8; DIFFERENT VALUES OF REP



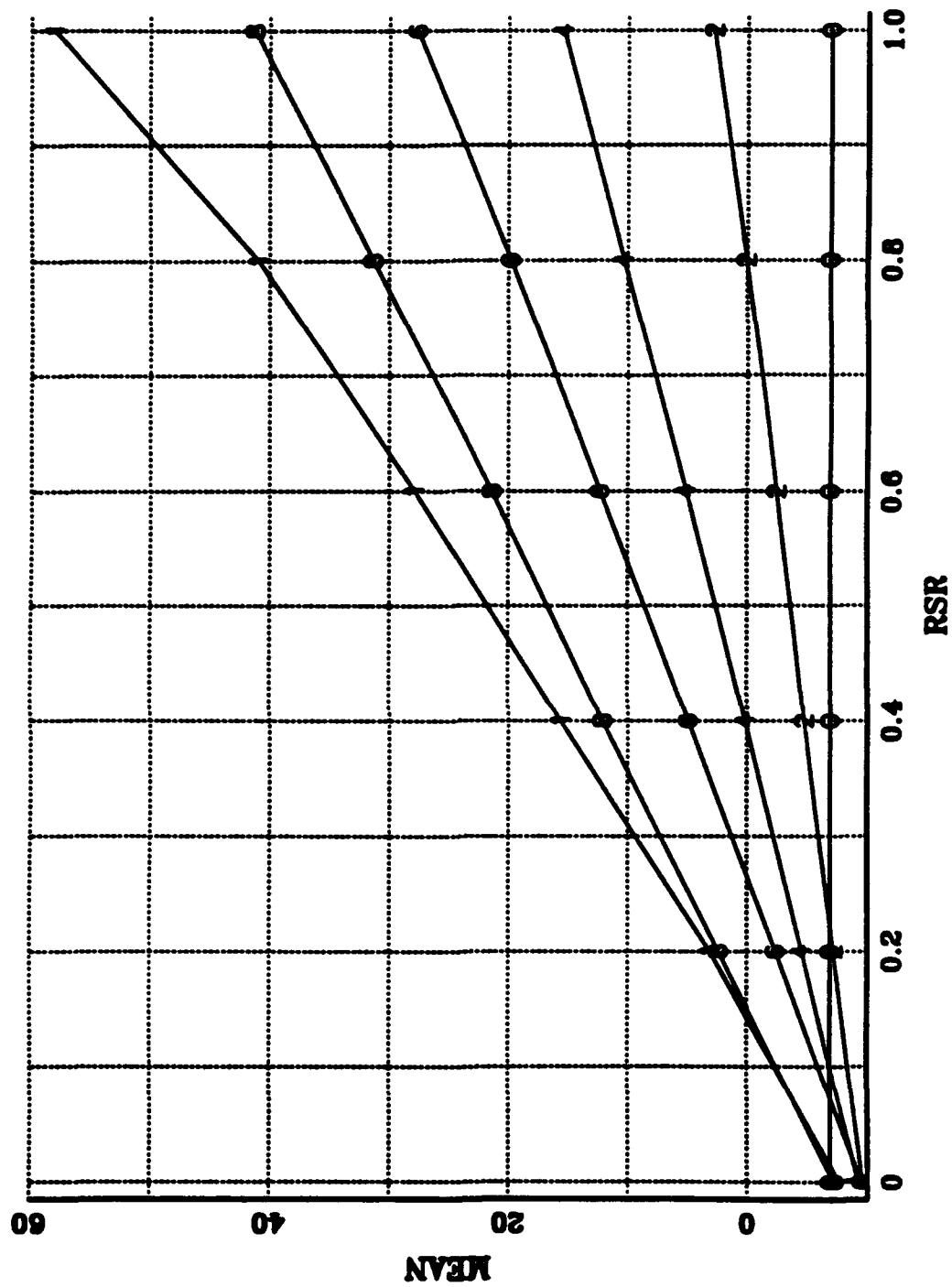
CRR IS FIXED AT 1.0
REP IS VARIED FOR EACH LINE



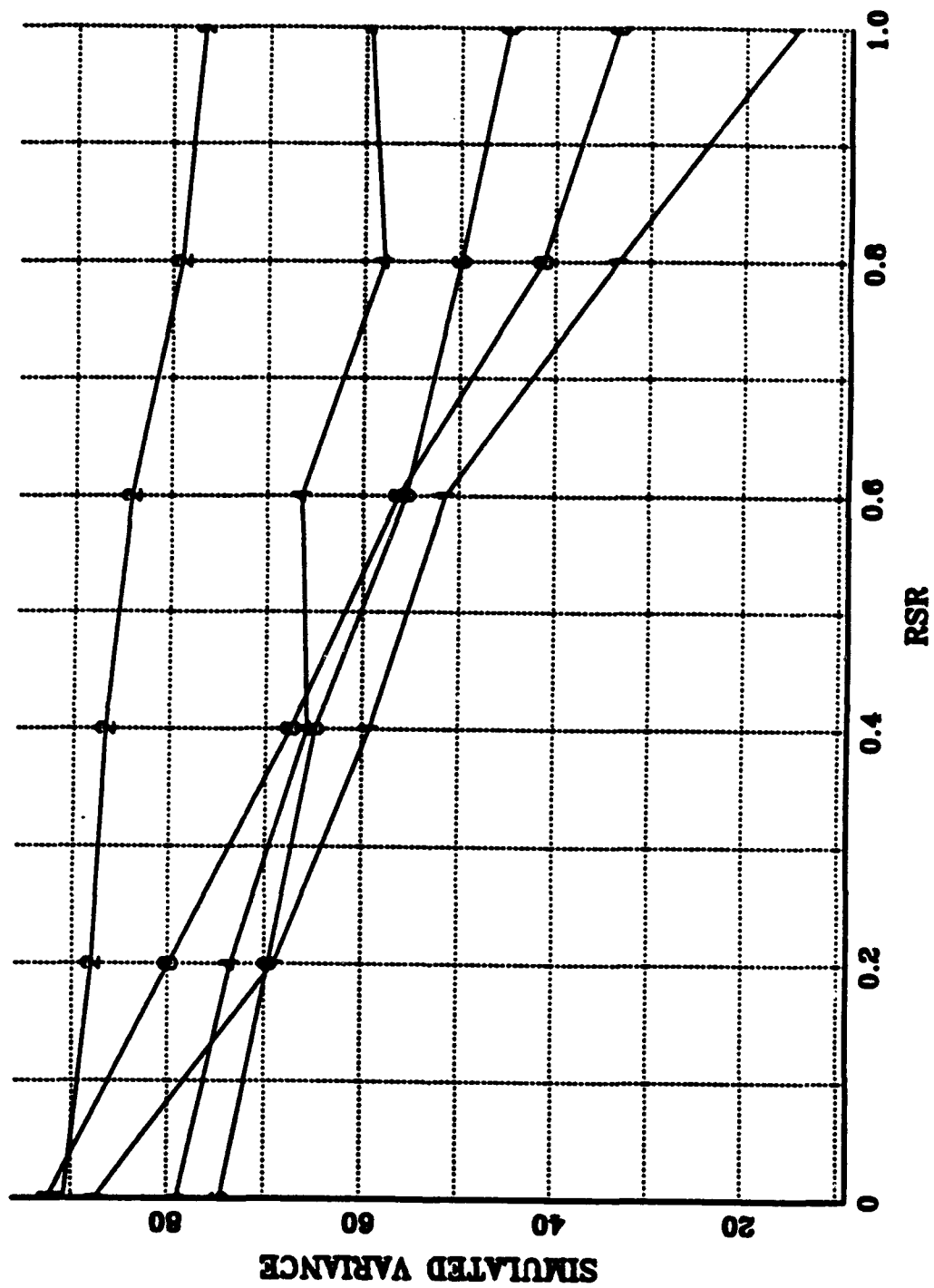
RSR VS SIMULATED VARIANCE
CRR IS FIXED AT 1.0; DIFFERENT VALUES OF REP



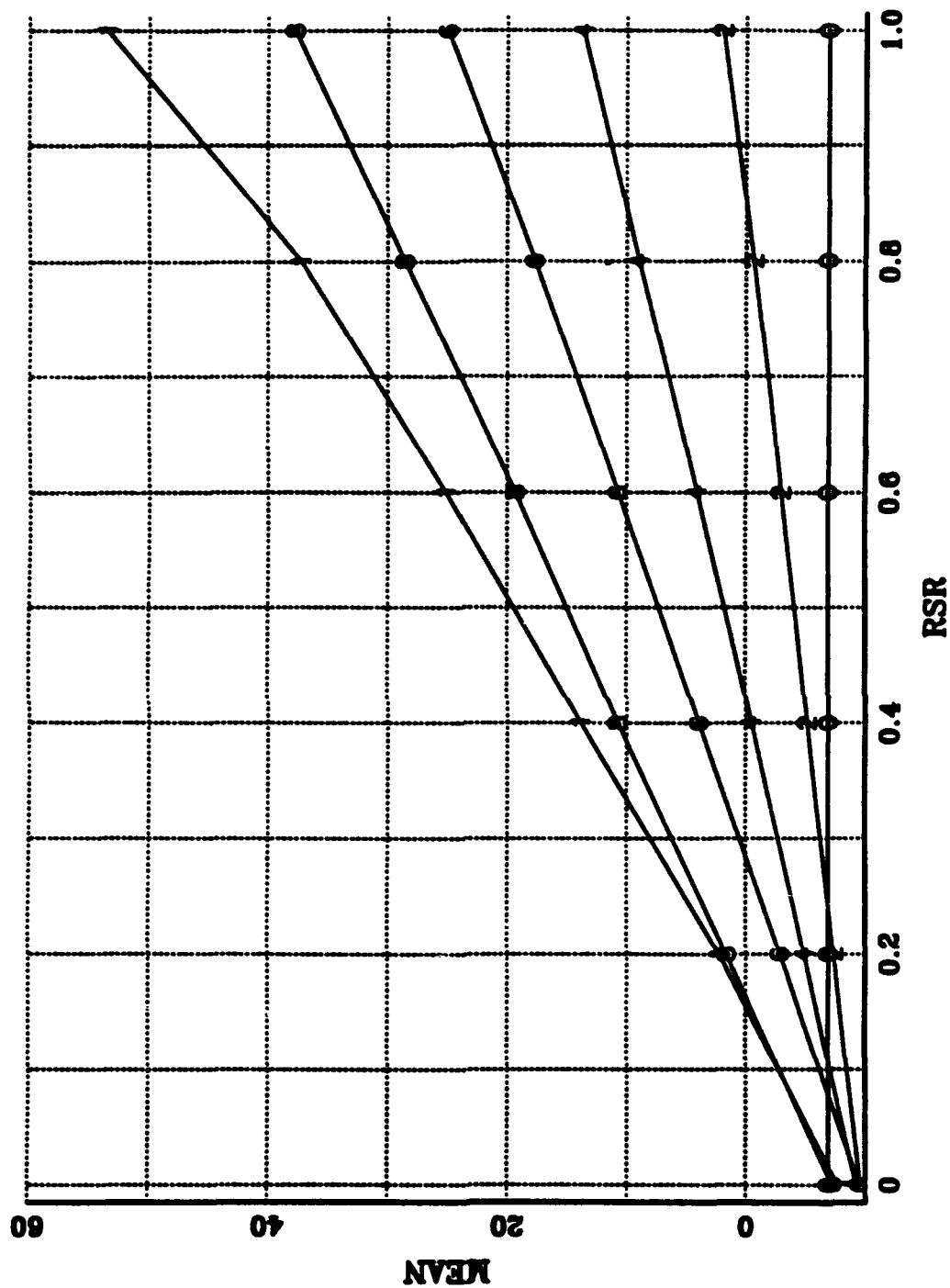
REP IS FIXED AT 0.0
CRR IS VARIED FOR EACH LINE



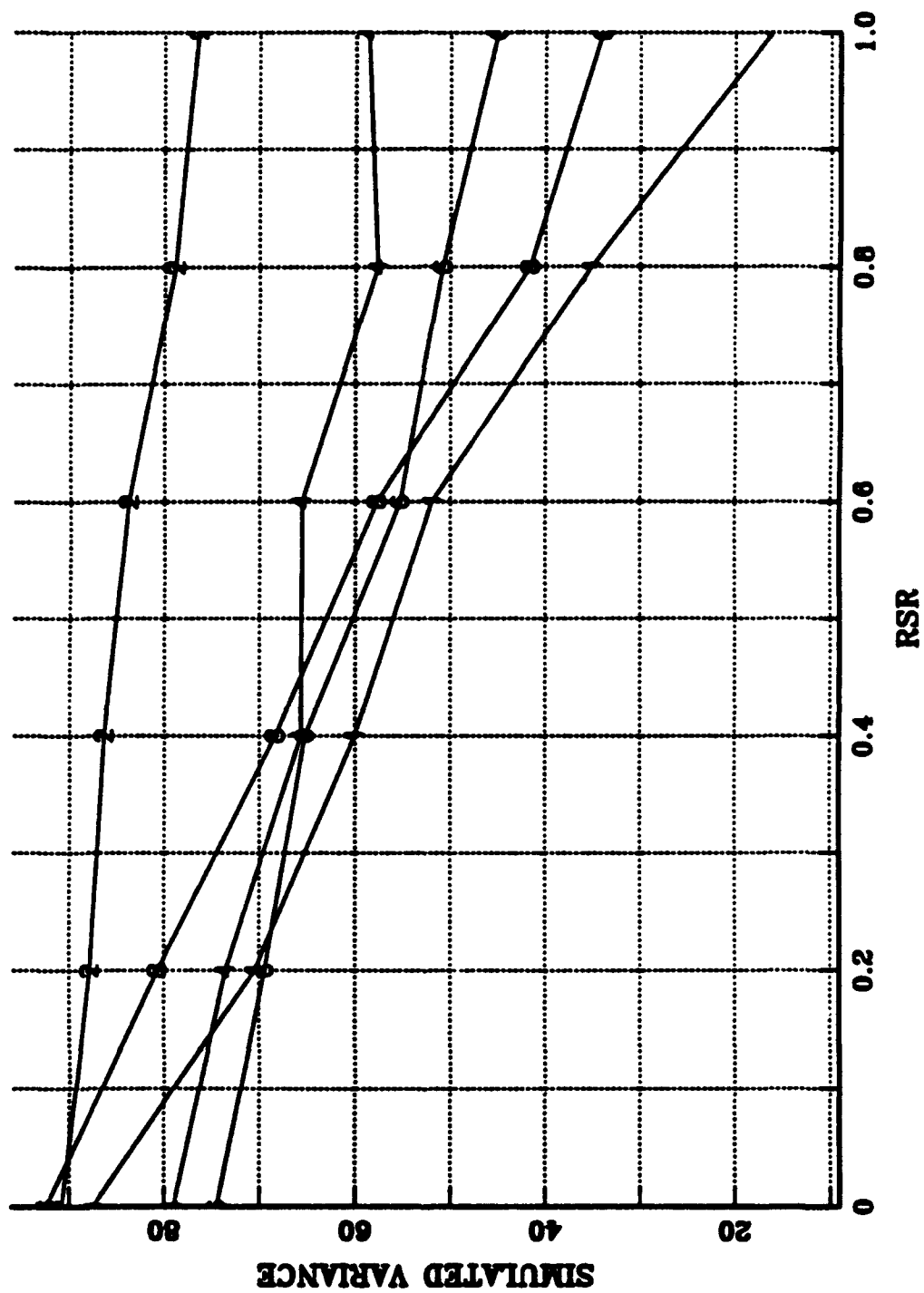
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 0.0; DIFFERENT VALUES OF CRR



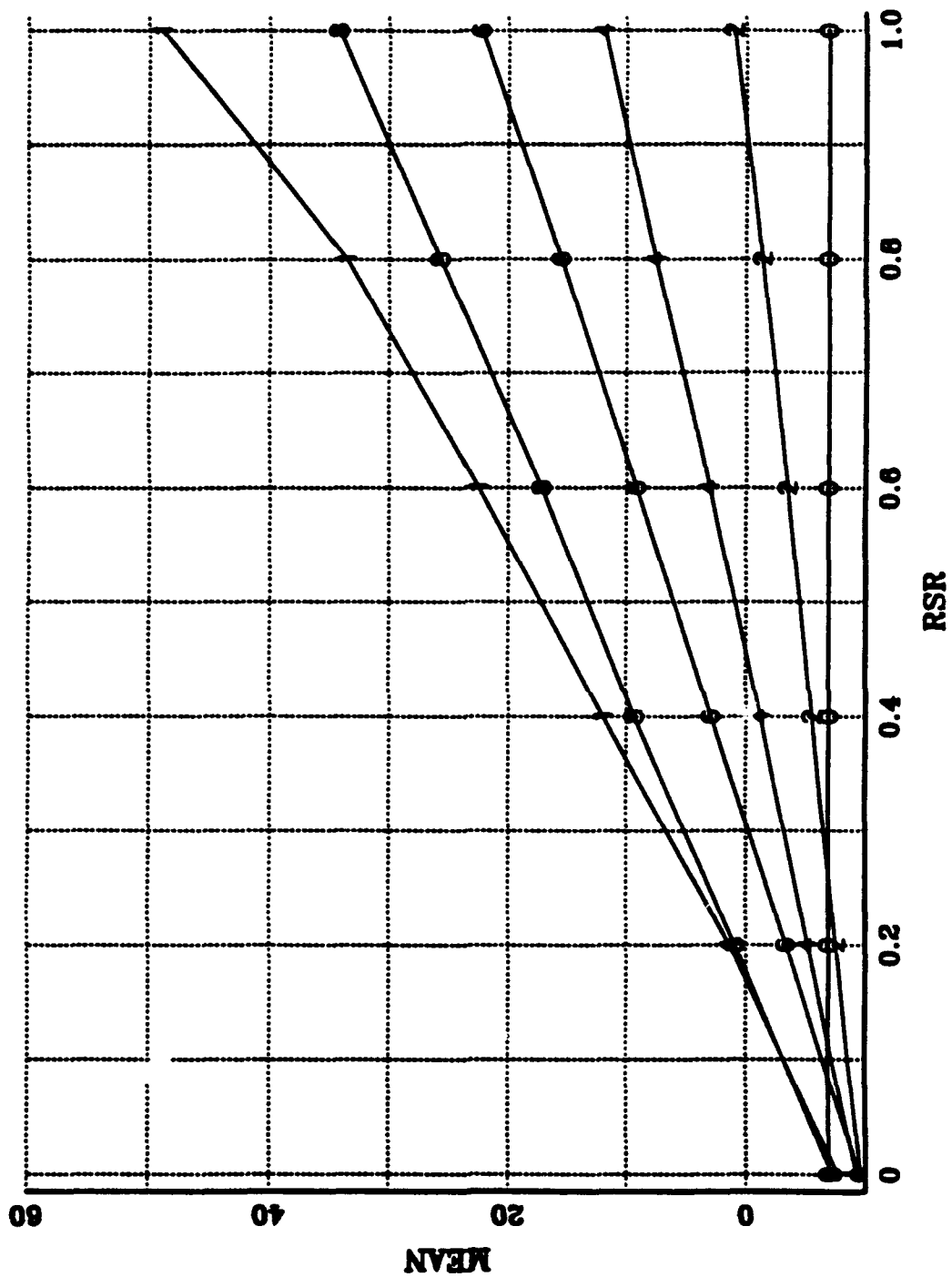
REP IS FIXED AT 0.25
CRR IS VARIED FOR EACH LINE



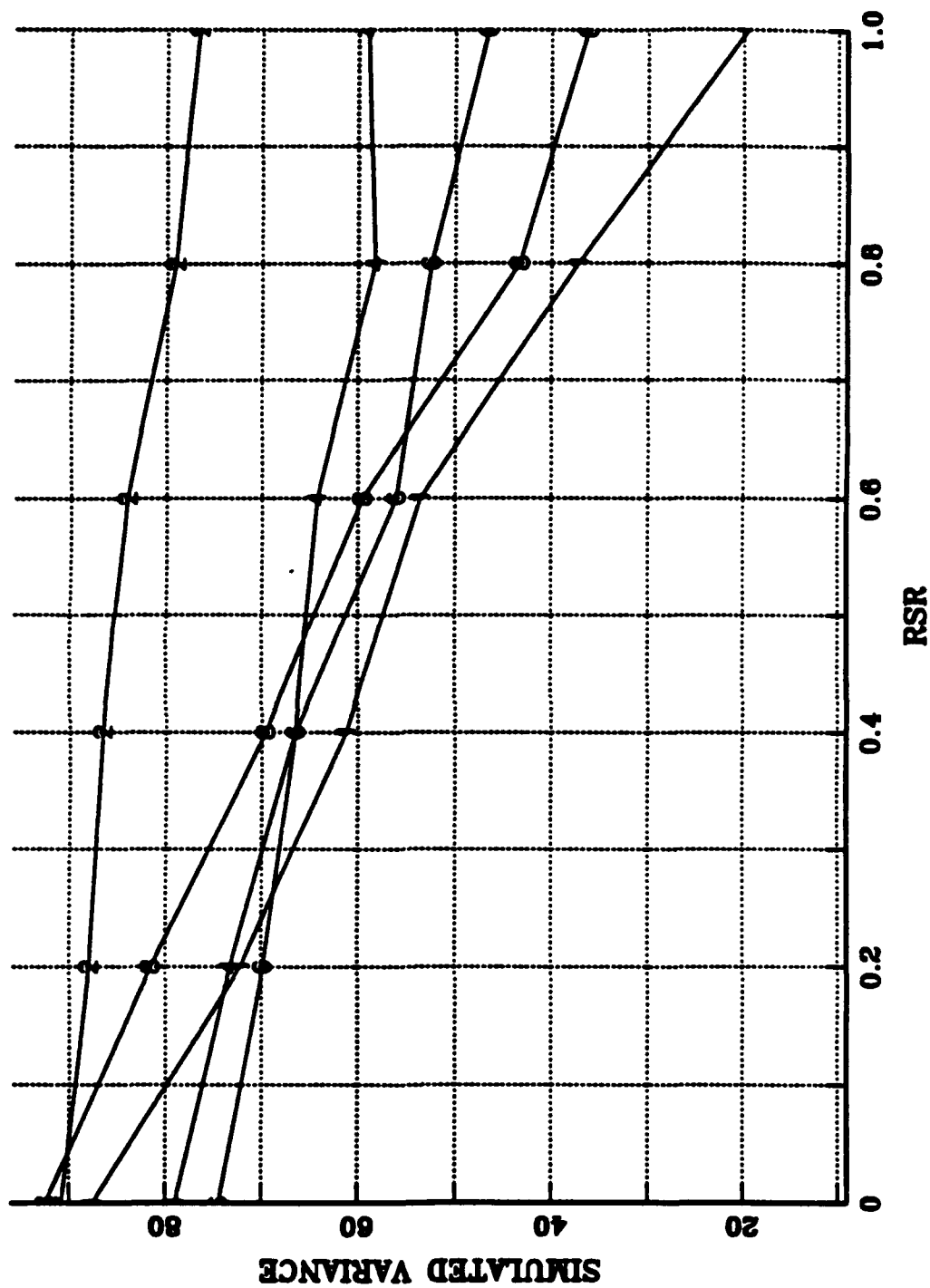
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 0.25; DIFFERENT VALUES OF CRR



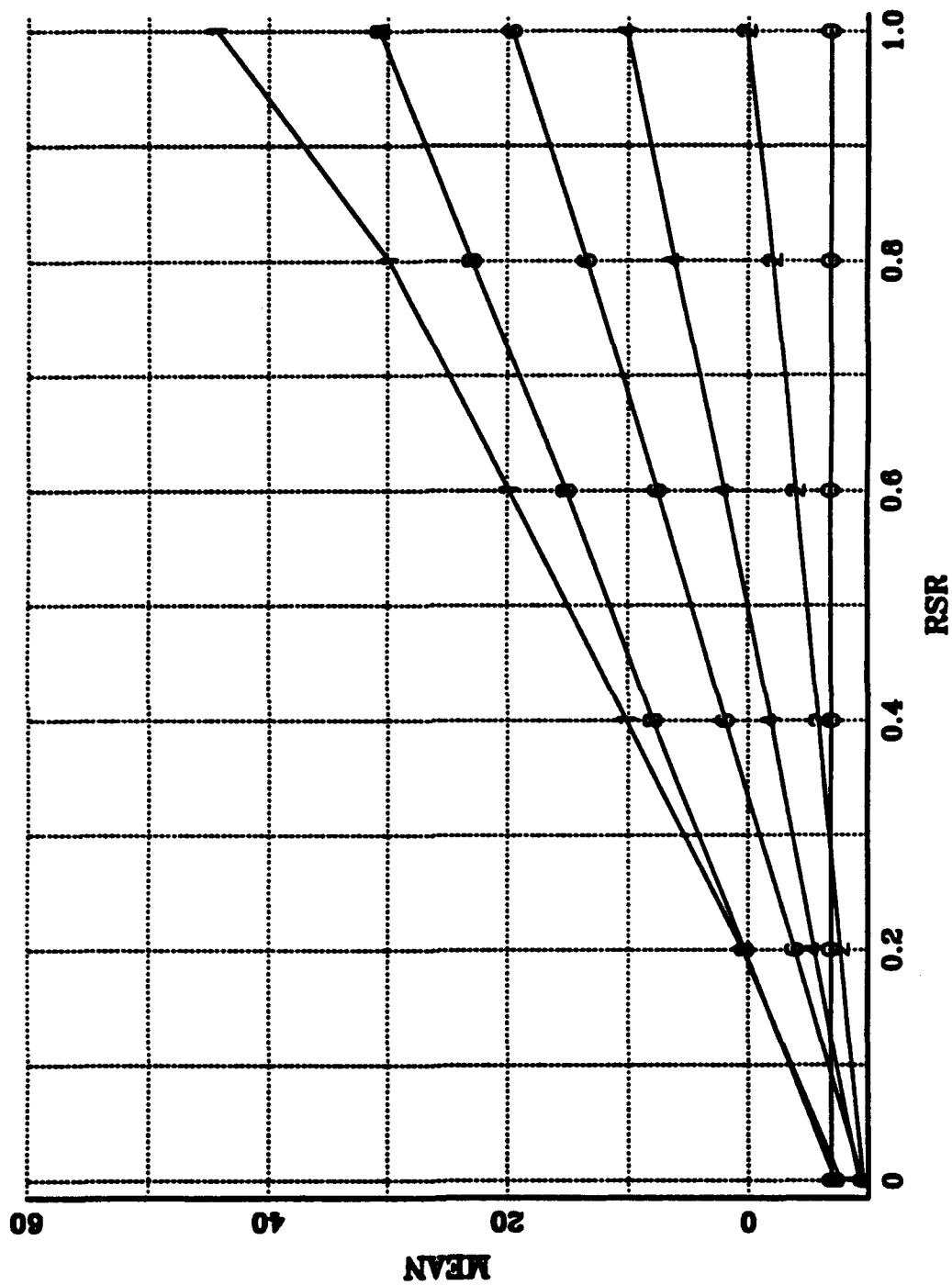
REP IS FIXED AT 0.50
CRR IS VARIED FOR EACH LINE



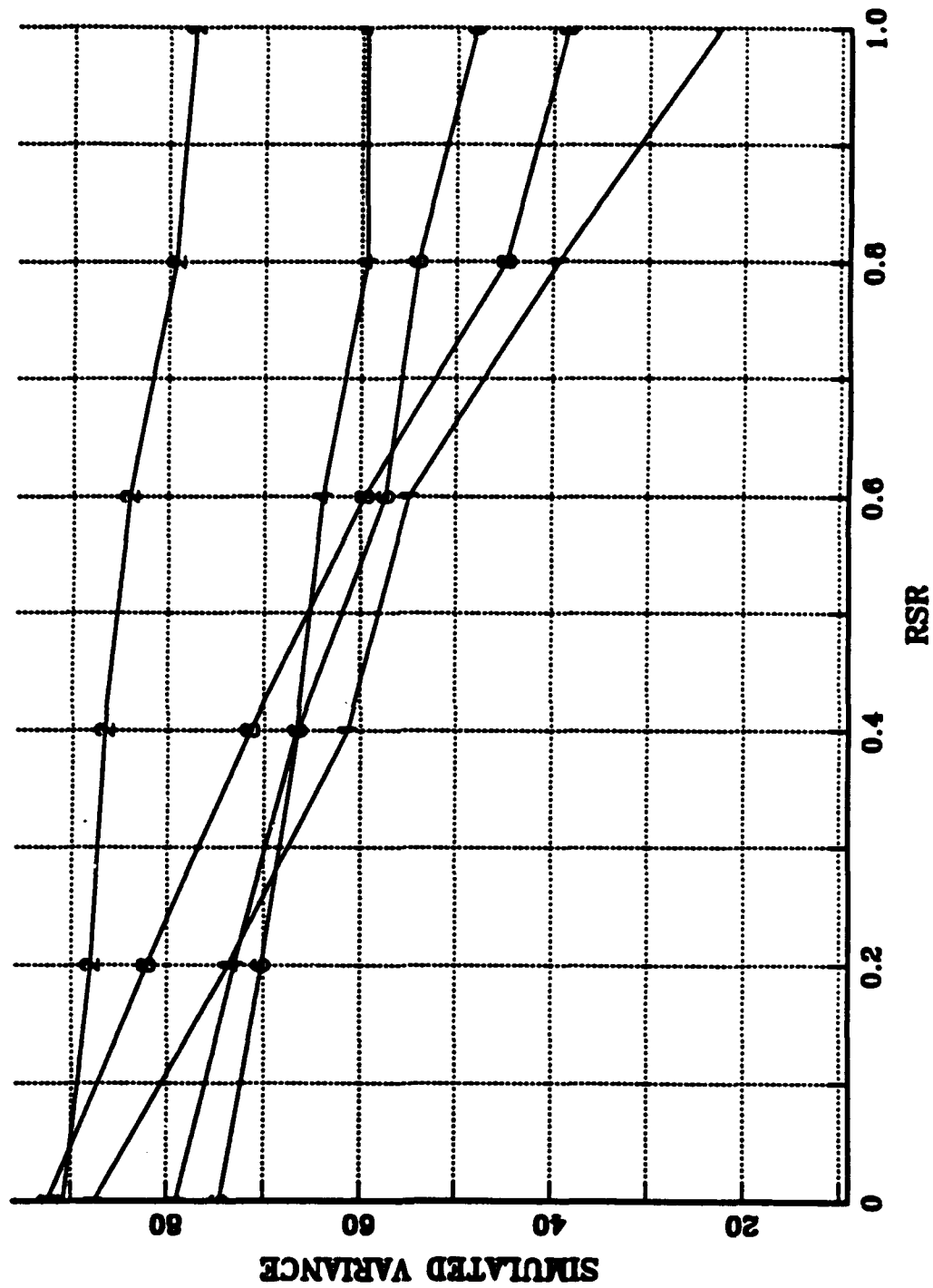
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 0.50; DIFFERENT VALUES OF CRR



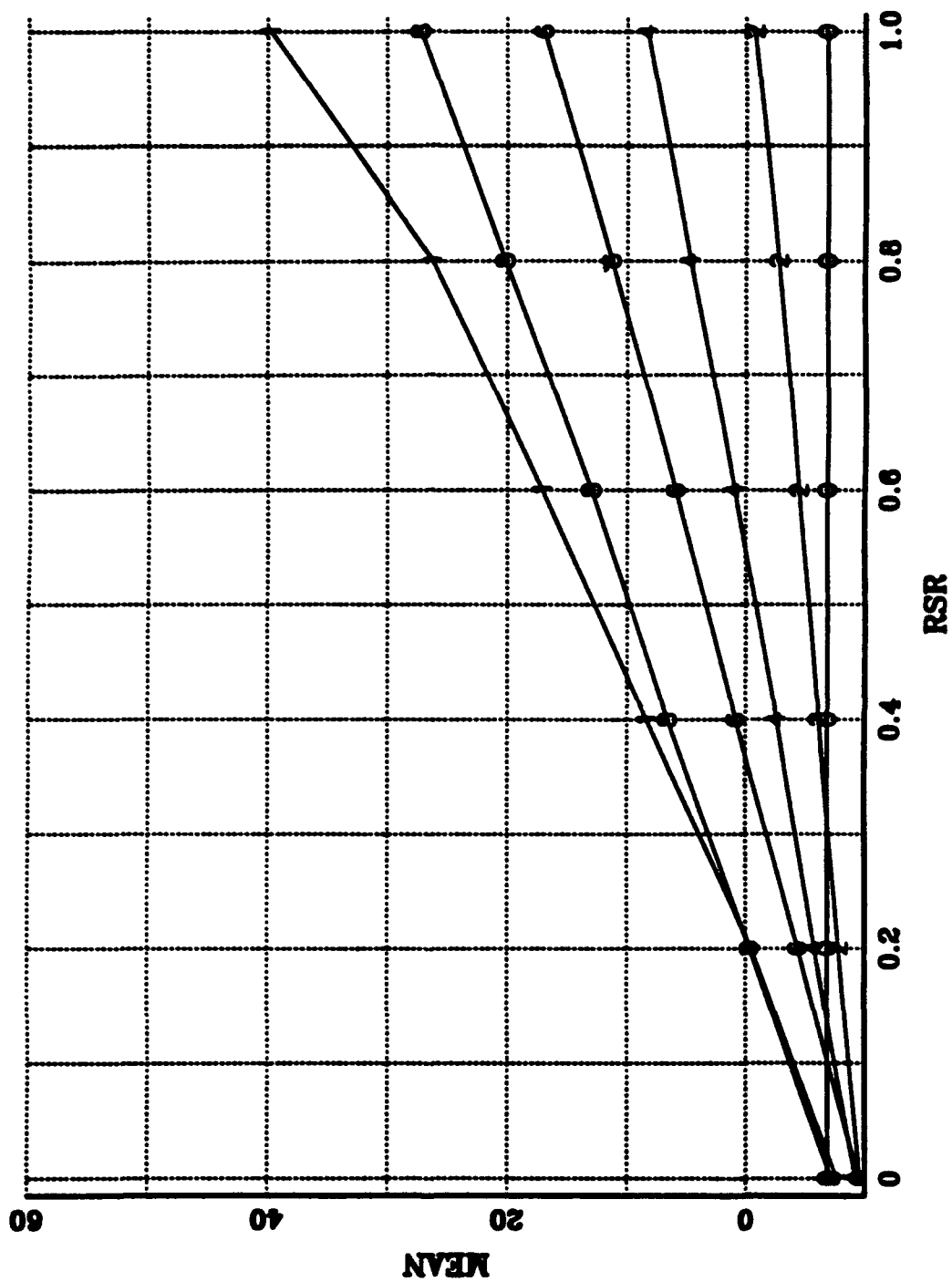
REP IS FIXED AT 0.75
CRR IS VARIED FOR EACH LINE



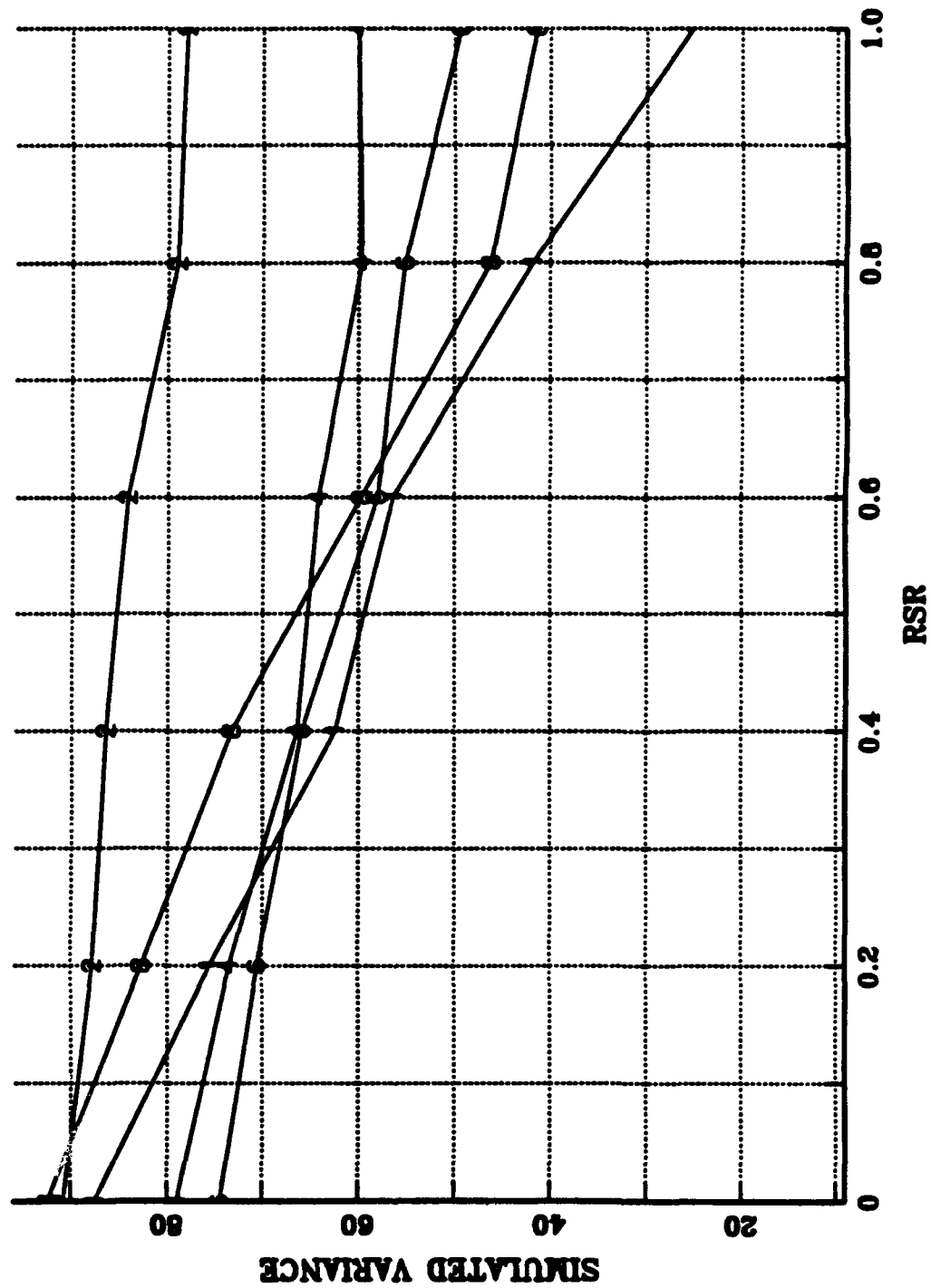
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 0.75; DIFFERENT VALUES OF CRR



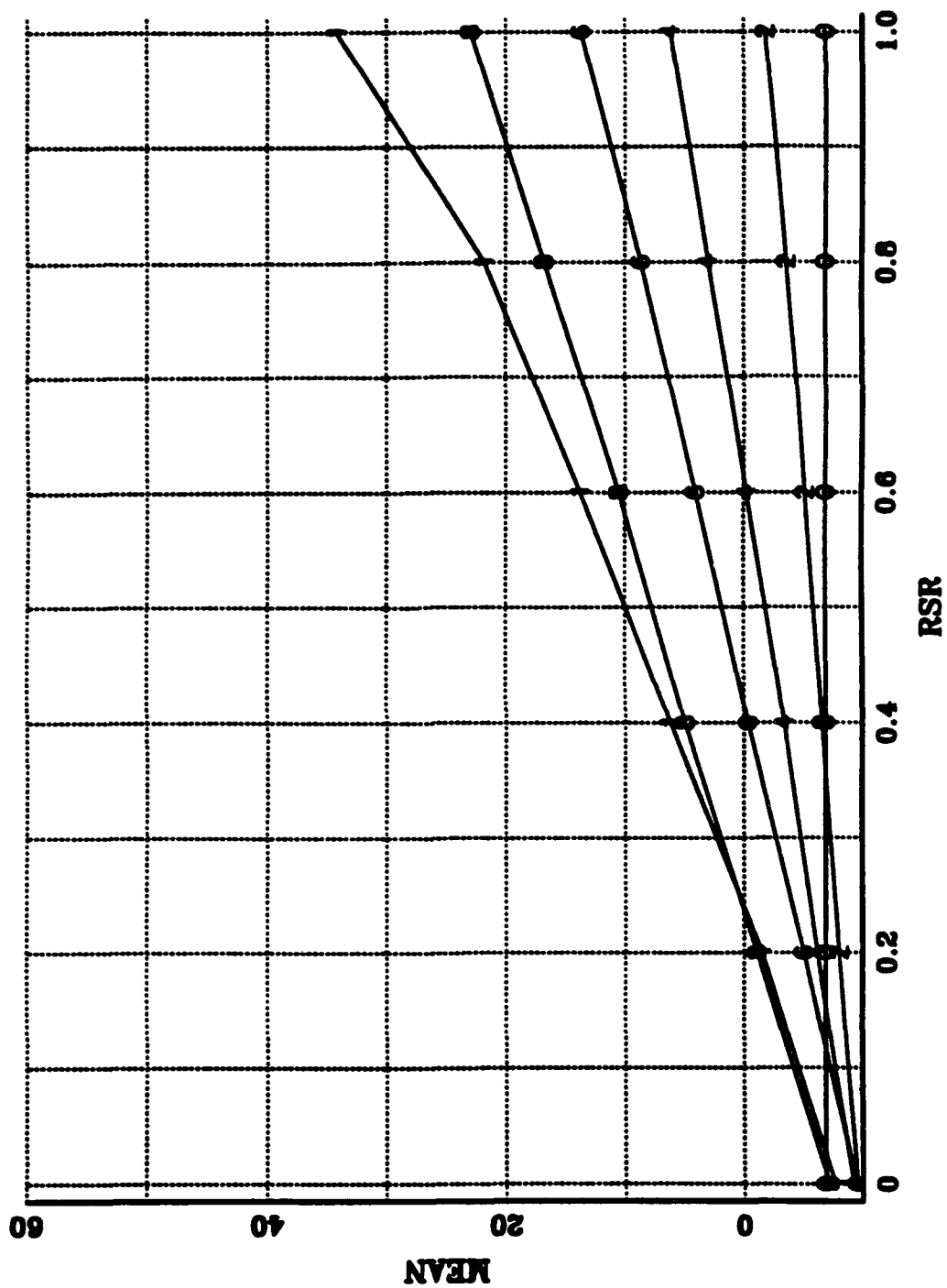
REP IS FIXED AT 1.0
CRR IS VARIED FOR EACH LINE



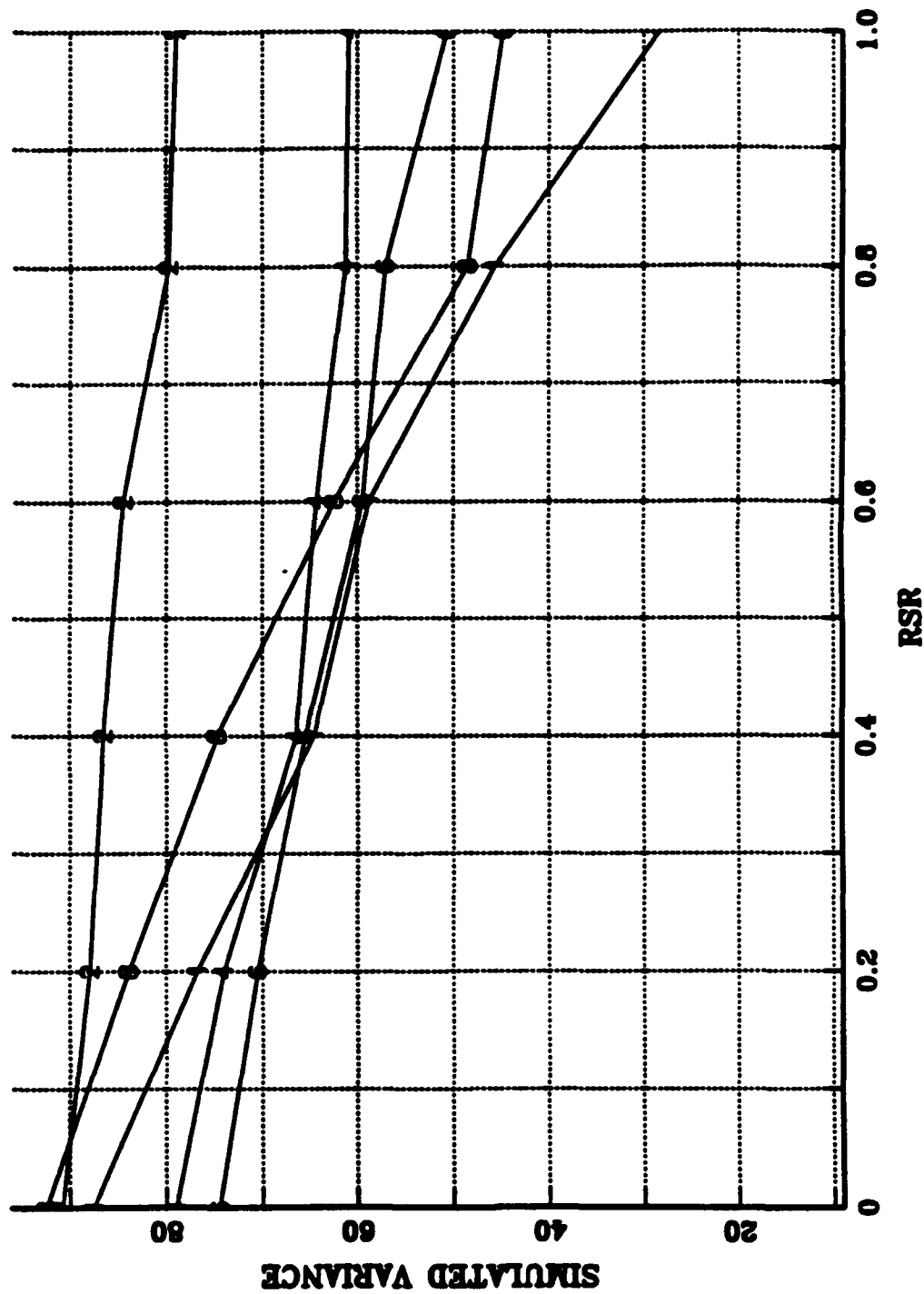
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 1.0; DIFFERENT VALUES OF CRR



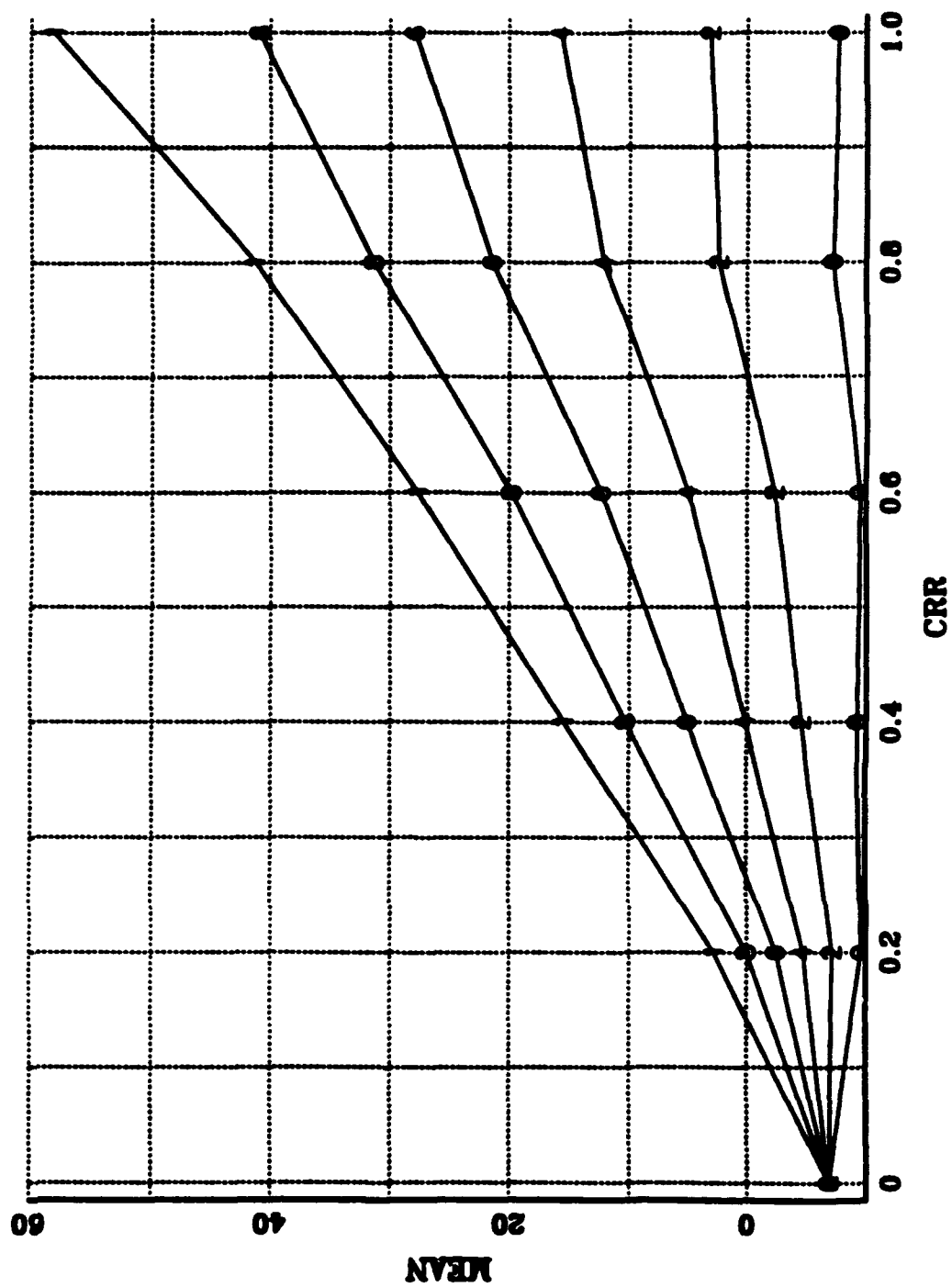
REP IS FIXED AT 1.3
CRR IS VARIED FOR EACH LINE



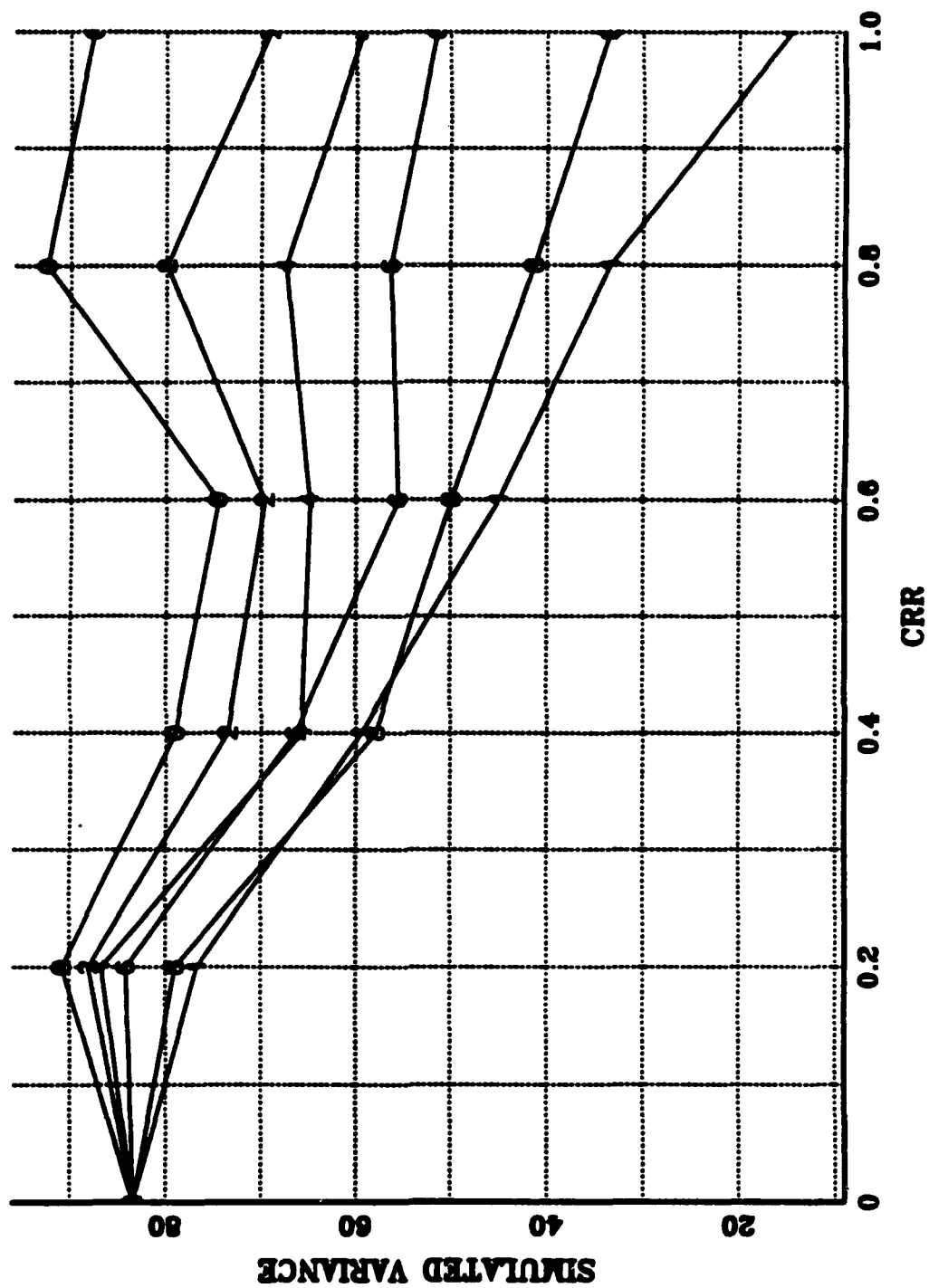
RSR VS SIMULATED VARIANCE
REP IS FIXED AT 1.3; DIFFERENT VALUES OF CRR



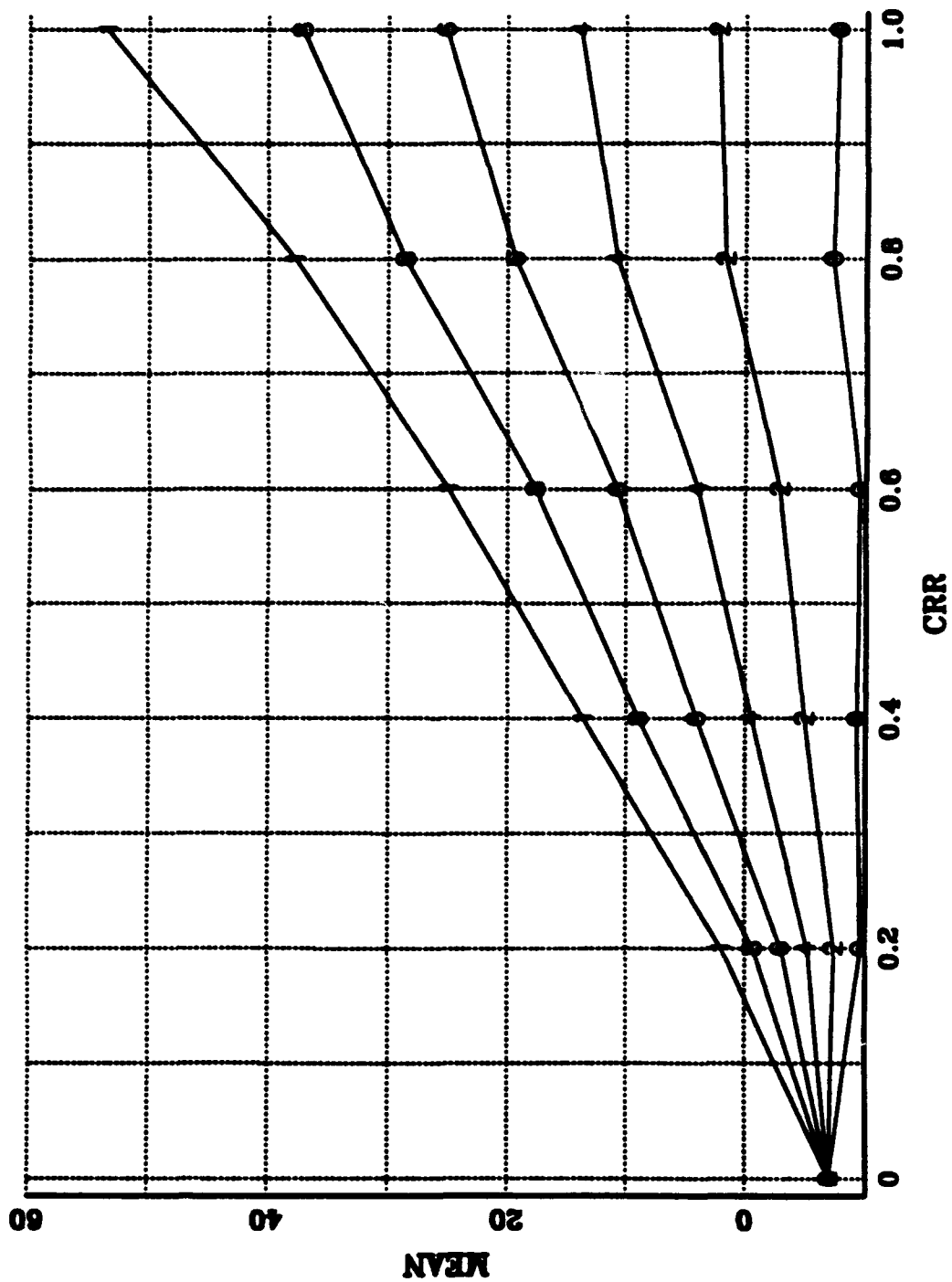
REP IS FIXED AT 0.0
RSR IS VARIED FOR EACH LINE



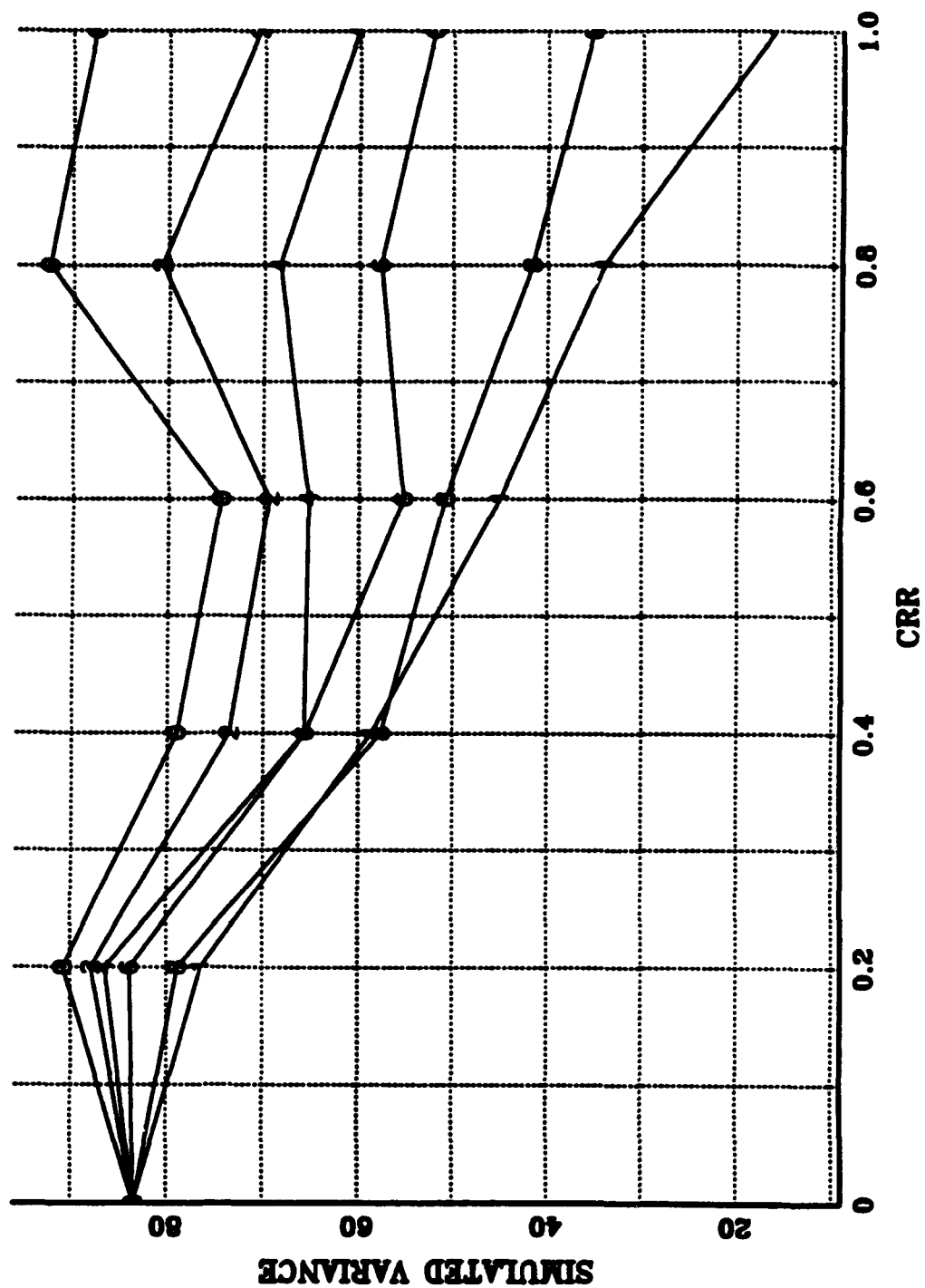
CRR VS SIMULATED VARIANCE
REP IS FIXED AT 0.0; DIFFERENT VALUES OF RSR



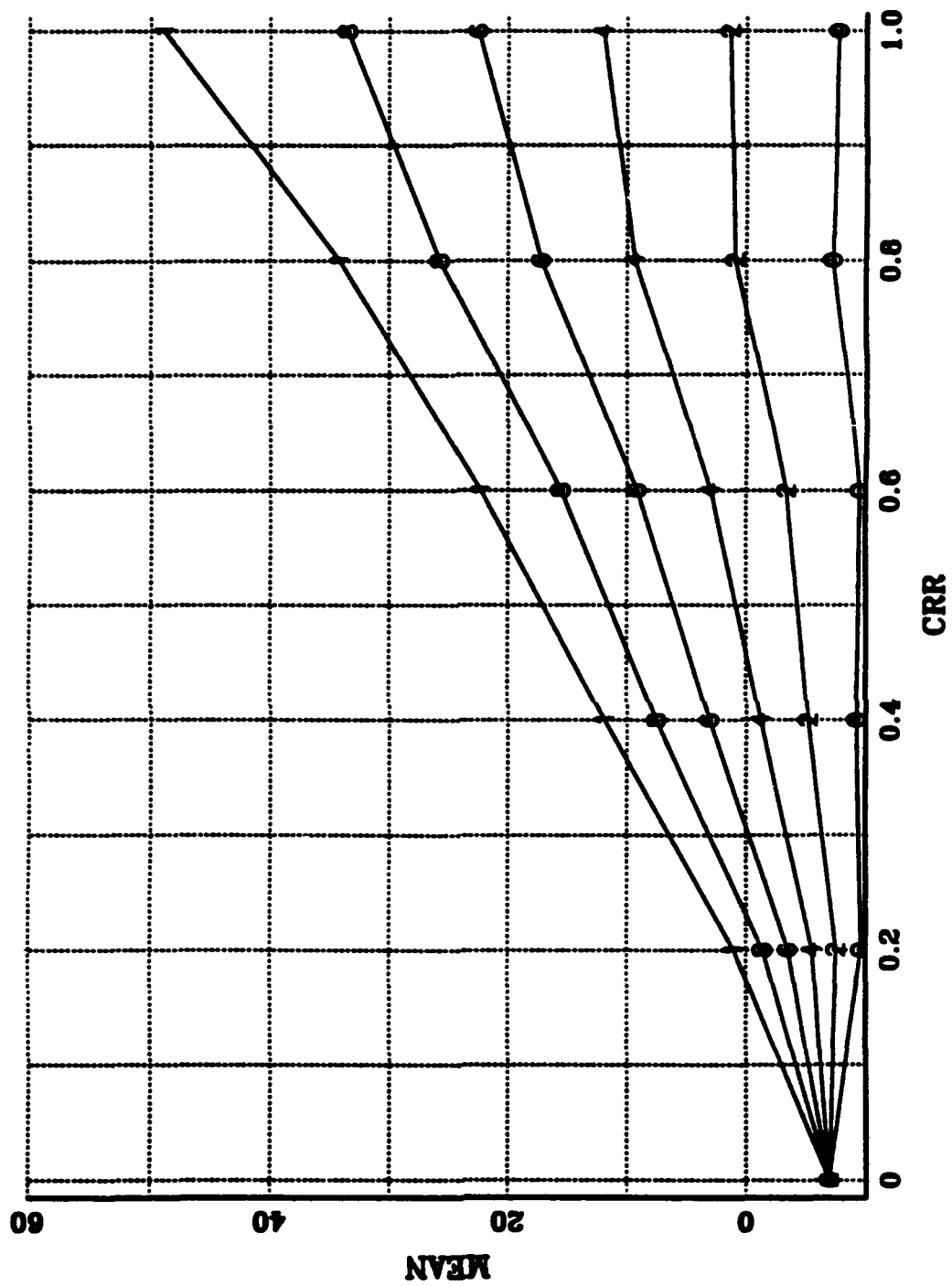
REP IS FIXED AT 0.25
RSR IS VARIED FOR EACH LINE



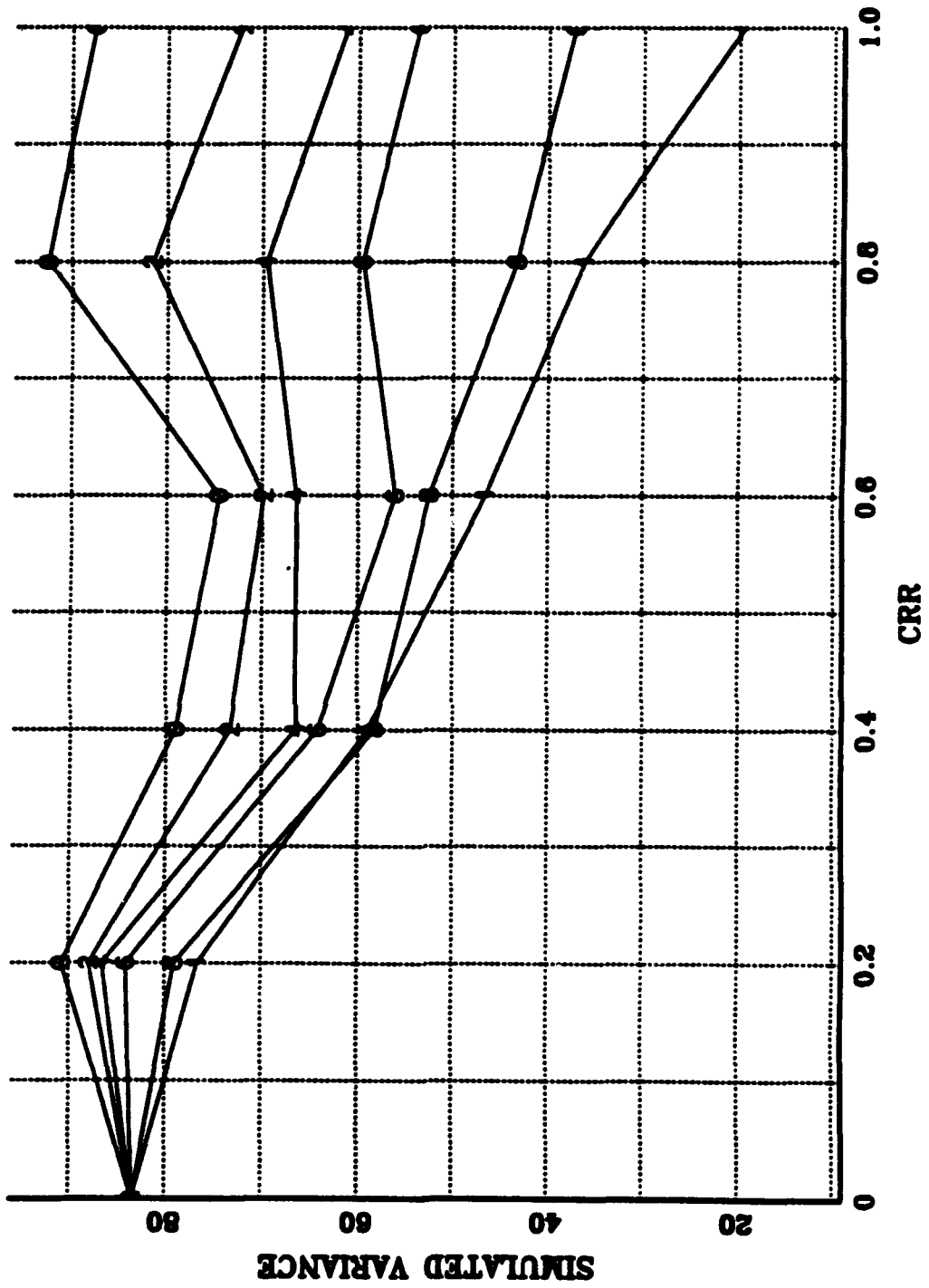
CRR VS SIMULATED VARIANCE
REP IS FIXED AT 0.25; DIFFERENT VALUES OF RSR



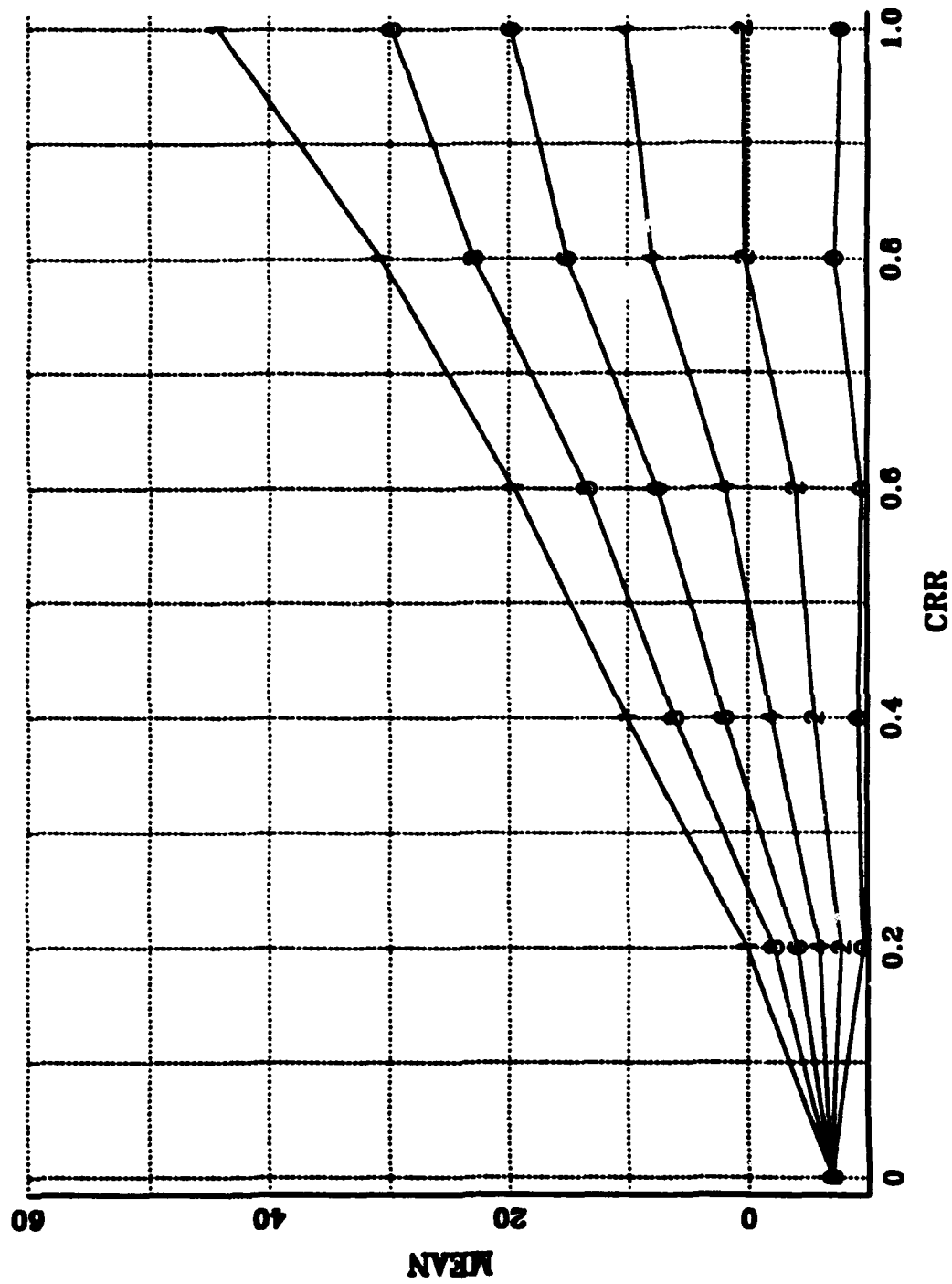
REP IS FIXED AT 0.50
RSR IS VARIED FOR EACH LINE



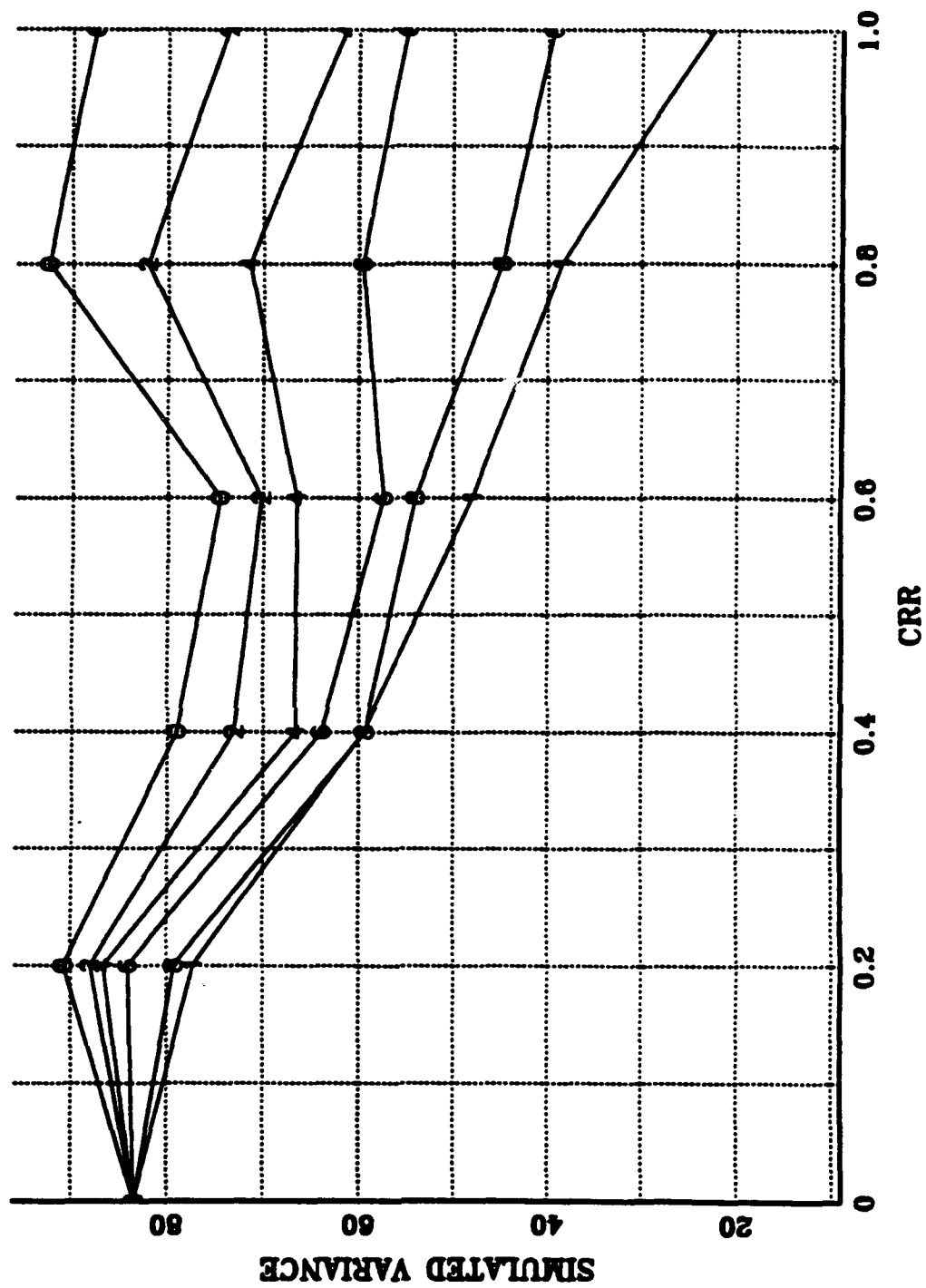
CRR VS SIMULATED VARIANCE
REP IS FIXED AT 0.50; DIFFERENT VALUES OF RSR



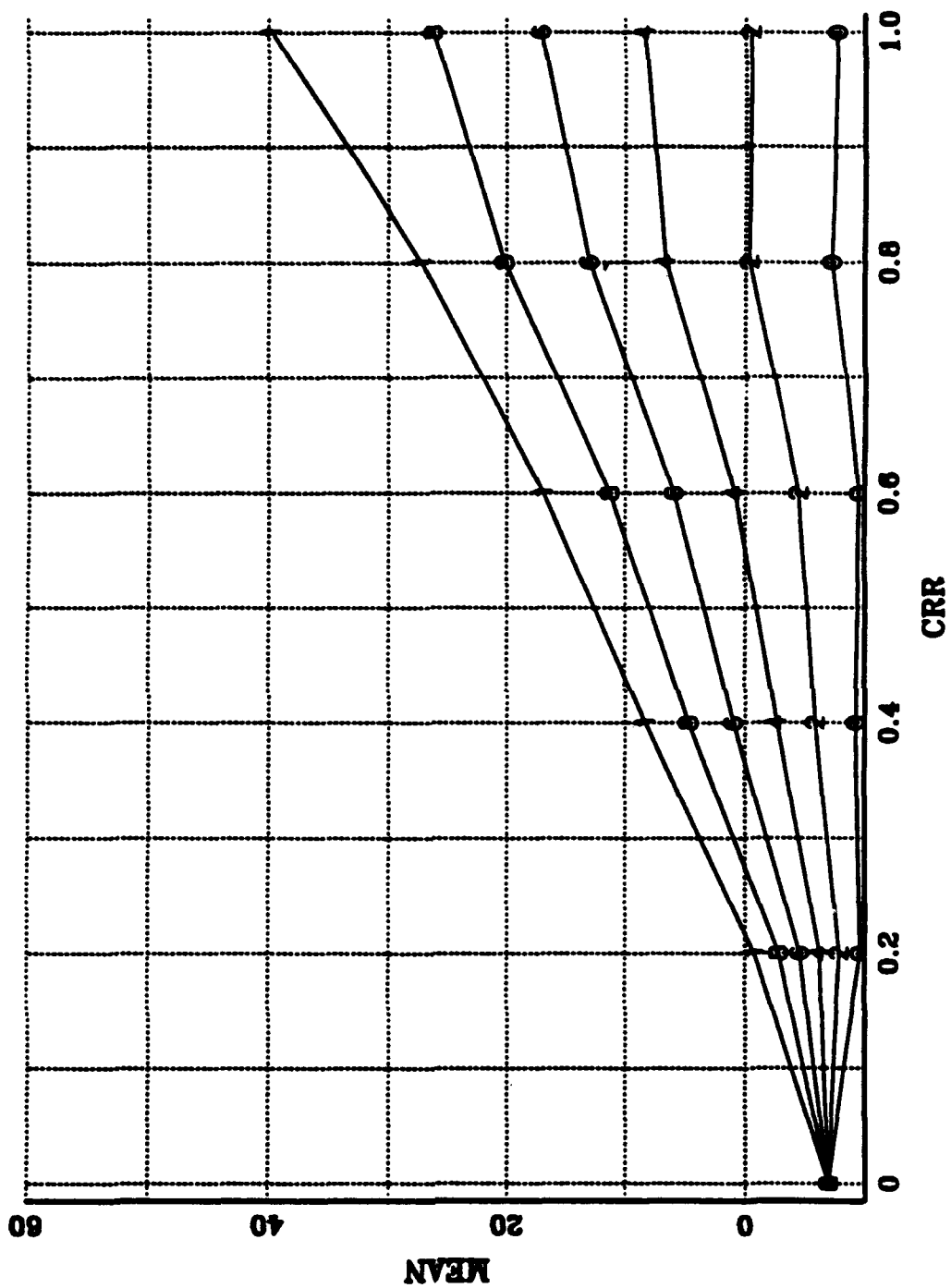
REP IS FIXED AT 0.75
RSR IS VARIED FOR EACH LINE



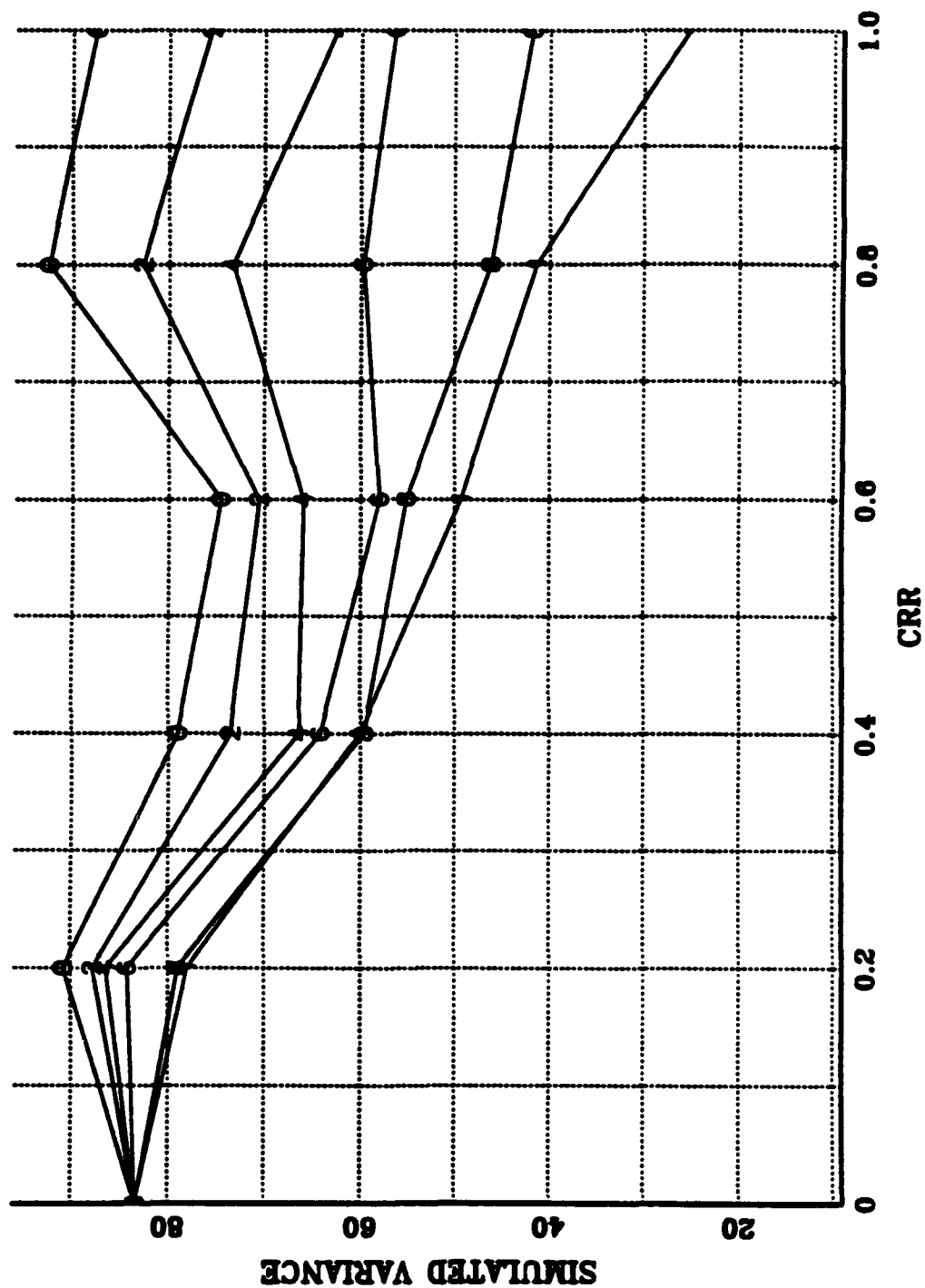
CRR VS SIMULATED VARIANCE
REP IS FIXED AT 0.75; DIFFERENT VALUES OF RSR



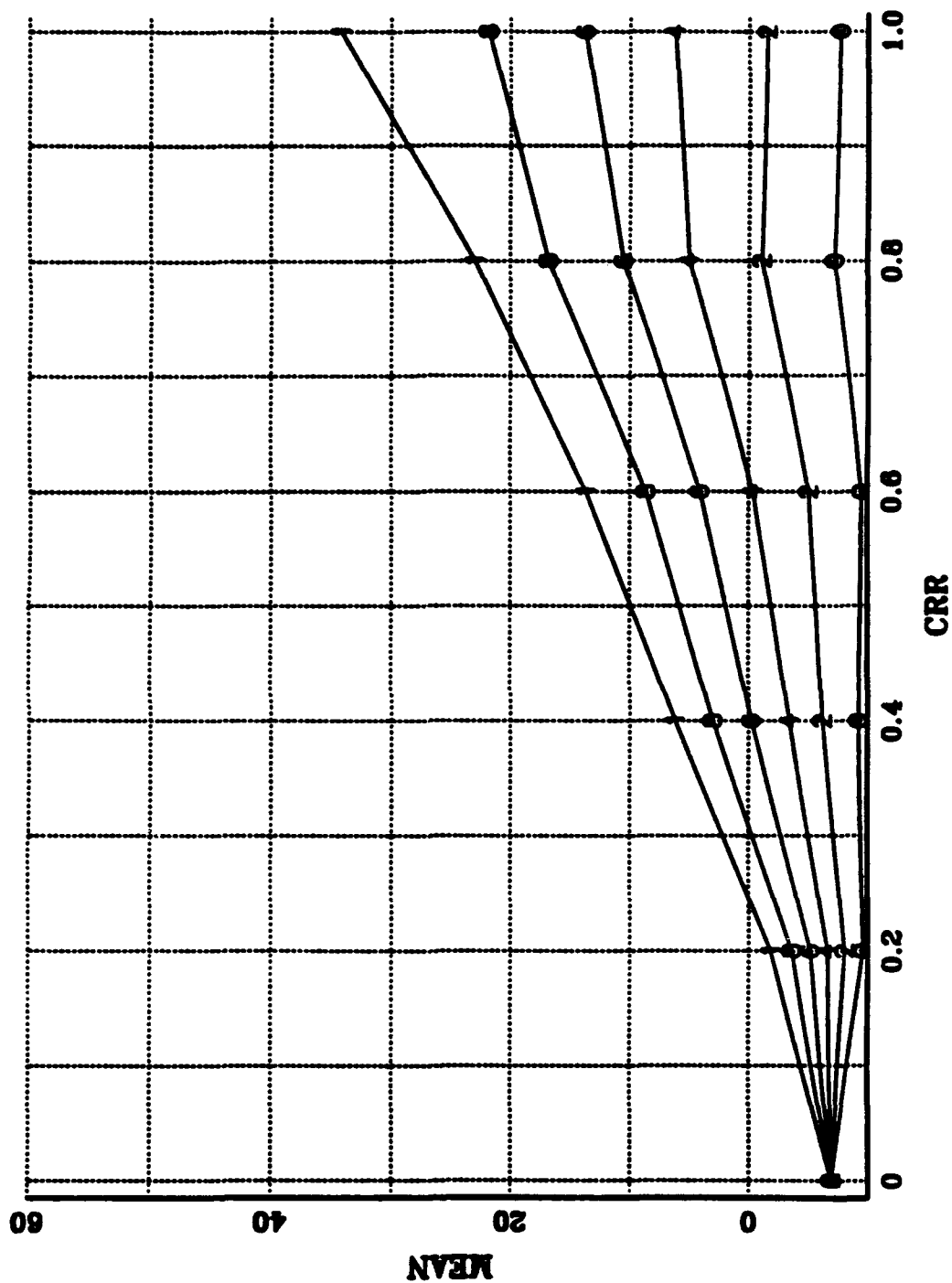
REP IS FIXED AT 1.0
RSR IS VARIED FOR EACH LINE



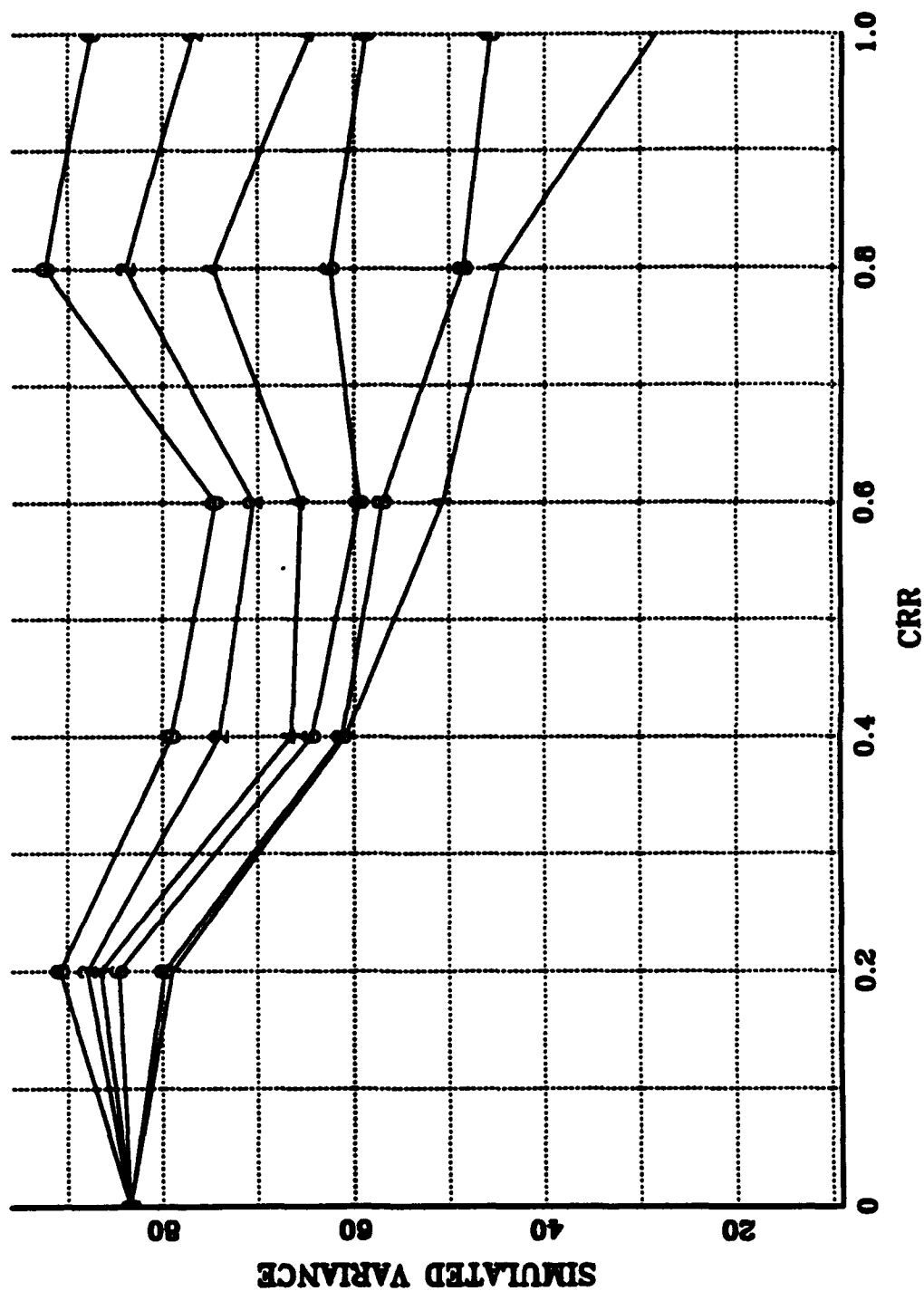
CRR VS SIMULATED VARIANCE
REP IS FIXED AT 1.0; DIFFERENT VALUES OF RSR



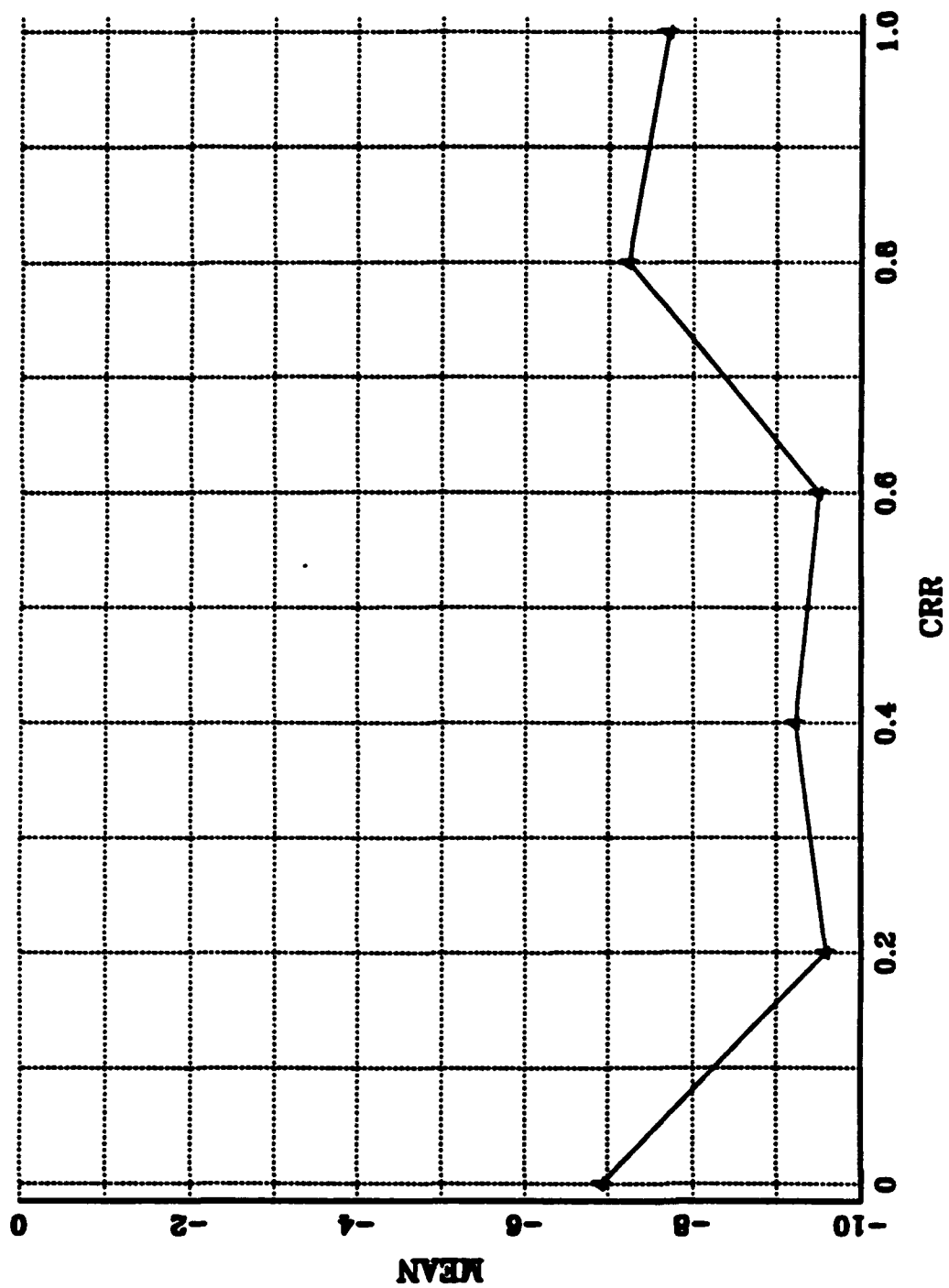
REP IS FIXED AT 1.3
RSR IS VARIED FOR EACH LINE



CRR VS SIMULATED VARIANCE
REP IS FIXED AT 1.3; DIFFERENT VALUES OF RSR

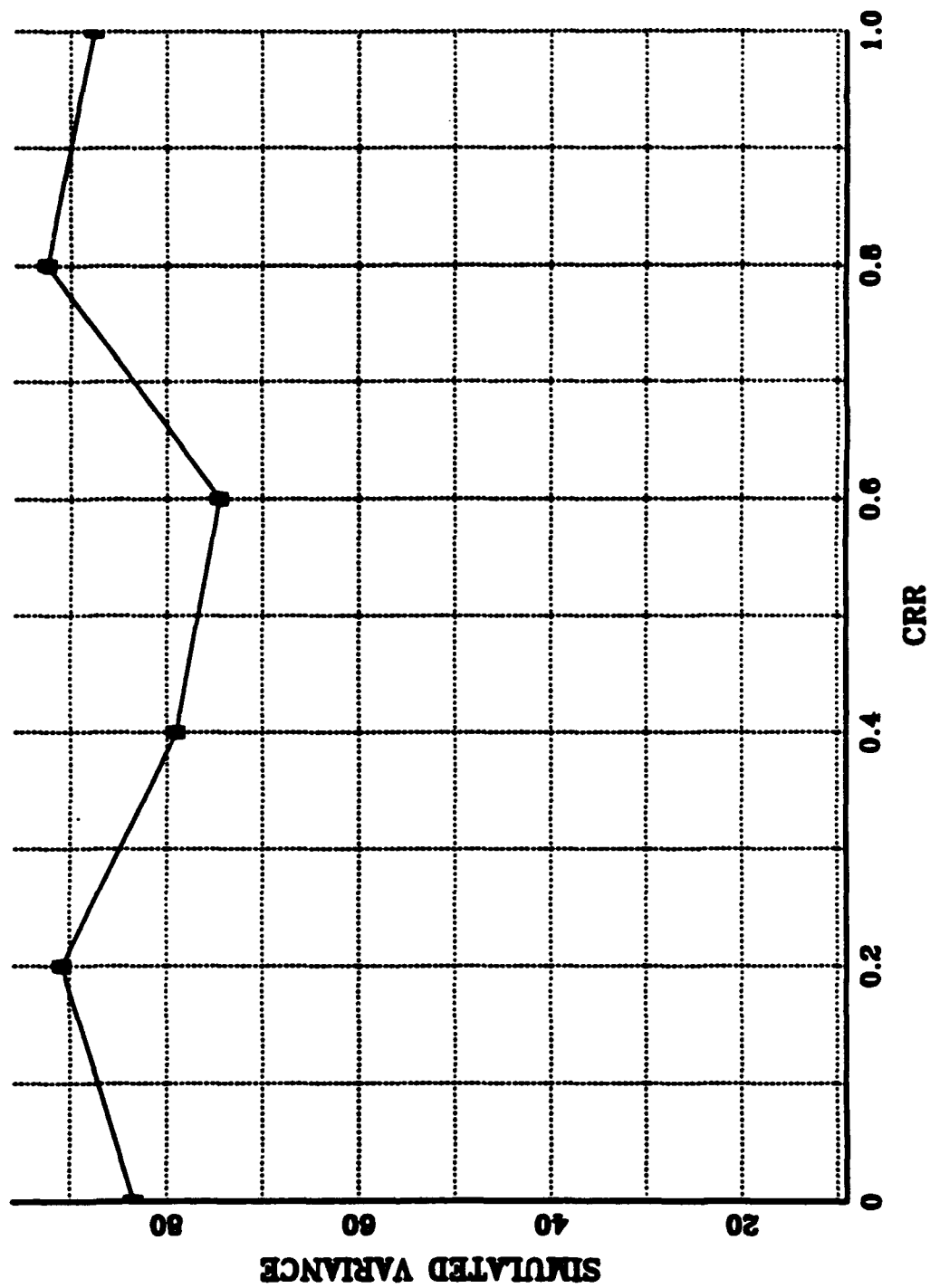


RSR IS FIXED AT 0.0
REP IS VARIED FOR EACH LINE

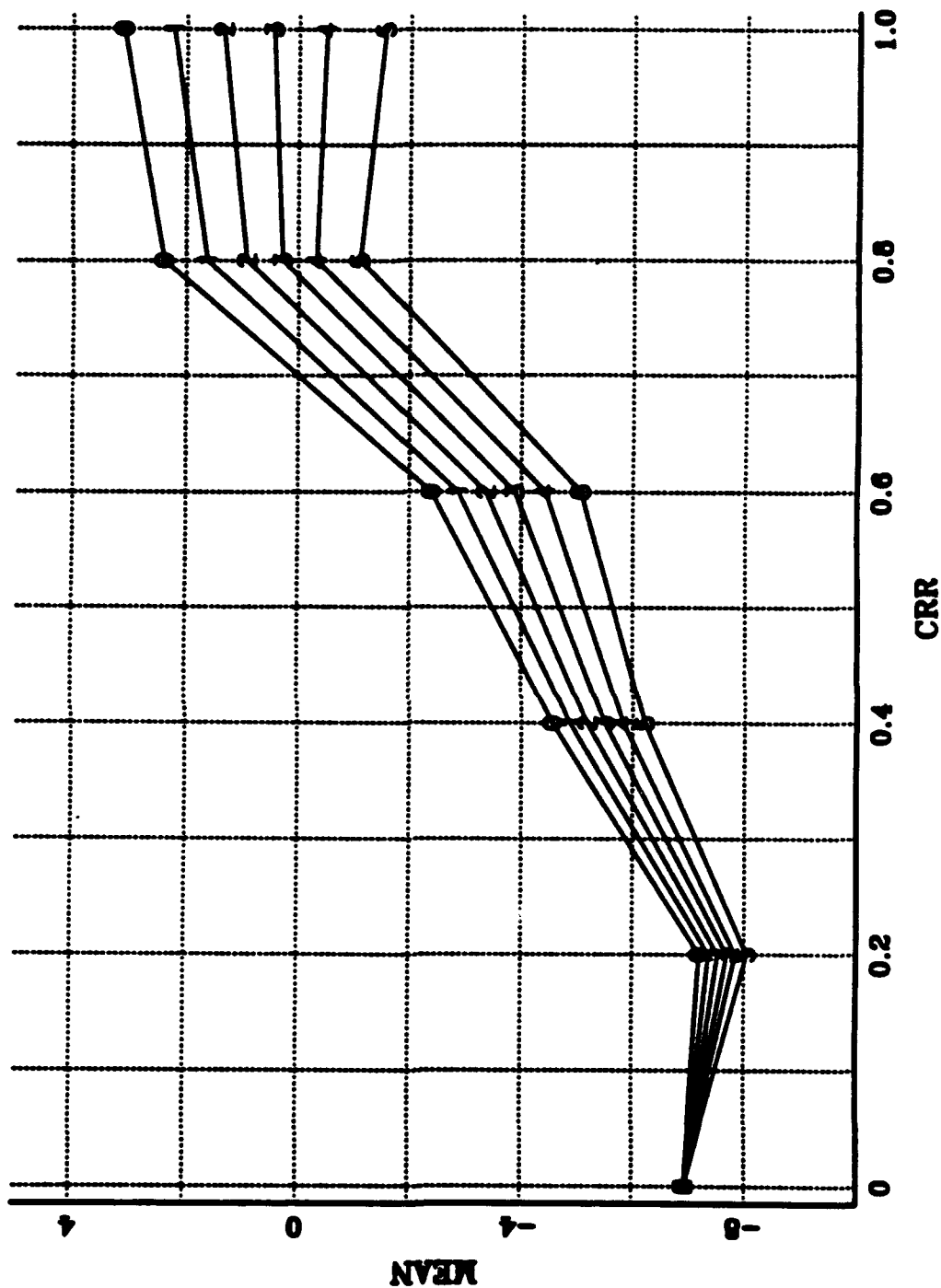


CRR VS SIMULATED VARIANCE

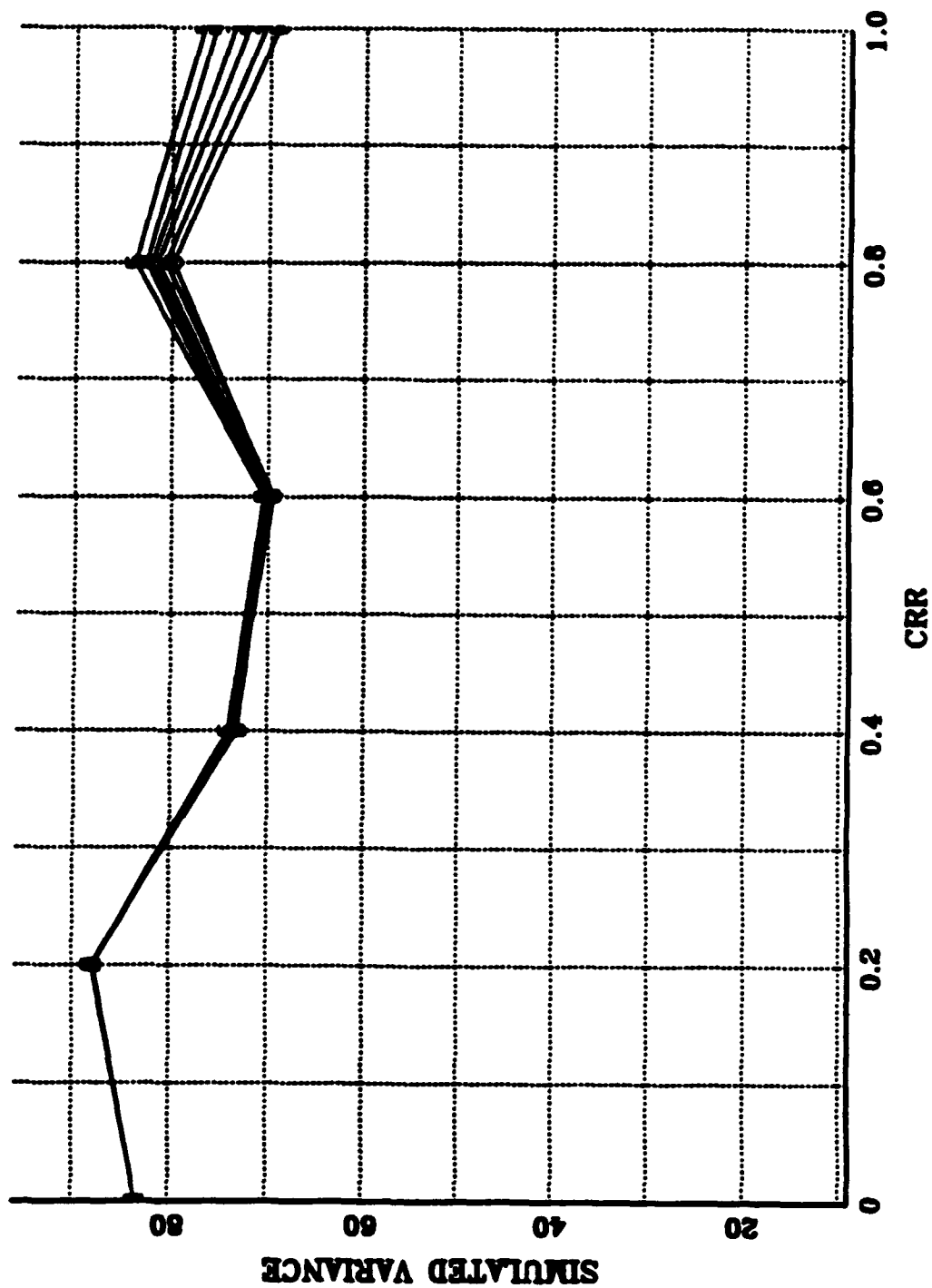
RSR IS FIXED AT 0.0; DIFFERENT VALUES OF REP



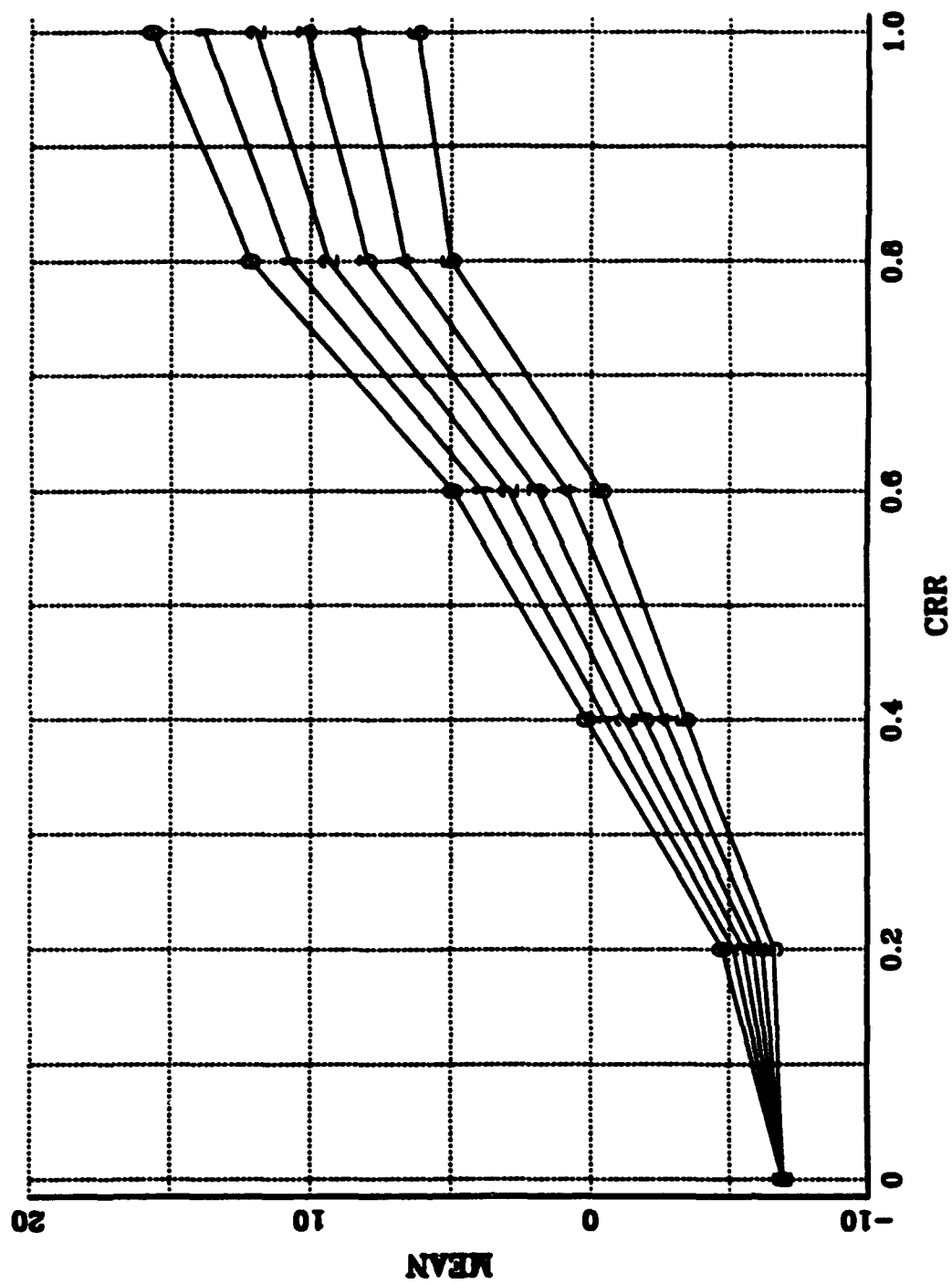
RSR IS FIXED AT 0.2
 REP IS VARIED FOR EACH LINE



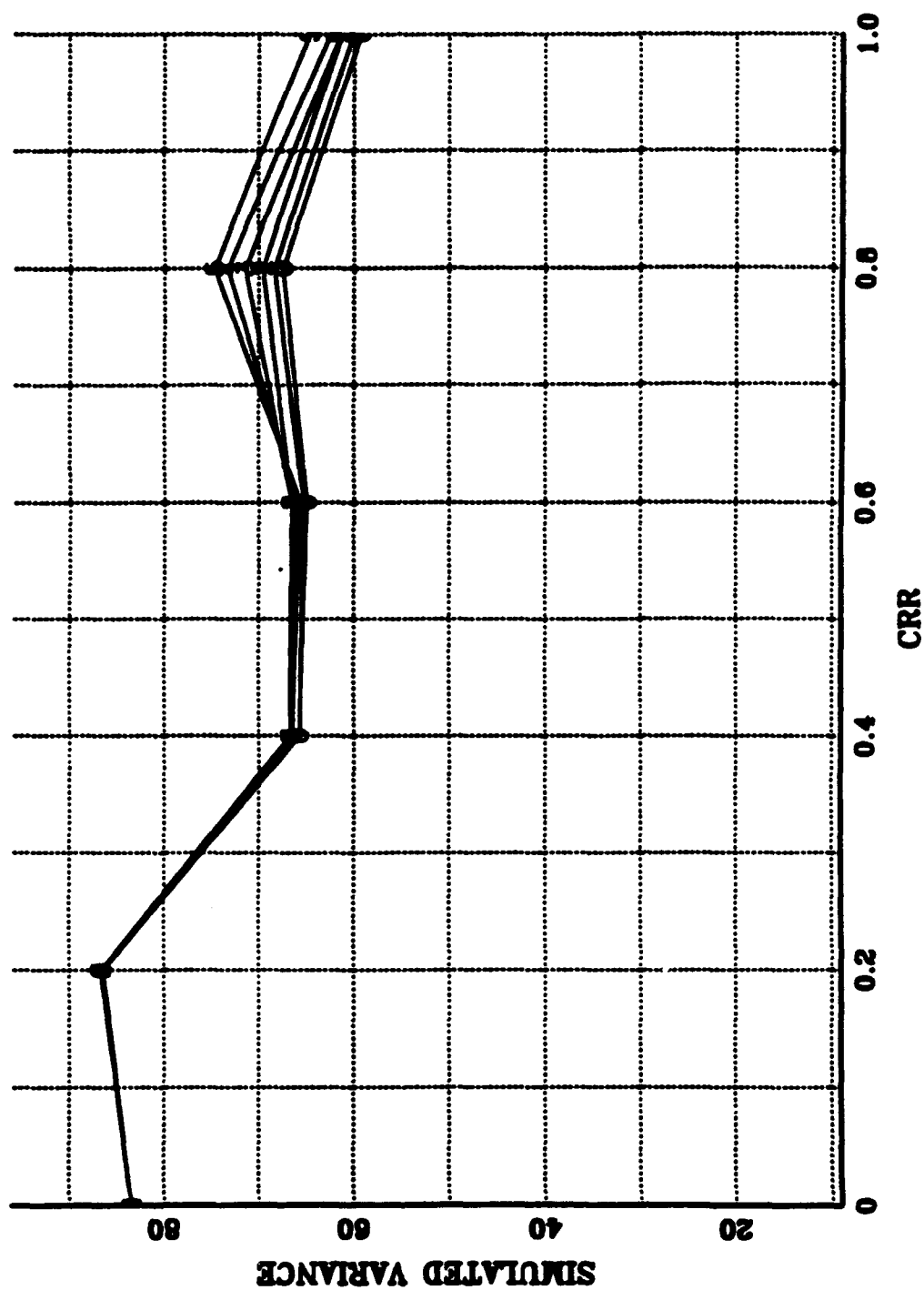
CRR VS SIMULATED VARIANCE
RSR IS FIXED AT 0.2; DIFFERENT VALUES OF REP



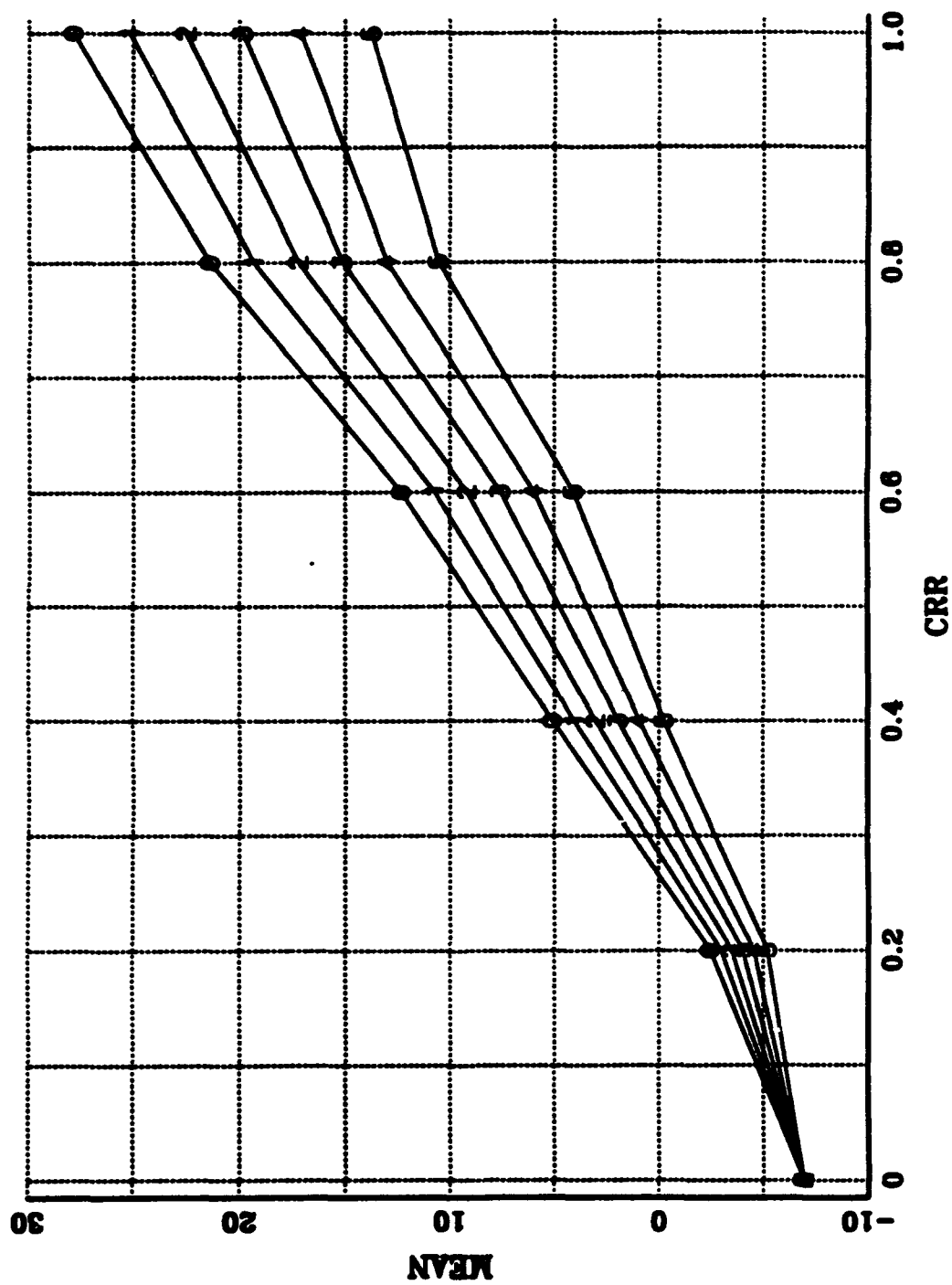
RSR IS FIXED AT 0.4
REP IS VARIED FOR EACH LINE



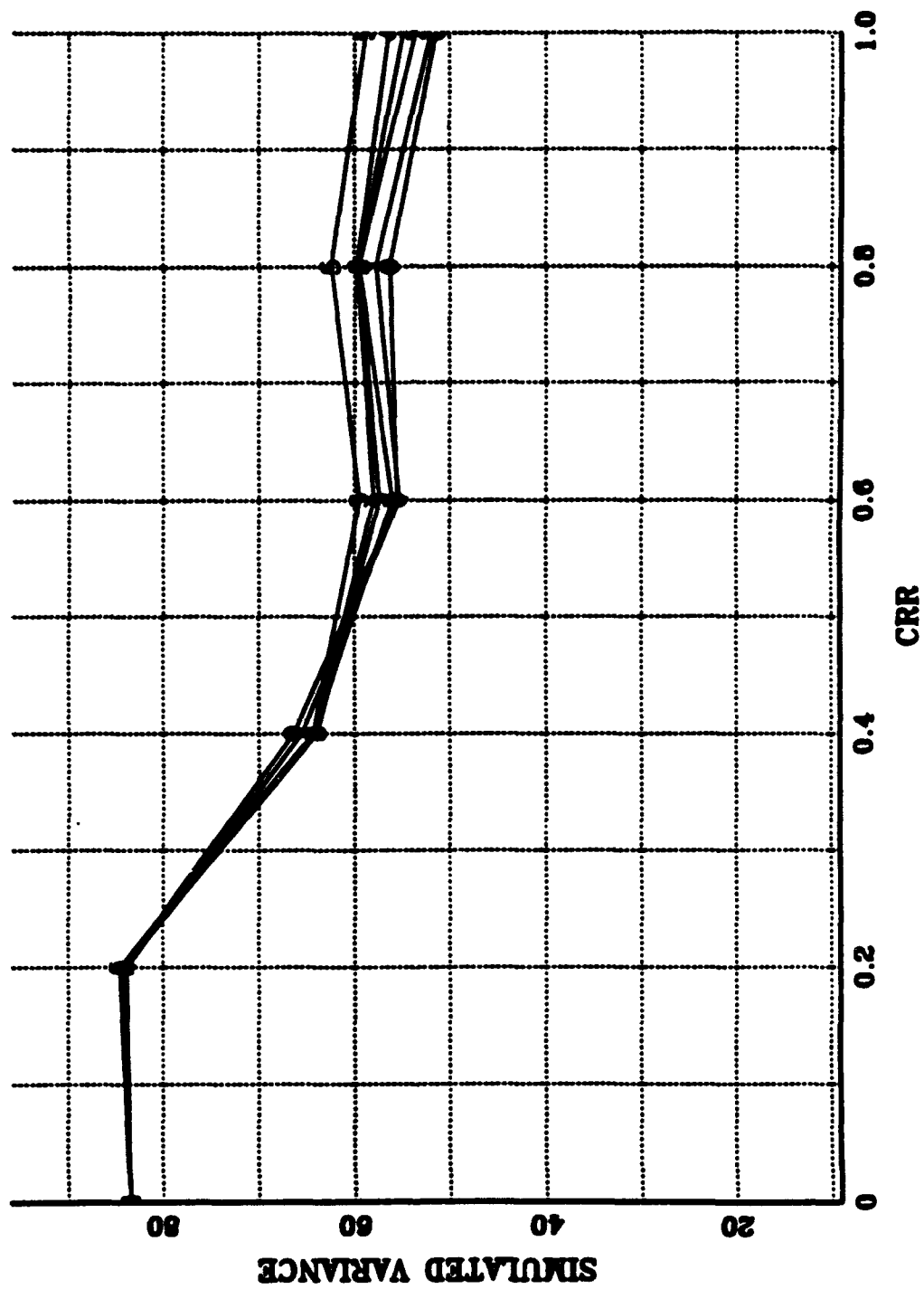
CRR VS SIMULATED VARIANCE **RSR IS FIXED AT 0.4; DIFFERENT VALUES OF REP**



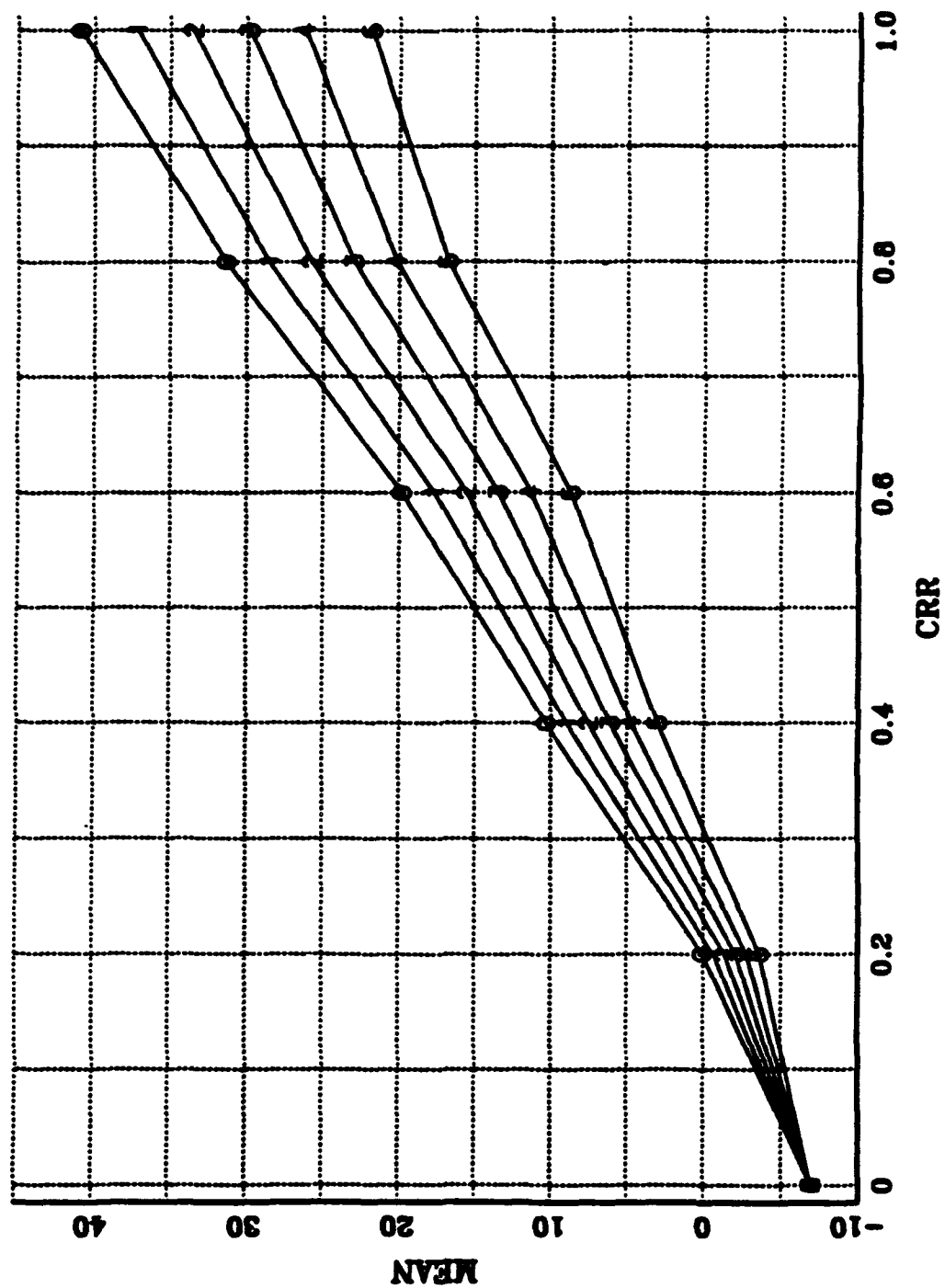
RSR IS FIXED AT 0.6
 REP IS VARIED FOR EACH LINE



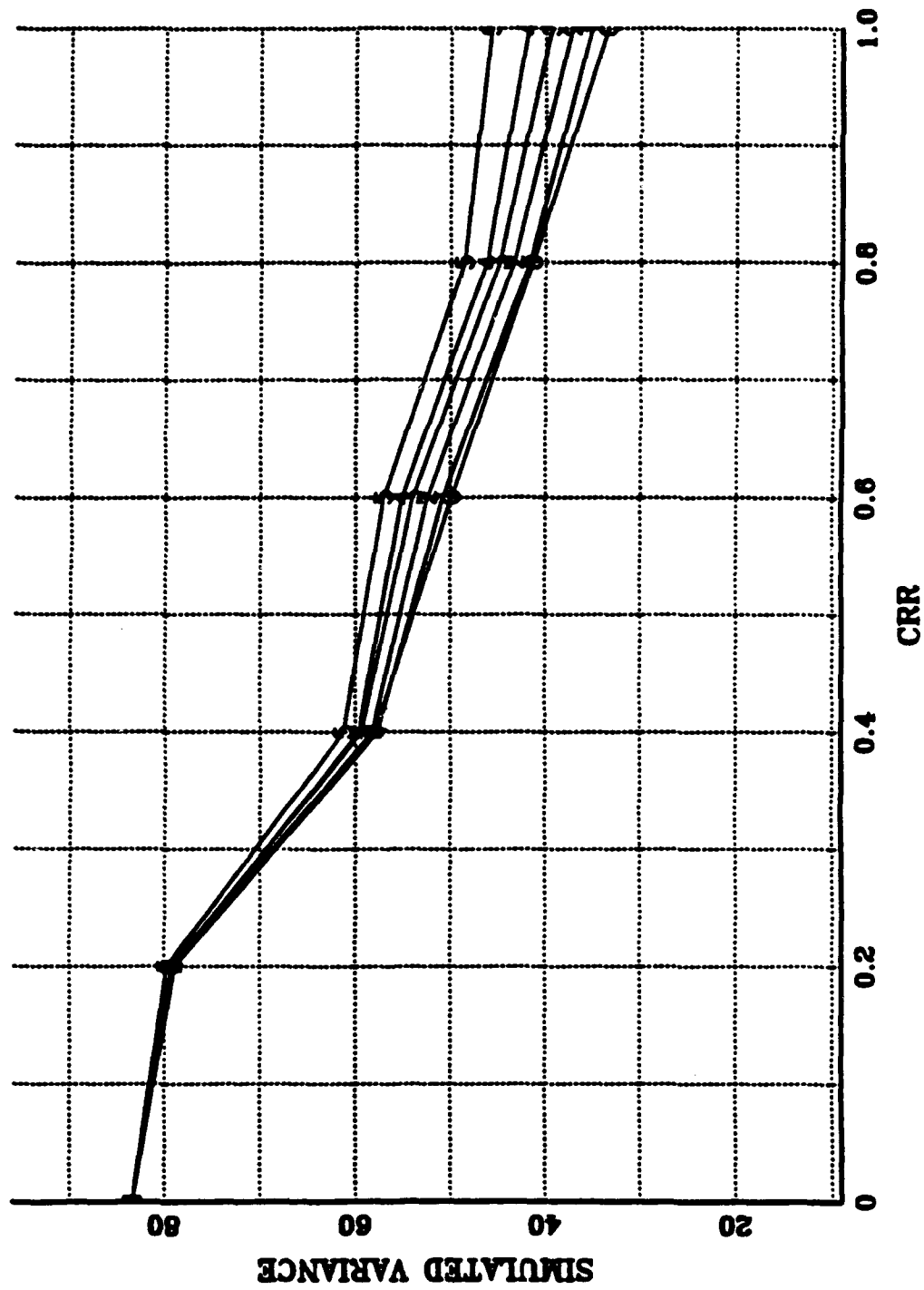
CRR VS SIMULATED VARIANCE
RSR IS FIXED AT 0.6; DIFFERENT VALUES OF REP



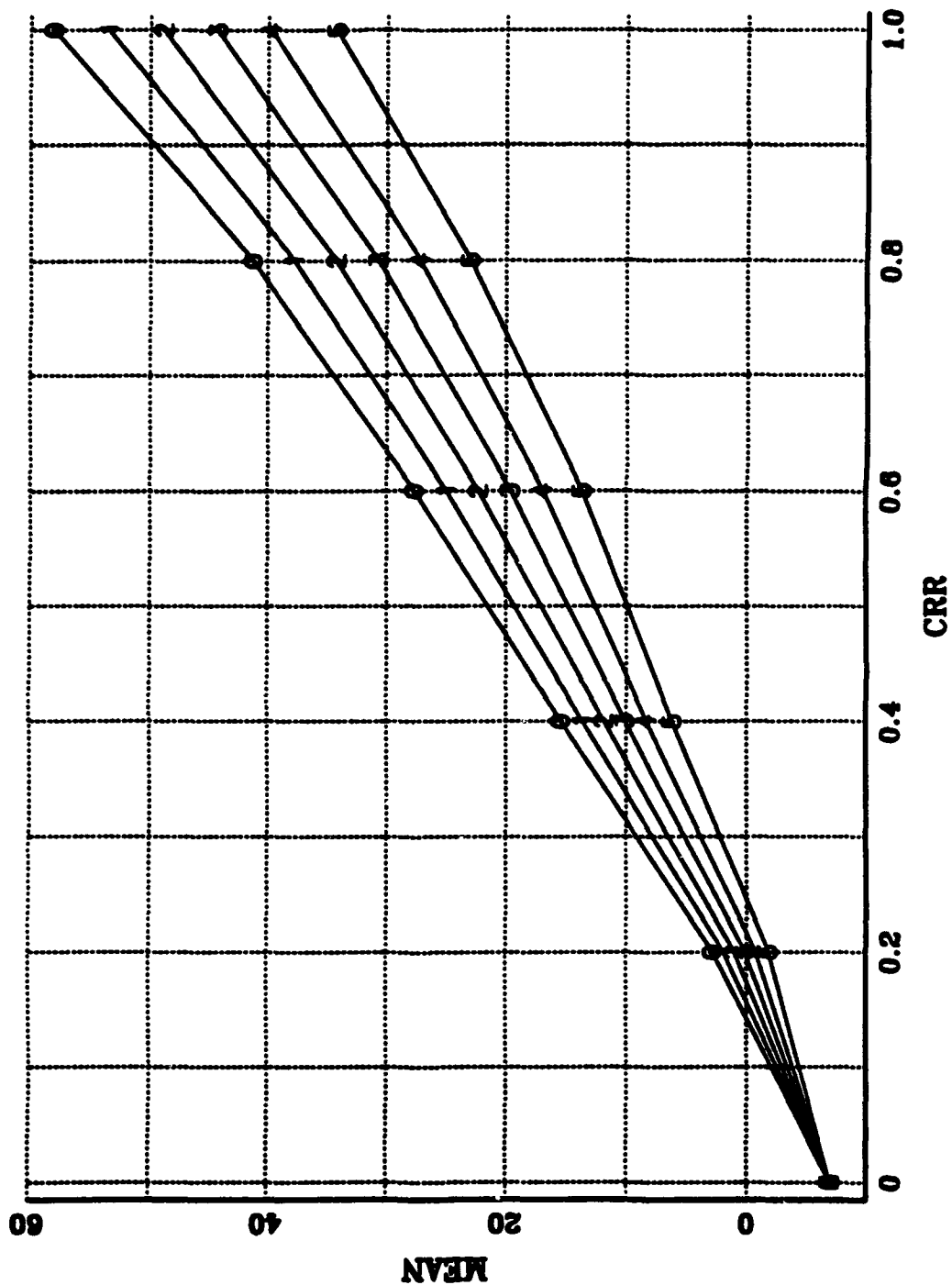
RSR IS FIXED AT 0.8
 REP IS VARIED FOR EACH LINE



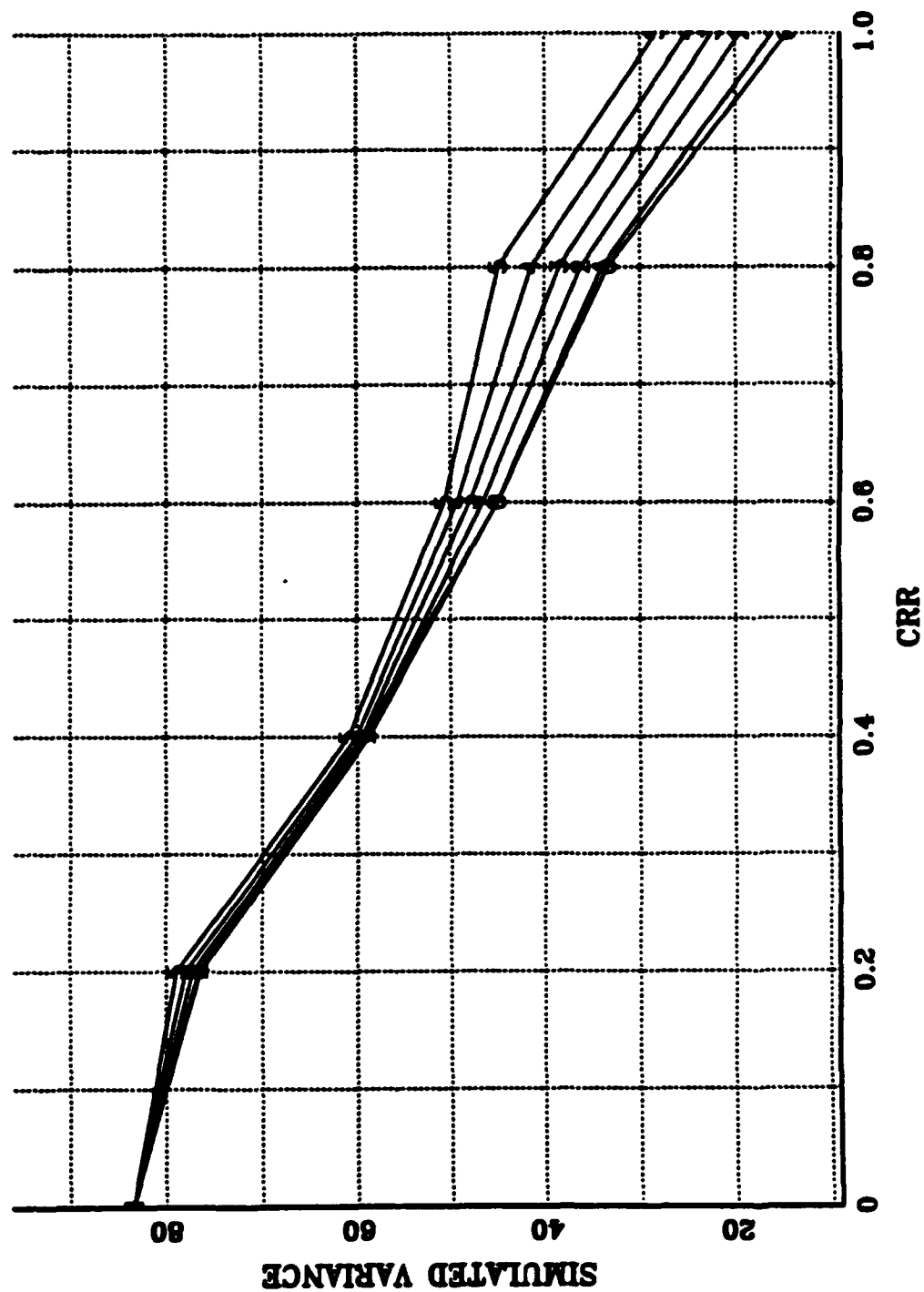
CRR VS SIMULATED VARIANCE
RSR IS FIXED AT 0.8; DIFFERENT VALUES OF REP



RSR IS FIXED AT 1.0
 REP IS VARIED FOR EACH LINE



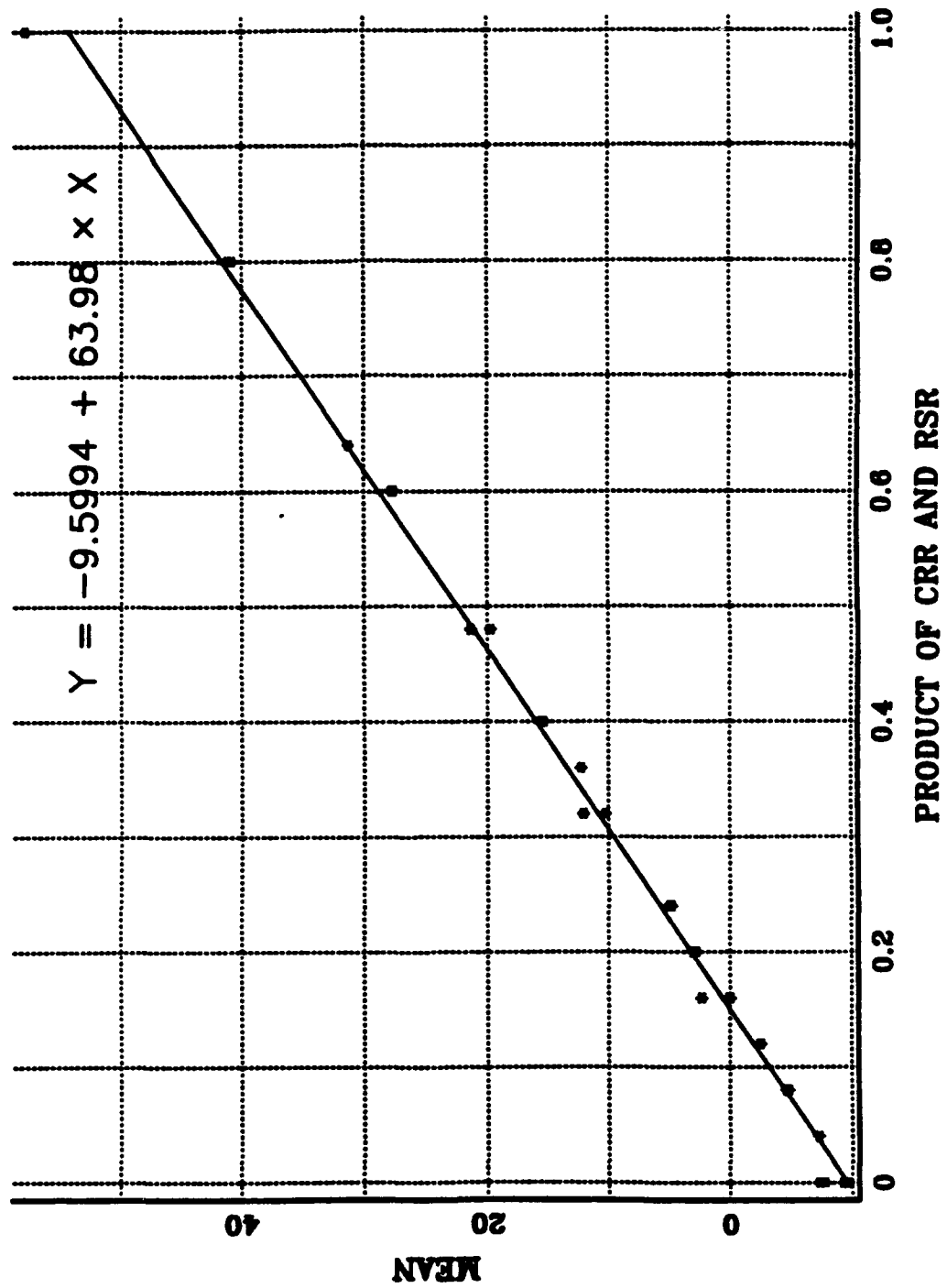
CRR VS SIMULATED VARIANCE
RSR IS FIXED AT 1.0; DIFFERENT VALUES OF REP



APPENDIX N

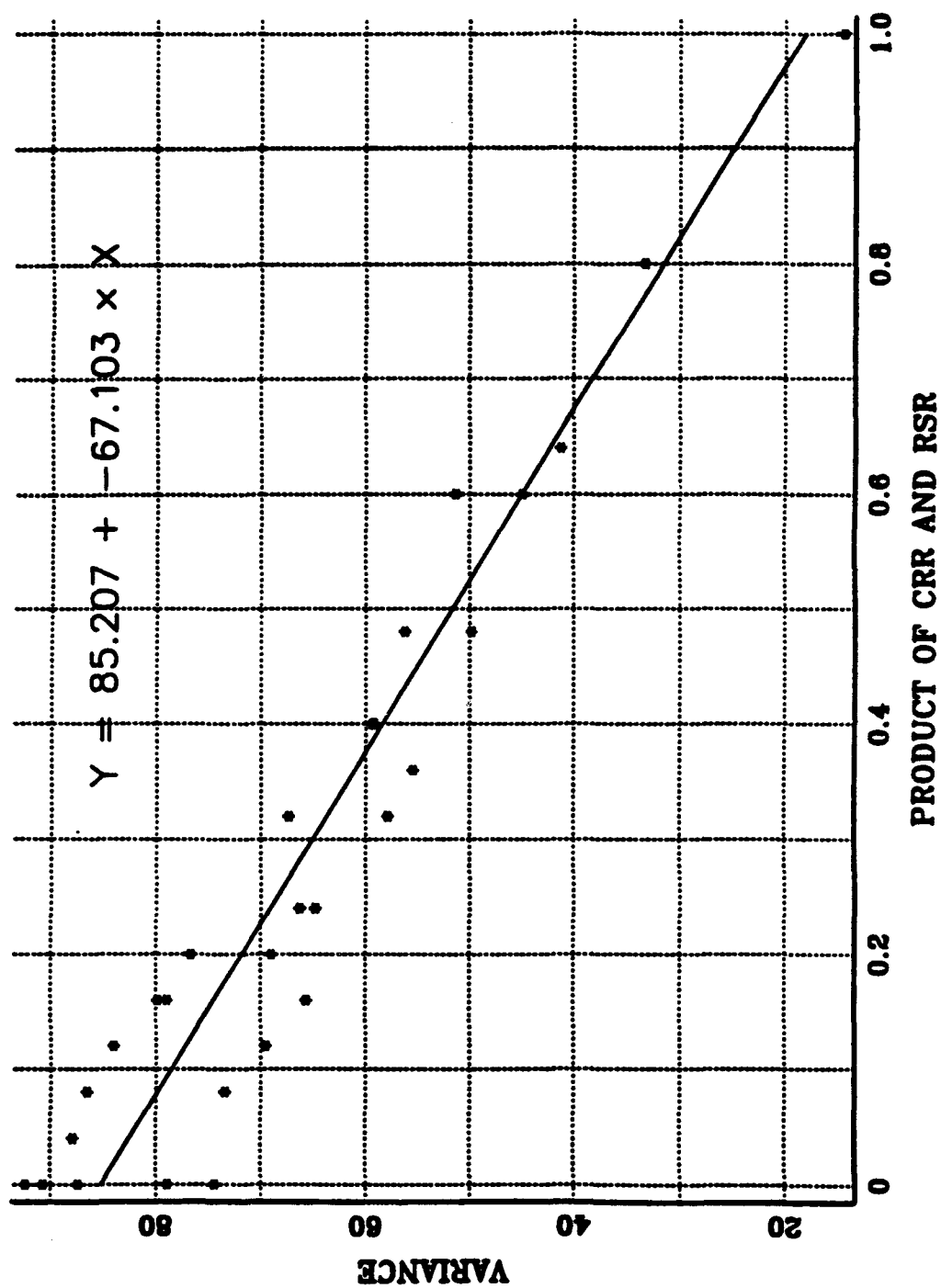
PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 0.0



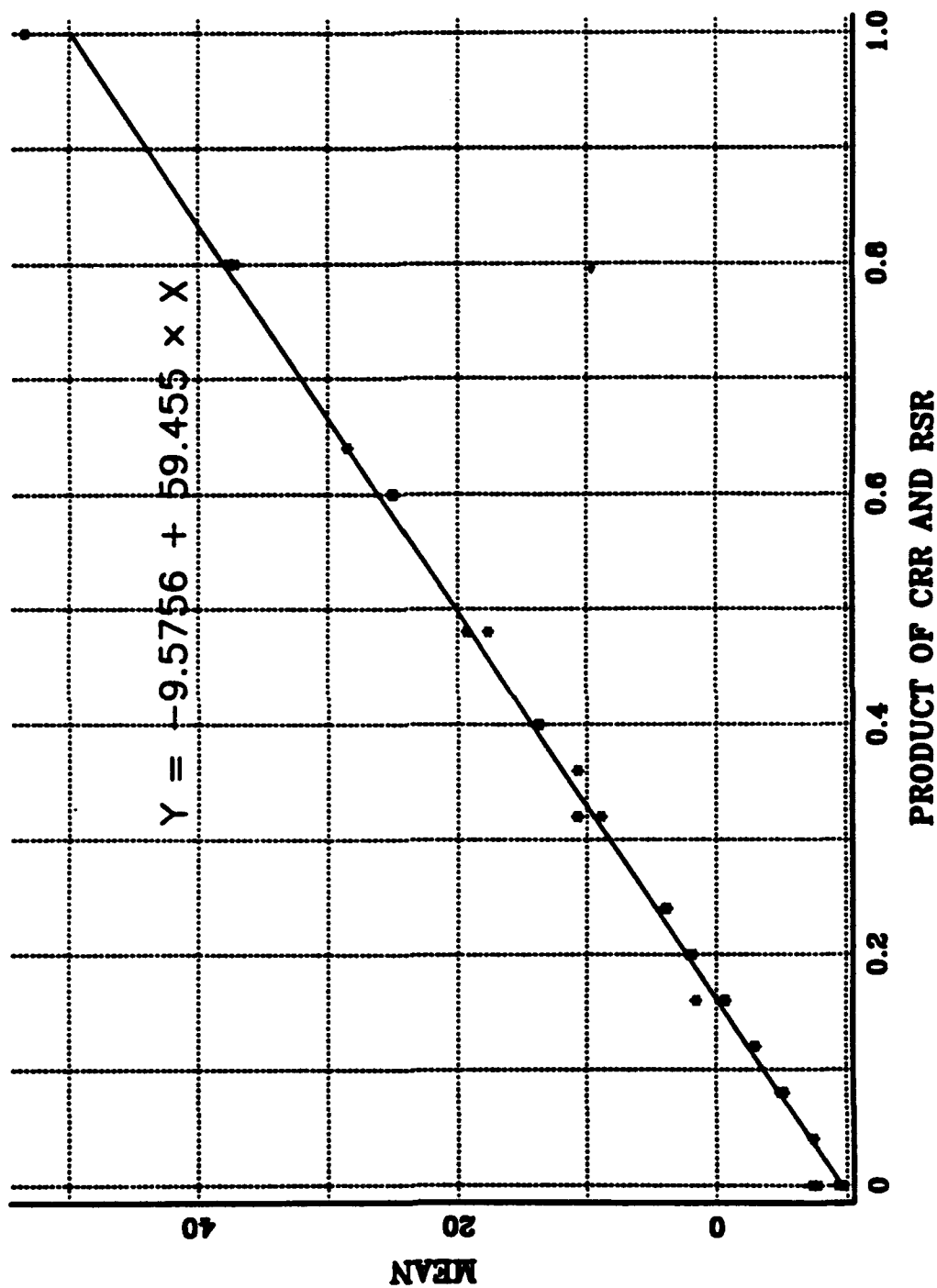
PRODUCT OF CRR AND RSR VS VARIANCE

REP = 0.00



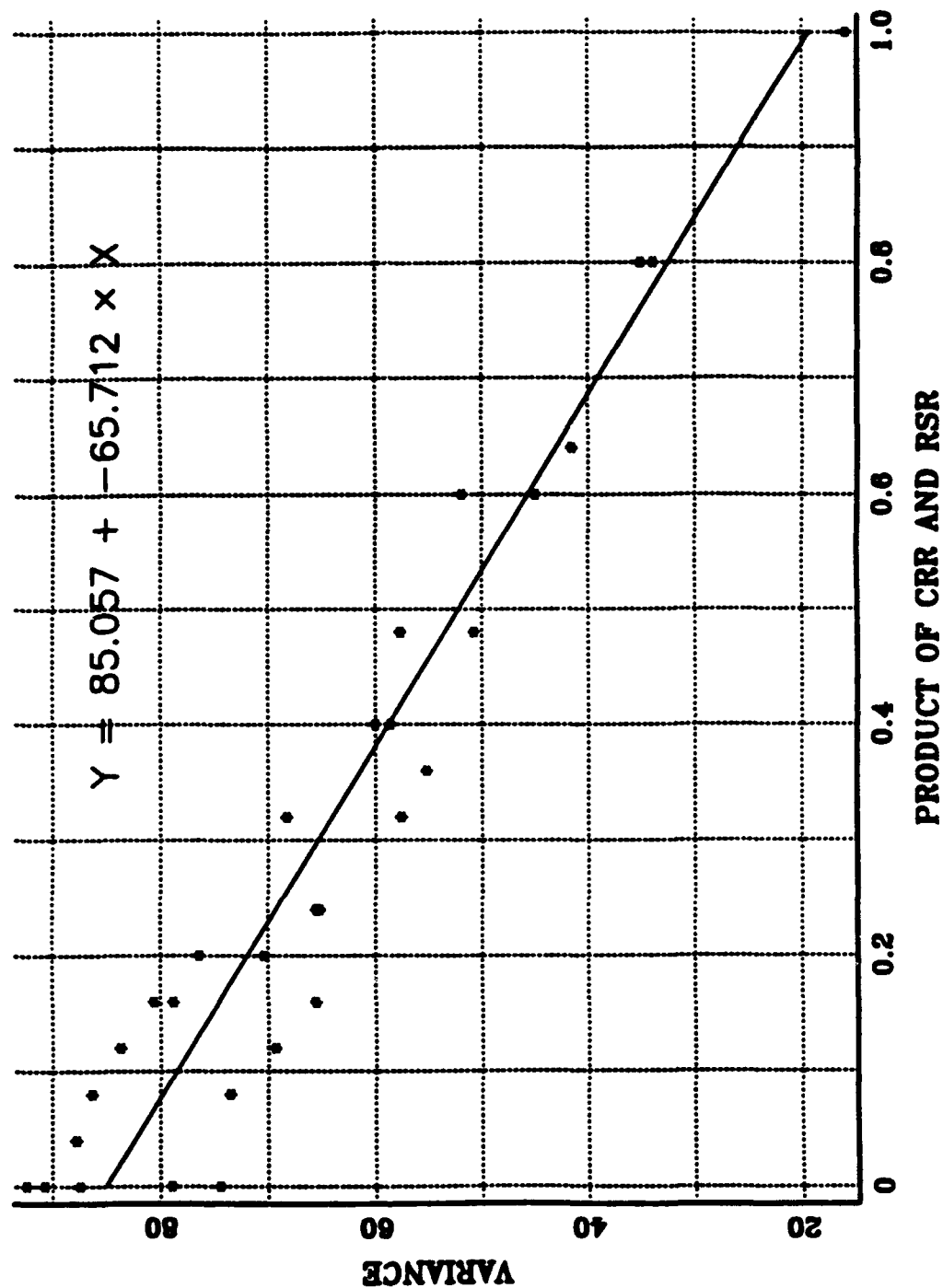
PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 0.25



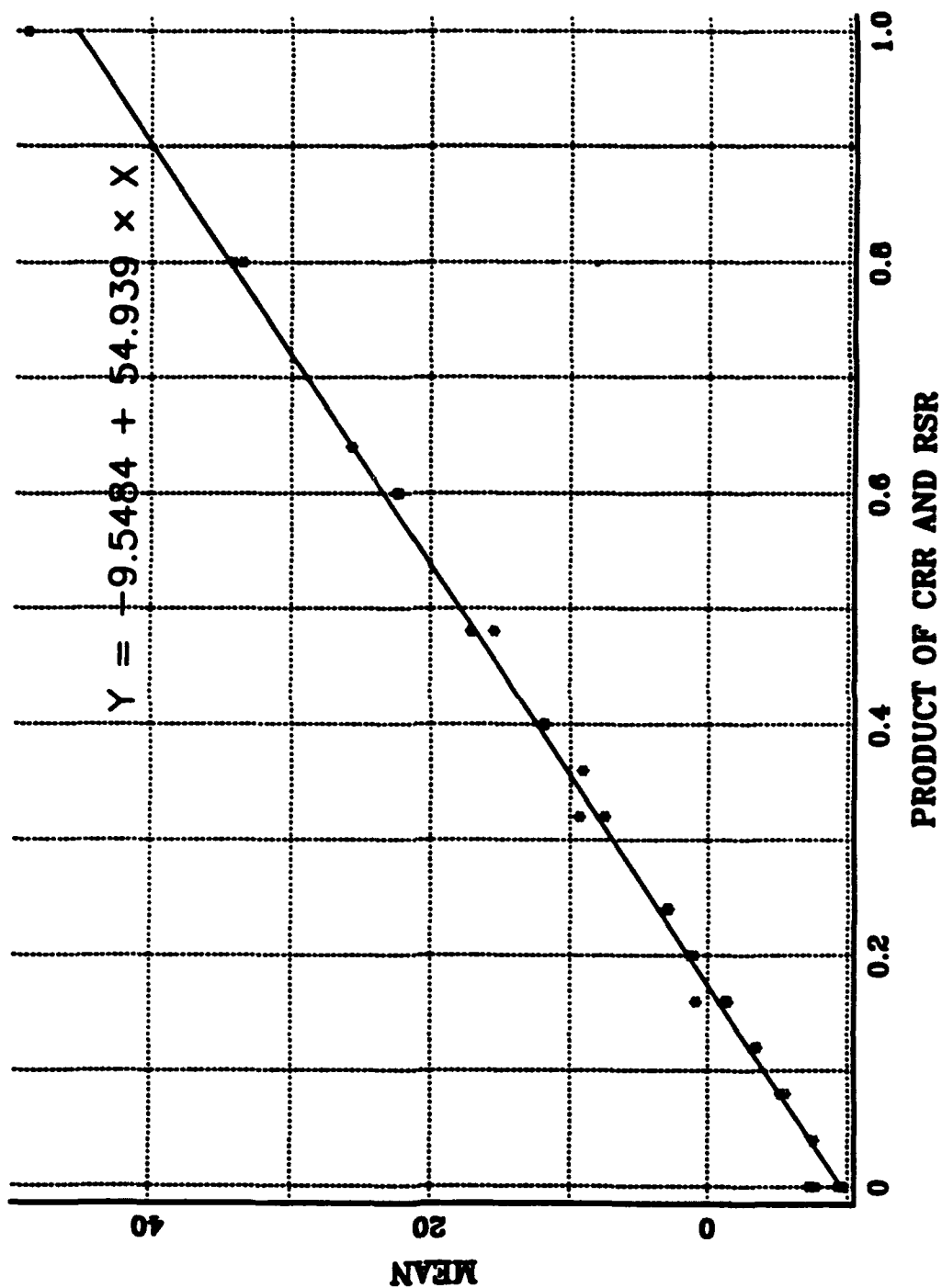
PRODUCT OF CRR AND RSR VS VARIANCE

REP = 0.25



PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 0.50

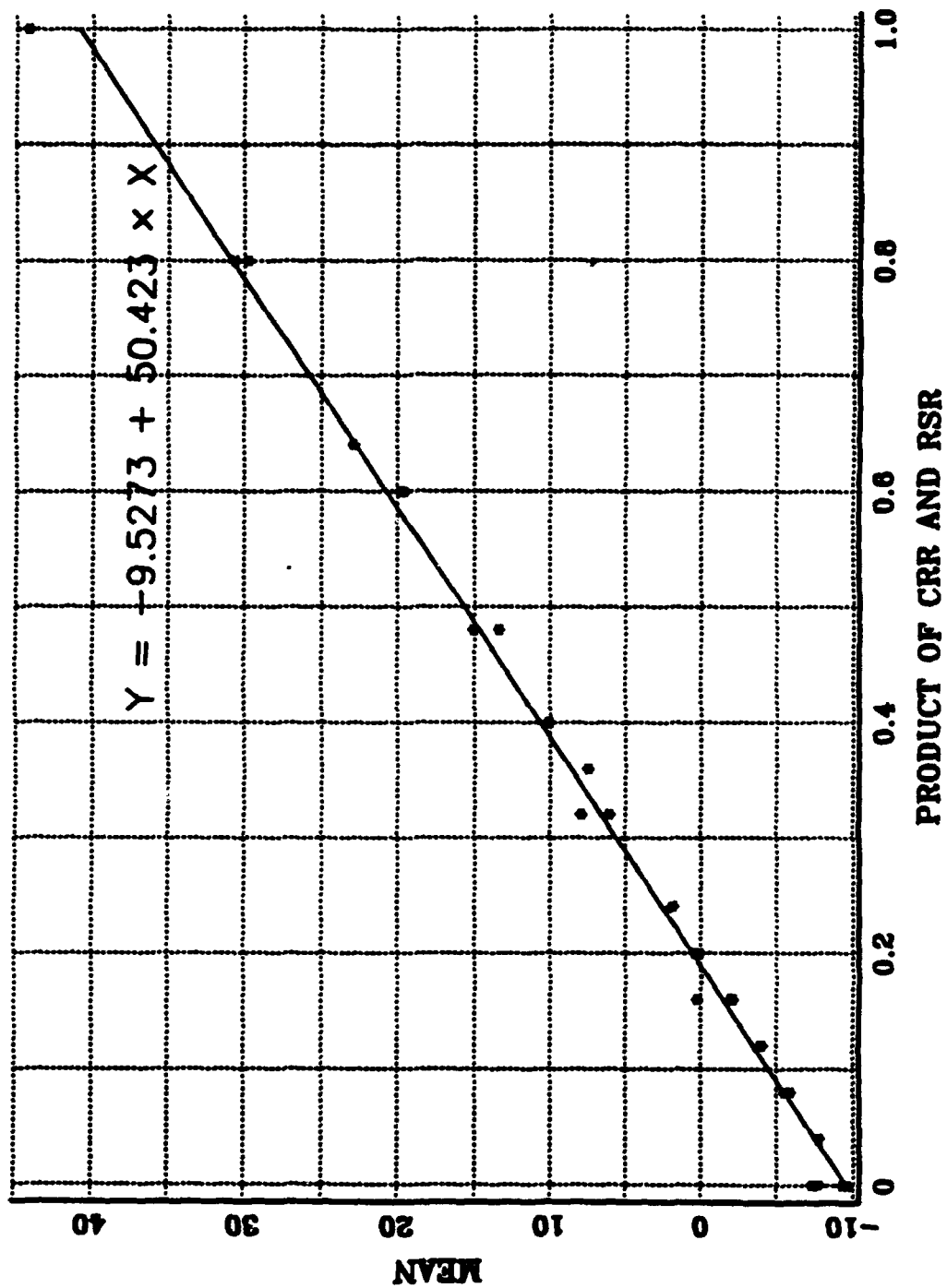


REP = 0.50



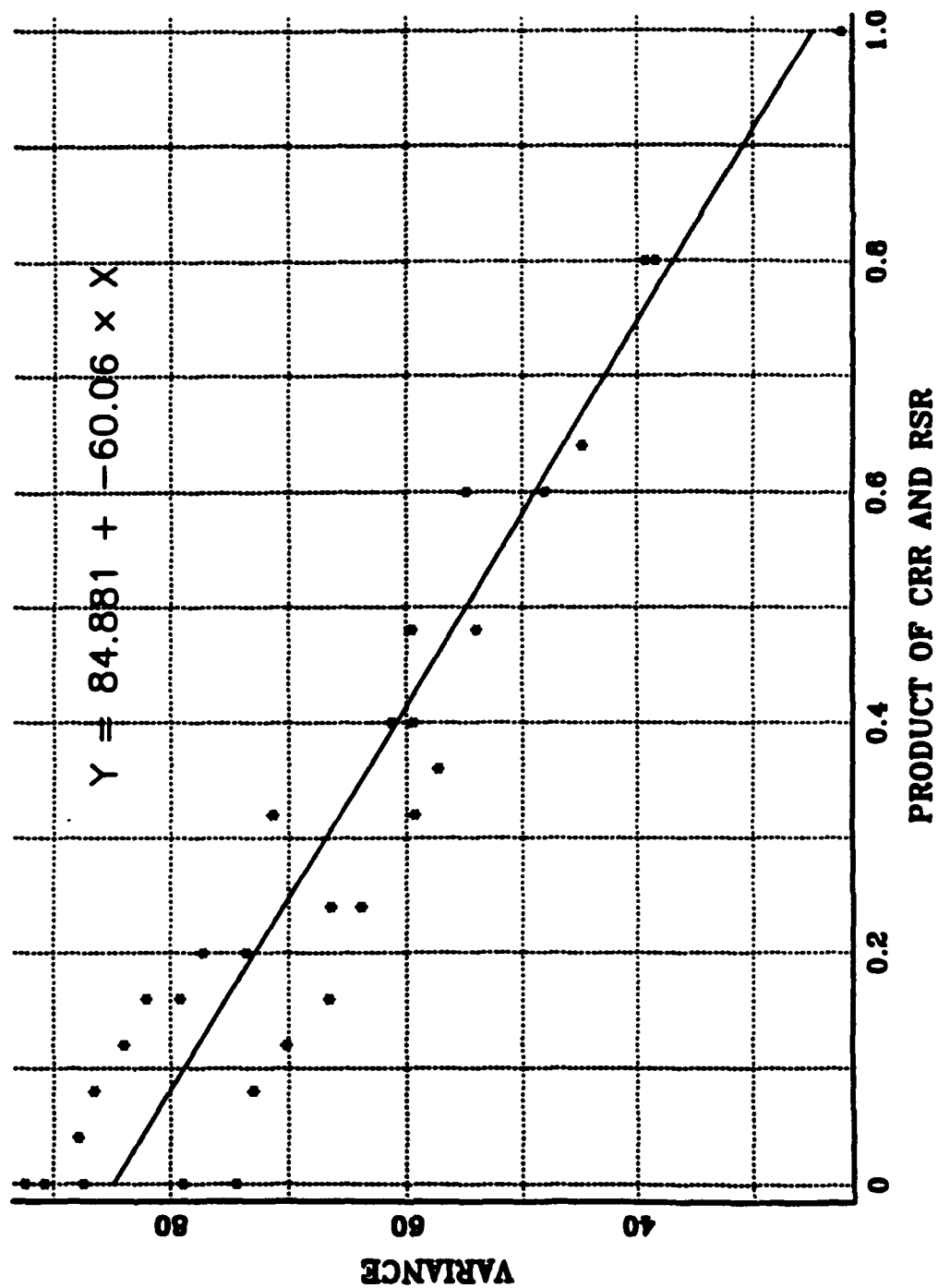
PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 0.75



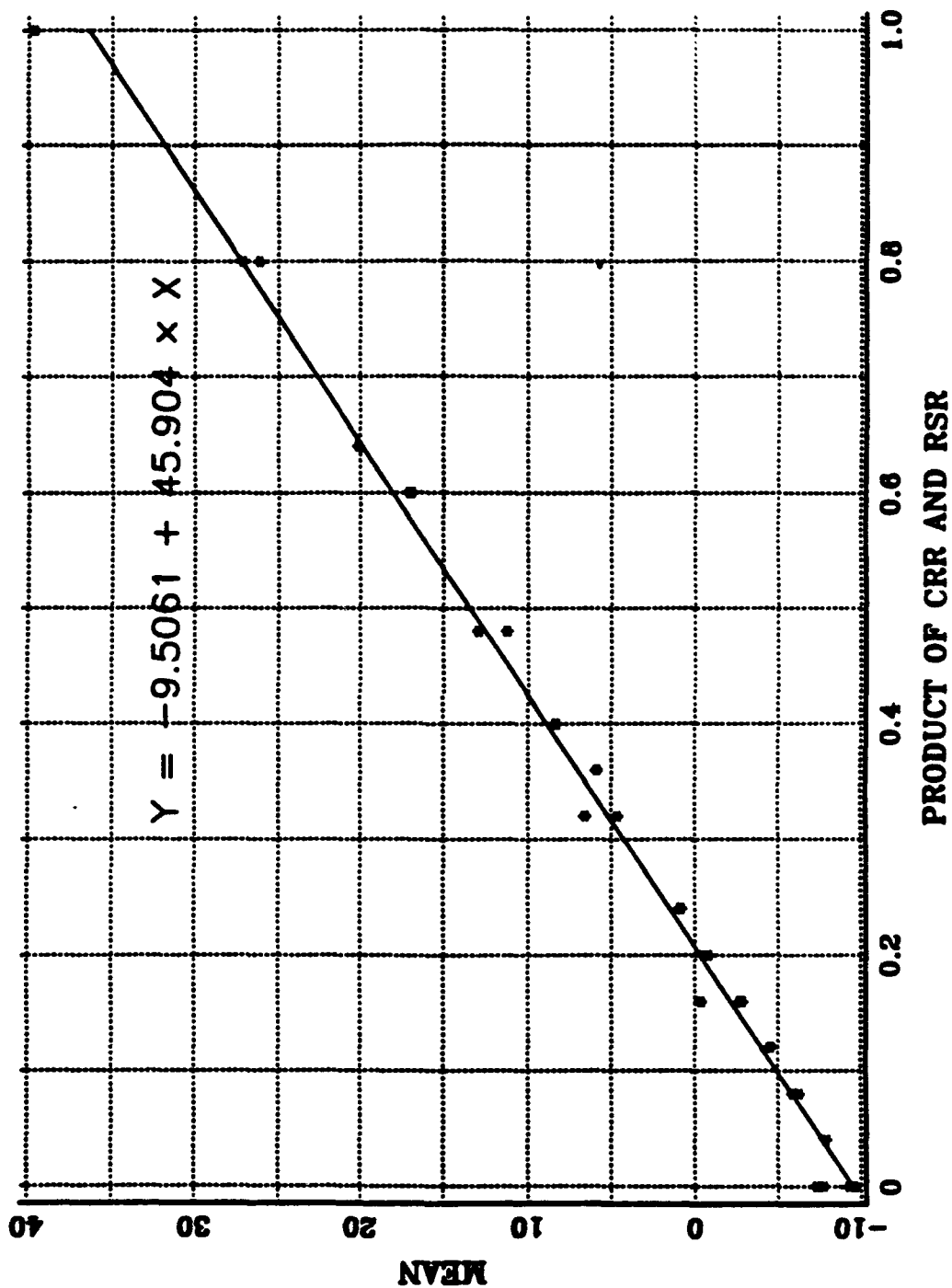
PRODUCT OF CRR AND RSR VS VARIANCE

REP = 0.75



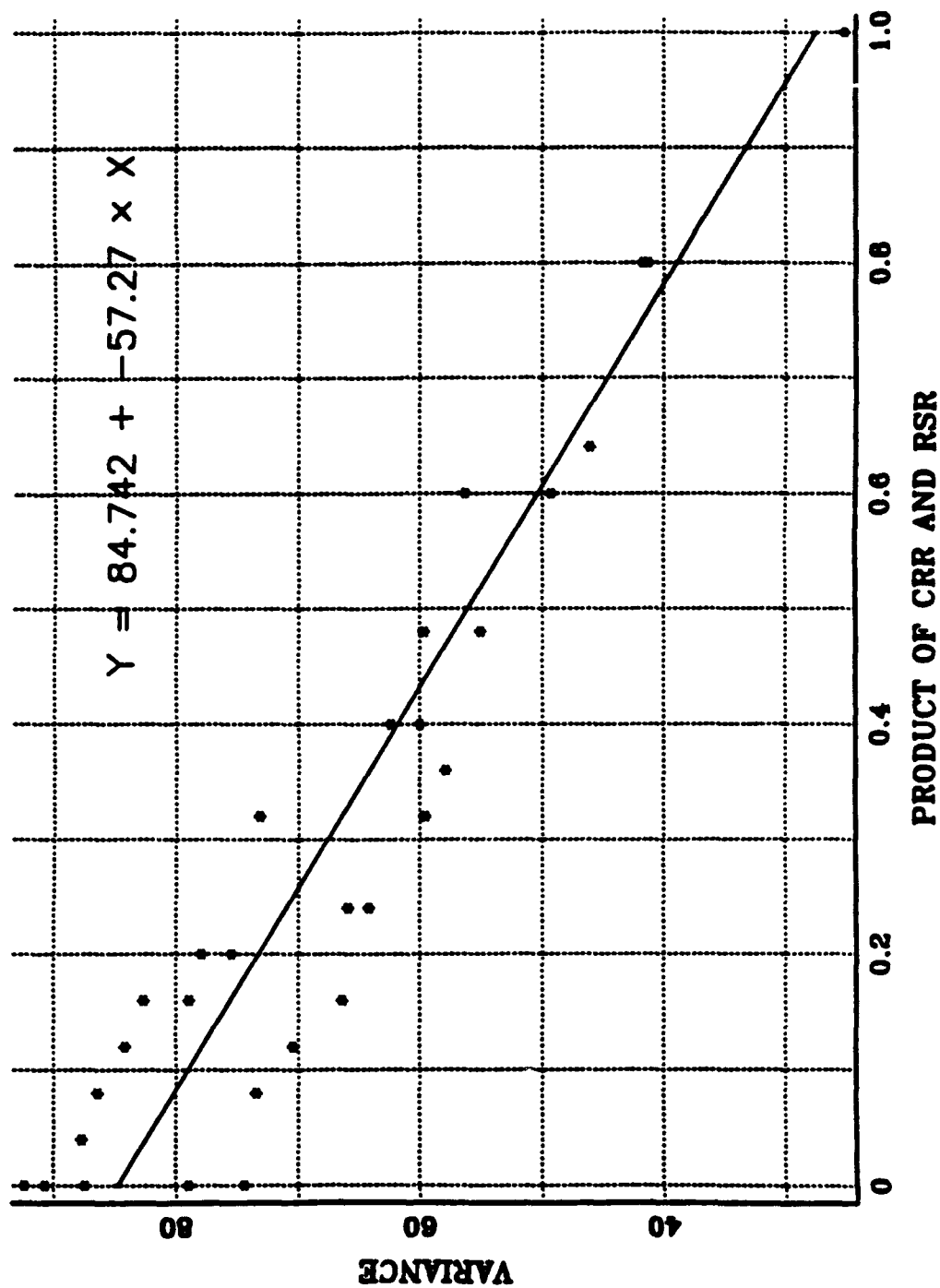
PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 1.00



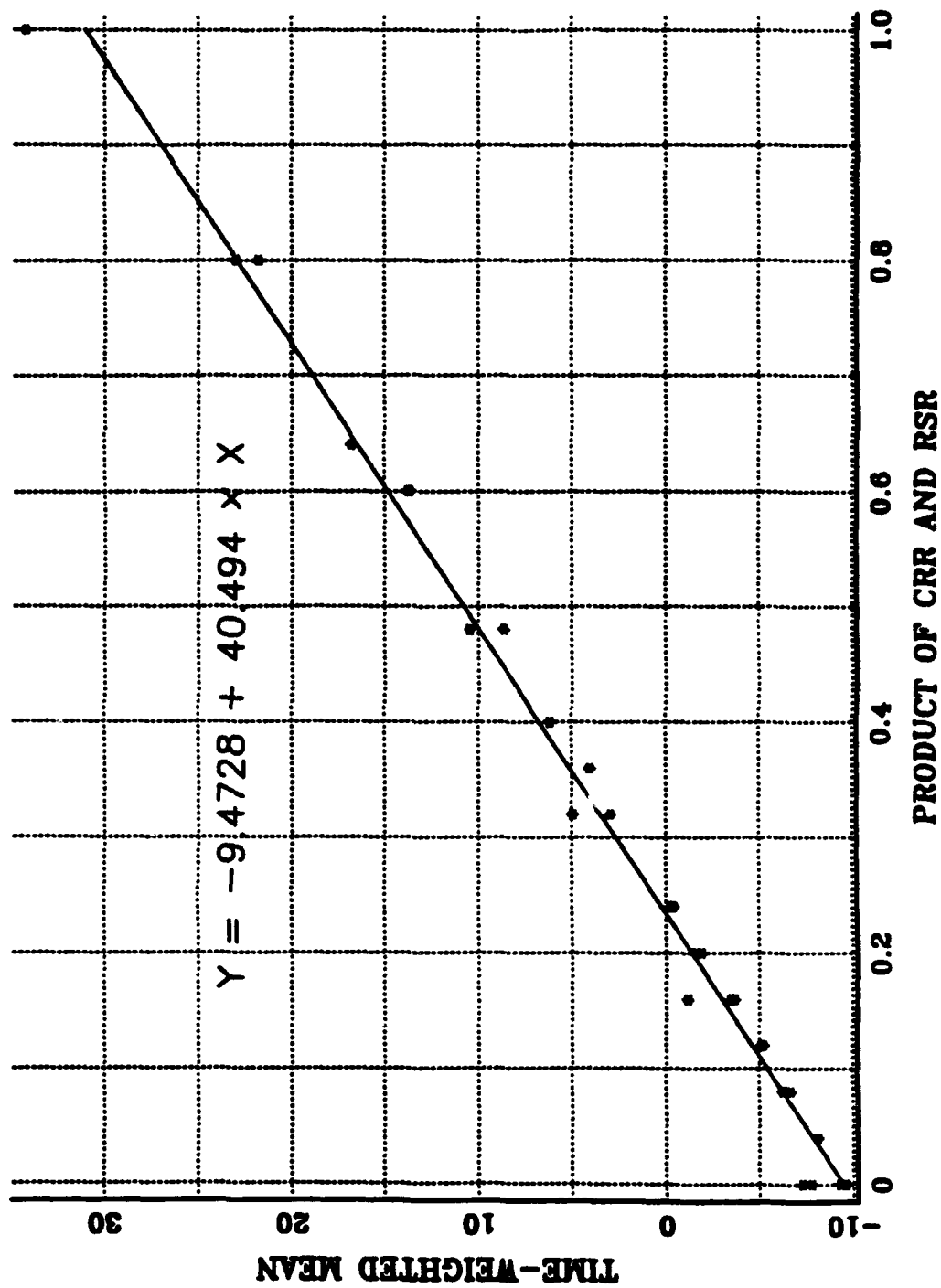
PRODUCT OF CRR AND RSR VS VARIANCE

REP = 1.0



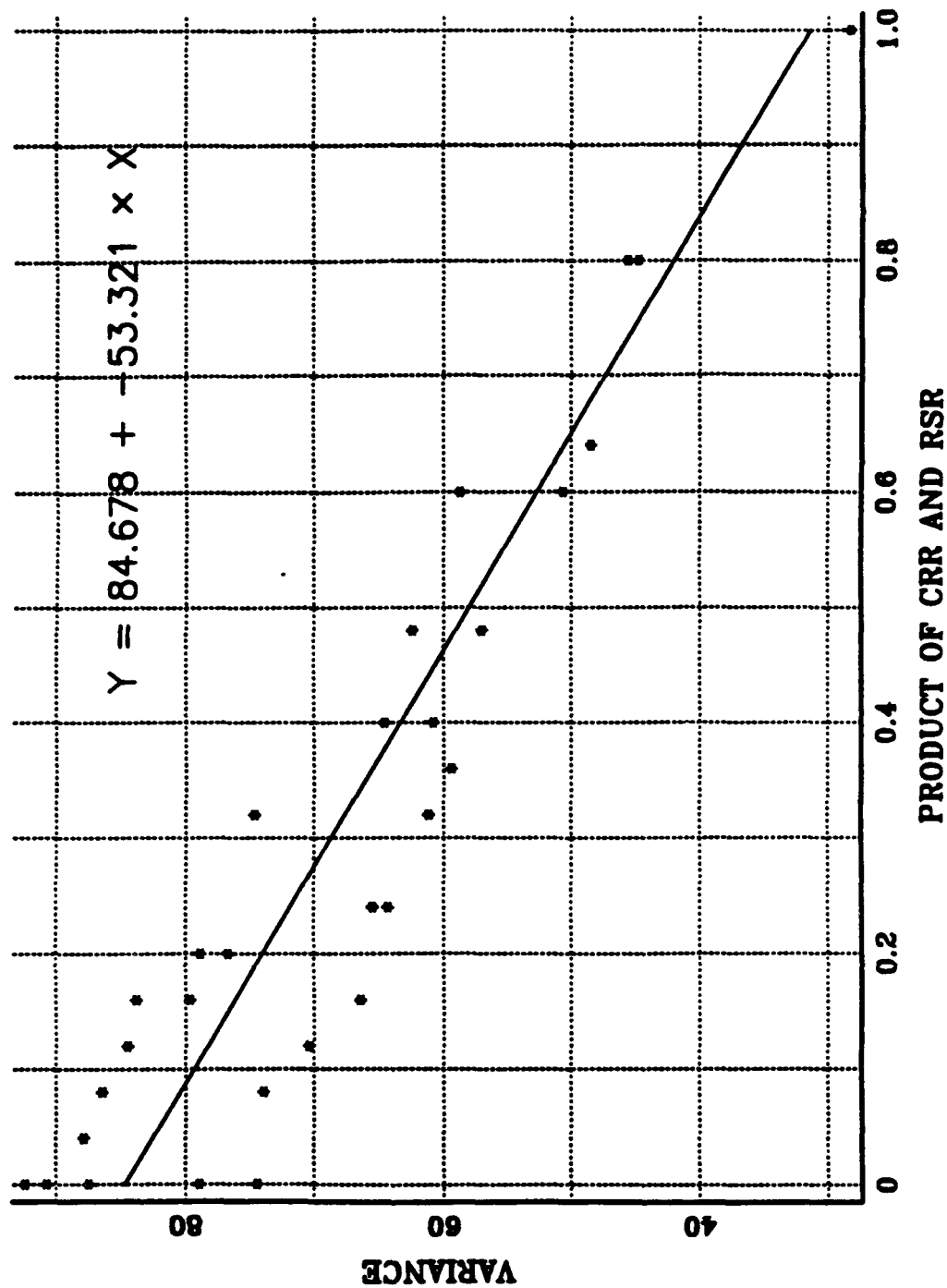
PRODUCT OF CRR AND RSR VS TIME WEIGHTED MEAN

REP = 1.3

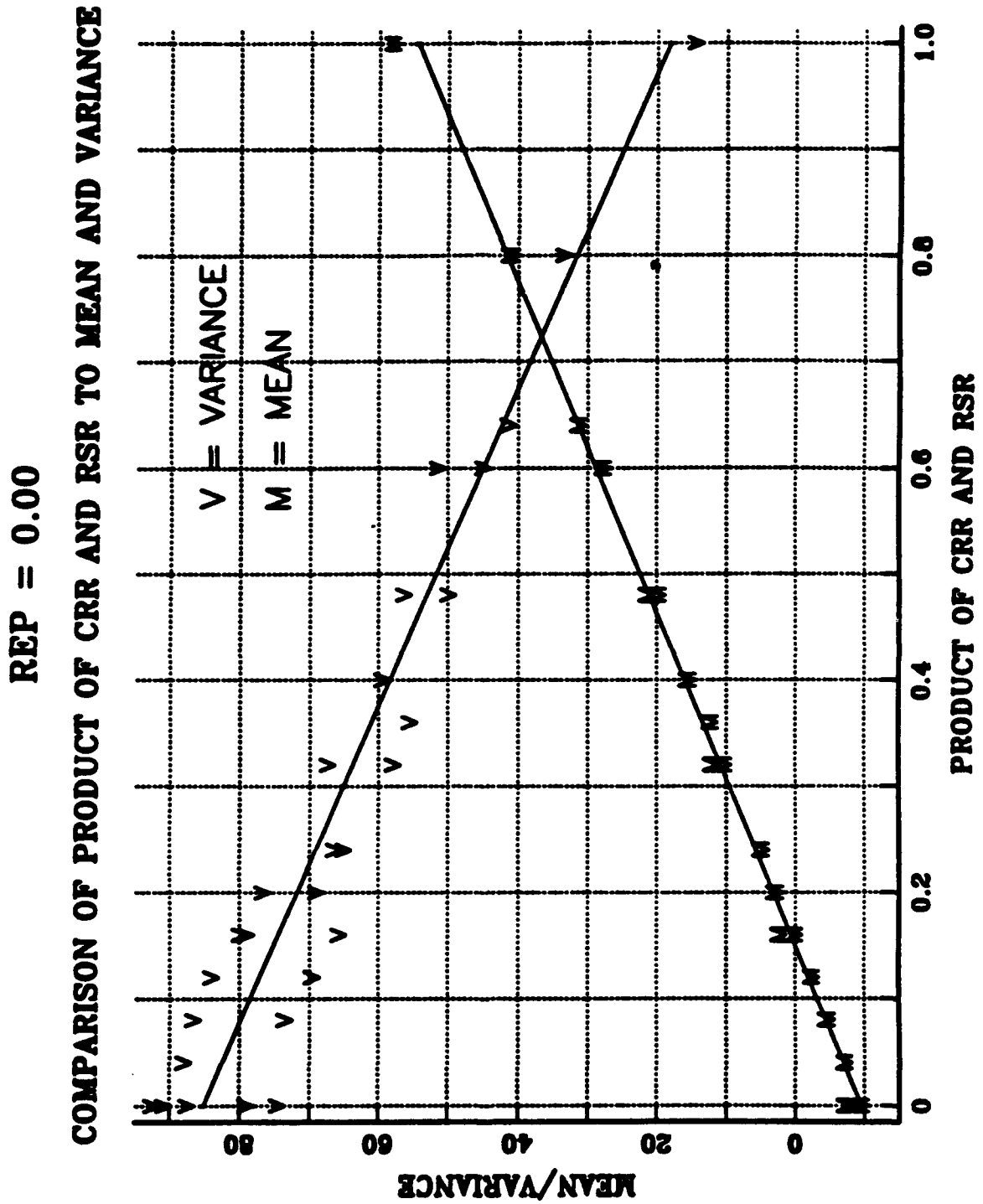


PRODUCT OF CRR AND RSR VS VARIANCE

REP = 1.3

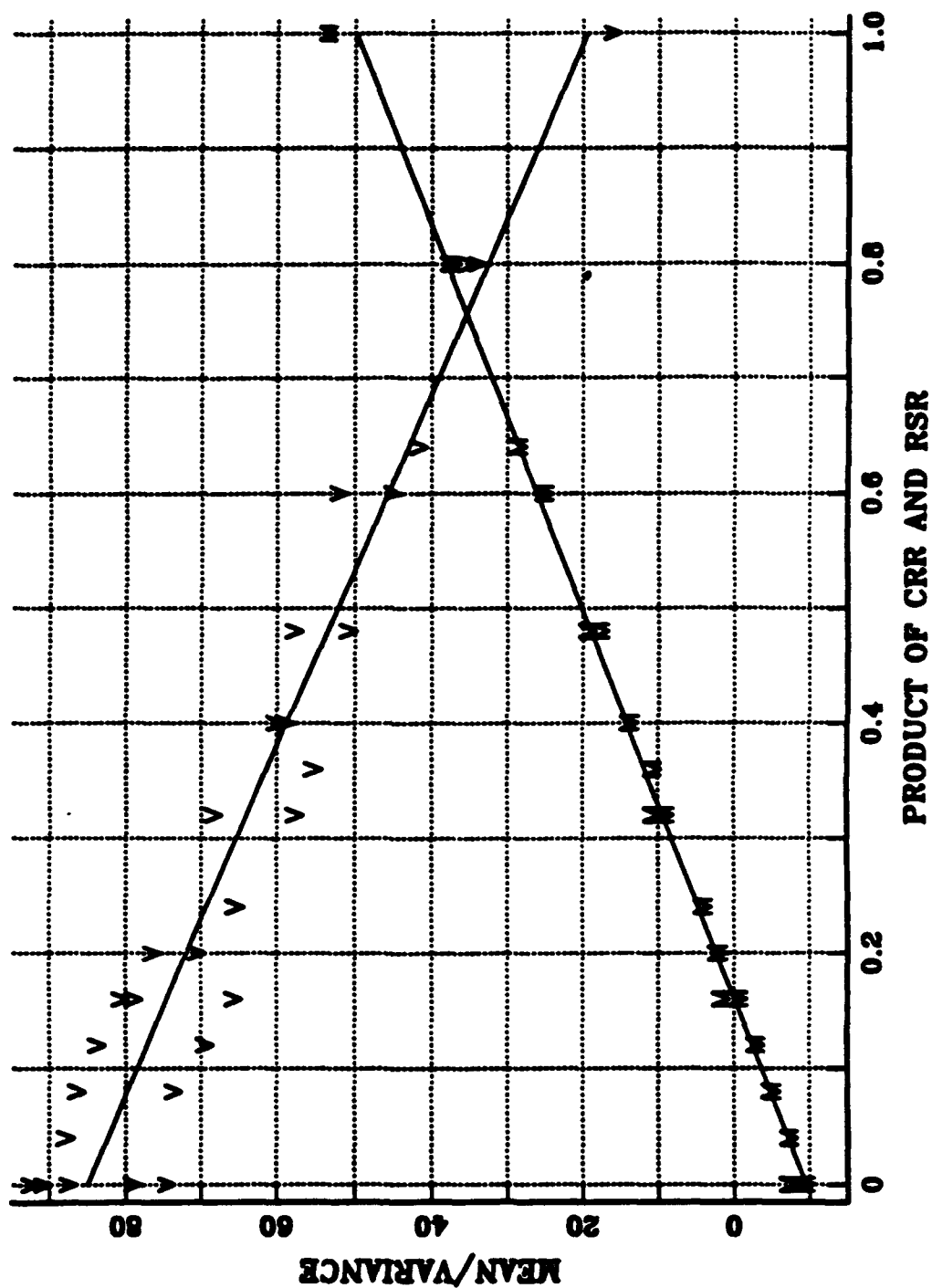


APPENDIX 0



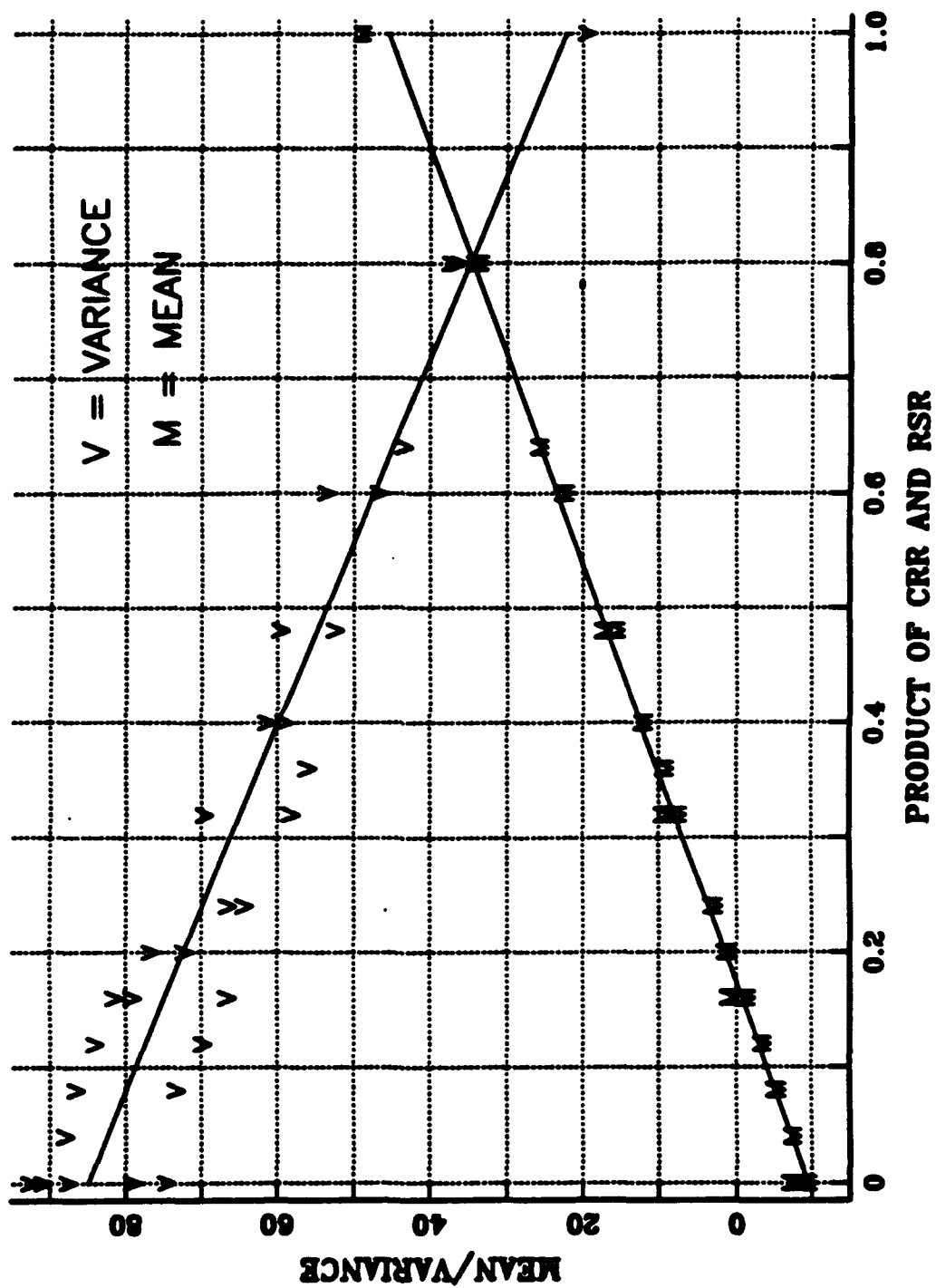
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COMPARISON OF PRODUCT OF CRR AND RSR TO MEAN AND VARIANCE



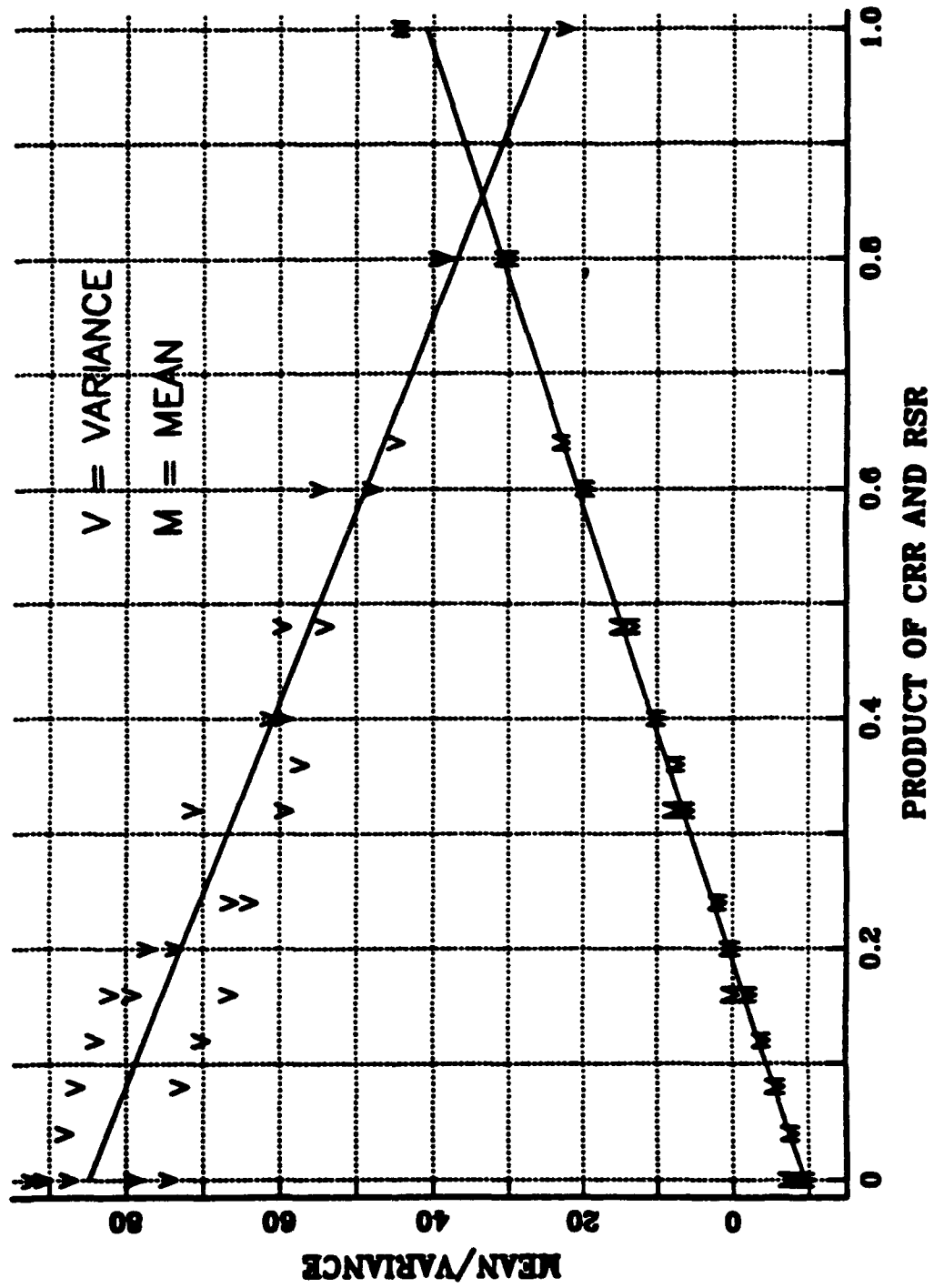
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COMPARISON OF PRODUCT OF CRR AND RSR TO MEAN AND VARIANCE



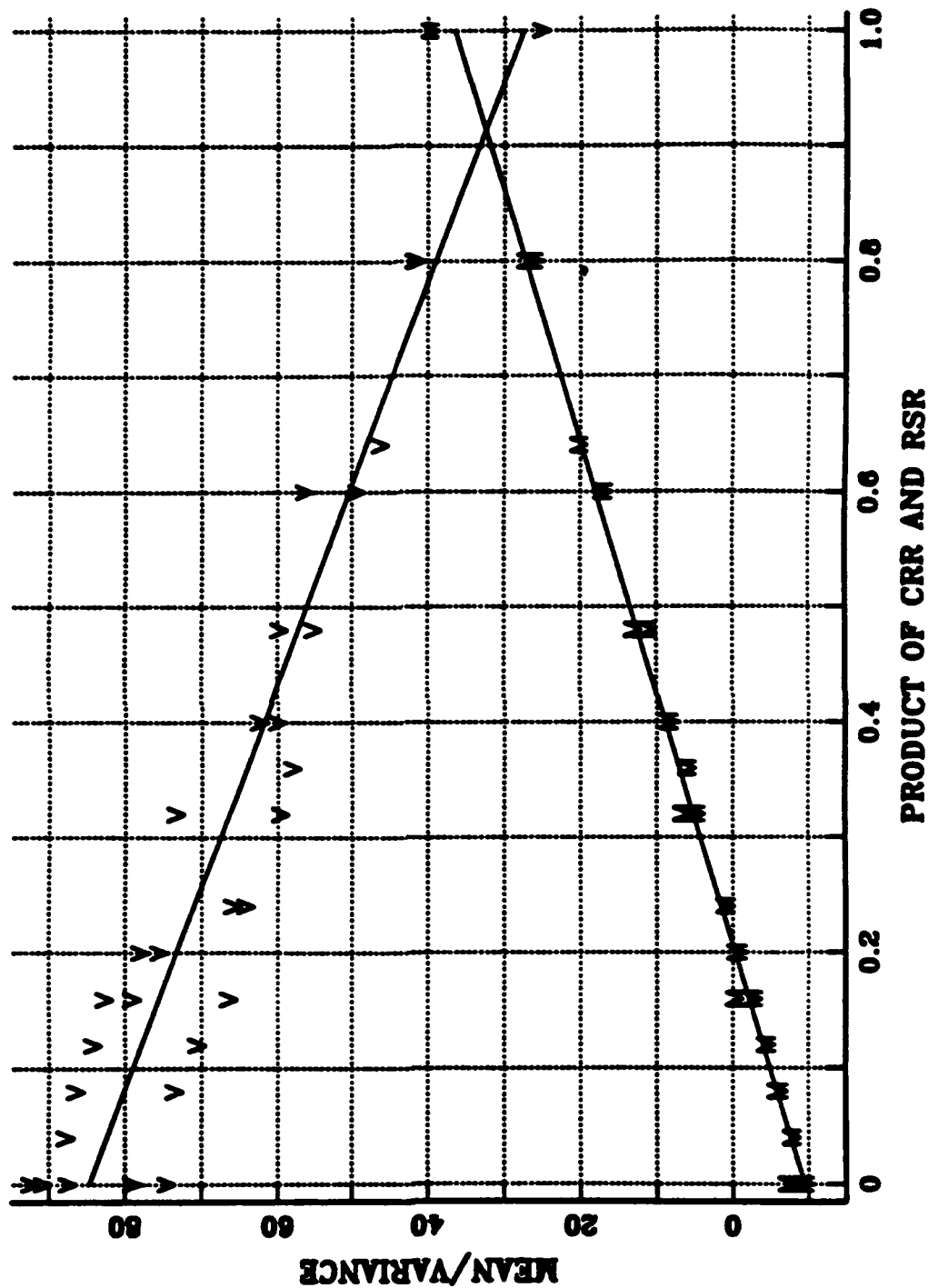
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COMPARISON OF PRODUCT OF CRR AND RSR TO MEAN AND VARIANCE



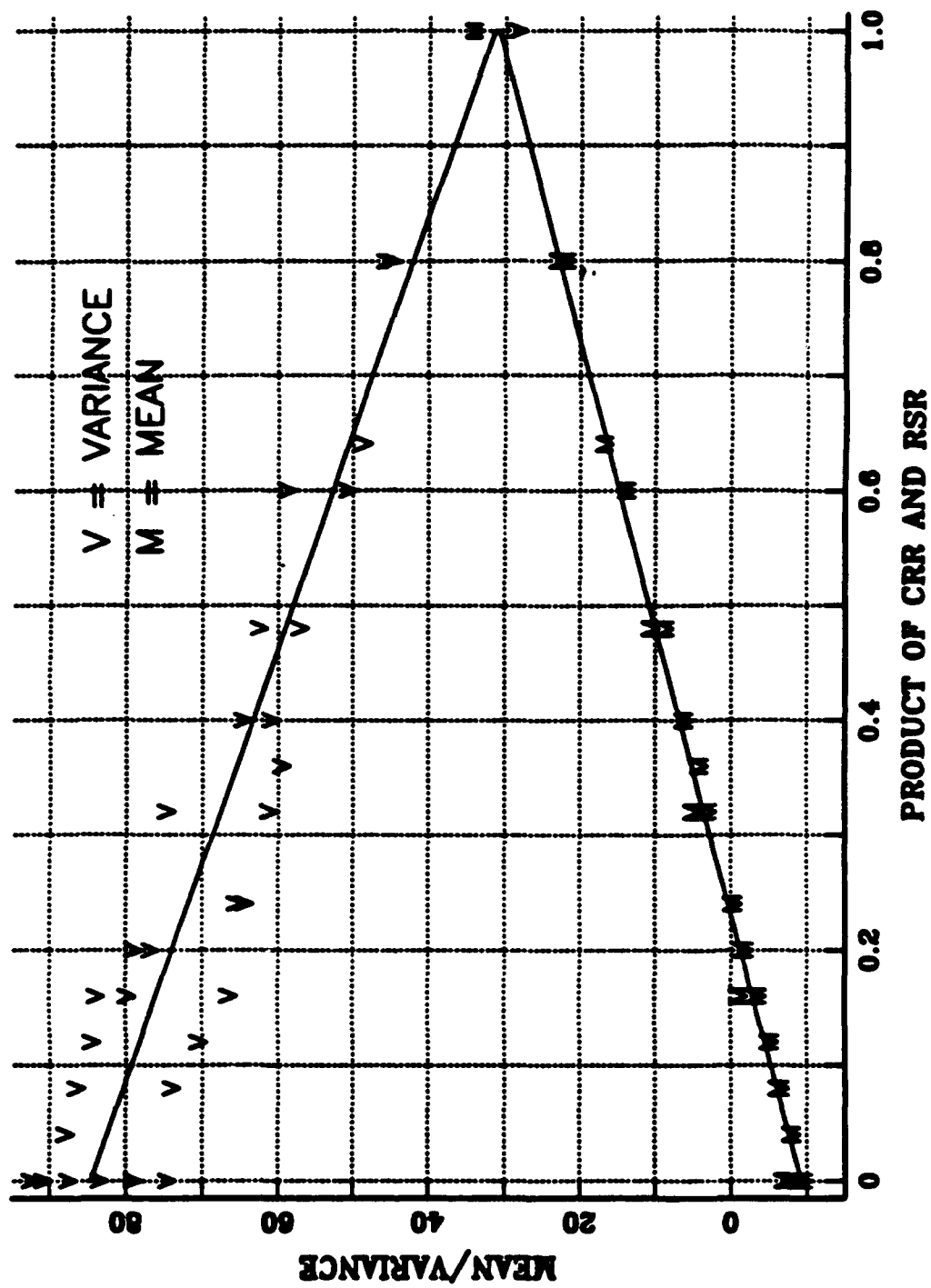
REP = 1.0

COMPARISON OF PRODUCT OF CRR AND RSR TO MEAN AND VARIANCE



REP = 1.3

COMPARISON OF PRODUCT OF CRR AND RSR TO MEAN AND VARIANCE



LIST OF REFERENCES

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